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TEXTBOOK TESTBANKS

Solutions Manual for Precalculus 12th Sullivan

SOLUTIONS MANUAL SAMPLE PREVIEW

PUBLISHER

Pearson

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RESOURCE TYPE

Solutions Manual

ISBN

9780138279226

INSTRUCTOR SOLUTIONS MANUAL

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PRECALCULUS

TWELFTH EDITION

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Pearson

Pearson

PPID: A103000347066

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Chapter 1

Graphs

Section 1.1

1. 0

2. $|5-(-3)| = |8| = 8$

3. $\sqrt{3^2+4^2} = \sqrt{25} = 5$

4. $11^2 + 60^2 = 121 + 3600 = 3721 = 61^2$
Since the sum of the squares of two of the sides of the triangle equals the square of the third side, the triangle is a right triangle.

5. $\frac{1}{2}bh$

6. true

7. x-coordinate or abscissa; y-coordinate or ordinate

8. quadrants

9. midpoint

10. False; the distance between two points is never negative.

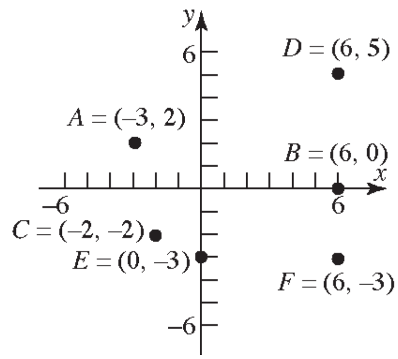
11. False; points that lie in Quadrant IV will have a positive x-coordinate and a negative y-coordinate. The point $(-1,4)$ lies in Quadrant II.

12. True; $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

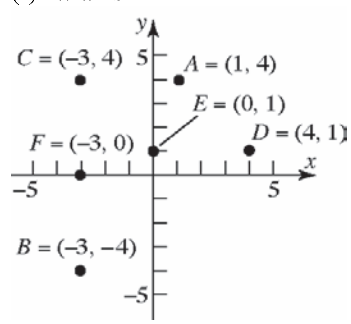
13. b

14. a

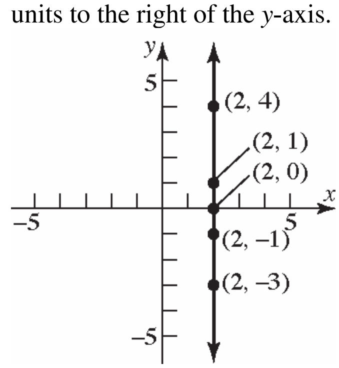
15. (a) Quadrant II
(b) x-axis
(c) Quadrant III
(d) Quadrant I
(e) y-axis
(f) Quadrant IV



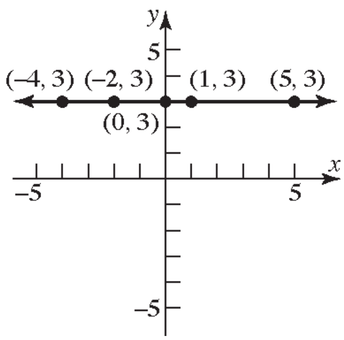
16. (a) Quadrant I
(b) Quadrant III
(c) Quadrant II
(d) Quadrant I
(e) y-axis
(f) x-axis



17. The points will be on a vertical line that is two units to the right of the y-axis.



18. The points will be on a horizontal line that is three units above the x-axis.



19. $d(P_1,P_2)=\sqrt{(2-0)^2+(1-0)^2}$
$=\sqrt{2^2+1^2}=\sqrt{4+1}=\sqrt{5}$

20. $d(P_1,P_2)=\sqrt{(-2-0)^2+(1-0)^2}$
$=\sqrt{(-2)^2+1^2}=\sqrt{4+1}=\sqrt{5}$

21. $d(P_1,P_2)=\sqrt{(-2-1)^2+(2-1)^2}$
$=\sqrt{(-3)^2+1^2}=\sqrt{9+1}=\sqrt{10}$

22. $d(P_1,P_2)=\sqrt{\left(2-(-1)\right)^2+(2-1)^2}$
$=\sqrt{3^2+1^2}=\sqrt{9+1}=\sqrt{10}$

23. $d(P_1,P_2)=\sqrt{(5-3)^2+\left(4-(-4)\right)^2}$
$=\sqrt{2^2+(8)^2}=\sqrt{4+64}=\sqrt{68}=2\sqrt{17}$

24. $d(P_1,P_2)=\sqrt{\left(2-(-1)\right)^2+\left(4-0\right)^2}$
$=\sqrt{(3)^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$

25. $d(P_1,P_2)=\sqrt{\left(4-(-7)\right)^2+(0-3)^2}$
$=\sqrt{11^2+(-3)^2}=\sqrt{121+9}=\sqrt{130}$

26. $d(P_1,P_2)=\sqrt{\left(4-2\right)^2+\left(2-(-3)\right)^2}$
$=\sqrt{2^2+5^2}=\sqrt{4+25}=\sqrt{29}$

27. $d(P_1,P_2)=\sqrt{(6-5)^2+\left(1-(-2)\right)^2}$
$=\sqrt{1^2+3^2}=\sqrt{1+9}=\sqrt{10}$

28. $d(P_1,P_2)=\sqrt{\left(6-(-4)\right)^2+\left(2-(-3)\right)^2}$
$=\sqrt{10^2+5^2}=\sqrt{100+25}$
$=\sqrt{125}=5\sqrt{5}$

29. $d(P_1,P_2)=\sqrt{\left(2.3-(-0.2)\right)^2+\left(1.1-(0.3)\right)^2}$
$=\sqrt{2.5^2+0.8^2}=\sqrt{6.25+0.64}$
$=\sqrt{6.89}\approx 2.62$

30. $d(P_1,P_2)=\sqrt{\left(-0.3-1.2\right)^2+\left(1.1-2.3\right)^2}$
$=\sqrt{(-1.5)^2+(-1.2)^2}=\sqrt{2.25+1.44}$
$=\sqrt{3.69}\approx 1.92$

31. $d(P_1,P_2)=\sqrt{(0-a)^2+(0-b)^2}$
$=\sqrt{(-a)^2+(-b)^2}=\sqrt{a^2+b^2}$

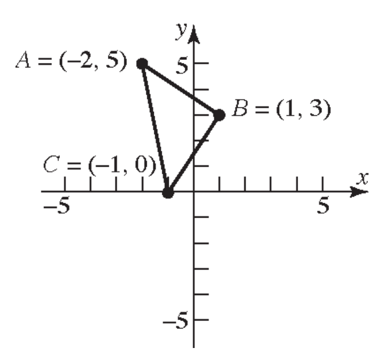
32. $d(P_1,P_2)=\sqrt{(0-a)^2+(0-a)^2}$
$=\sqrt{(-a)^2+(-a)^2}$
$=\sqrt{a^2+a^2}=\sqrt{2a^2}=|a|\sqrt{2}$

33. $A=(-2,5),\ \ B=(1,3),\ \ C=(-1,0)$

$d(A,B)=\sqrt{\left(1-(-2)\right)^2+(3-5)^2}$
$=\sqrt{3^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$

$d(B,C)=\sqrt{\left(-1-1\right)^2+(0-3)^2}$
$=\sqrt{(-2)^2+(-3)^2}=\sqrt{4+9}=\sqrt{13}$

$d(A,C)=\sqrt{\left(-1-(-2)\right)^2+(0-5)^2}$
$=\sqrt{1^2+(-5)^2}=\sqrt{1+25}=\sqrt{26}$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

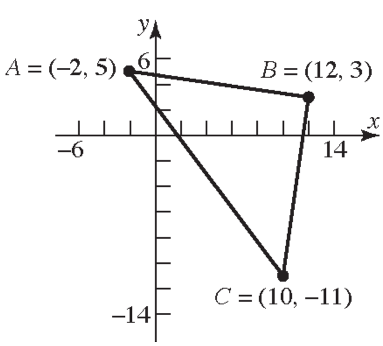
$$[d(A,B)]^2 + [d(B,C)]^2 = [d(A,C)]^2$$
$$\left(\sqrt{13}\right)^2 + \left(\sqrt{13}\right)^2 = \left(\sqrt{26}\right)^2$$
$$13 + 13 = 26$$
$$26 = 26$$

The area of a triangle is $A = \frac{1}{2} \cdot bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A,B)] \cdot [d(B,C)]$$
$$= \frac{1}{2} \cdot \sqrt{13} \cdot \sqrt{13} = \frac{1}{2} \cdot 13$$
$$= \frac{13}{2} \text{ square units}$$

34. $A = (-2, 5),\ B = (12, 3),\ C = (10, -11)$

$$d(A,B) = \sqrt{(12-(-2))^2 + (3-5)^2}$$
$$= \sqrt{14^2 + (-2)^2}$$
$$= \sqrt{196+4} = \sqrt{200}$$
$$= 10\sqrt{2}$$
$$d(B,C) = \sqrt{(10-12)^2 + (-11-3)^2}$$
$$= \sqrt{(-2)^2 + (-14)^2}$$
$$= \sqrt{4+196} = \sqrt{200}$$
$$= 10\sqrt{2}$$
$$d(A,C) = \sqrt{(10-(-2))^2 + (-11-5)^2}$$
$$= \sqrt{12^2 + (-16)^2}$$
$$= \sqrt{144+256} = \sqrt{400}$$
$$= 20$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

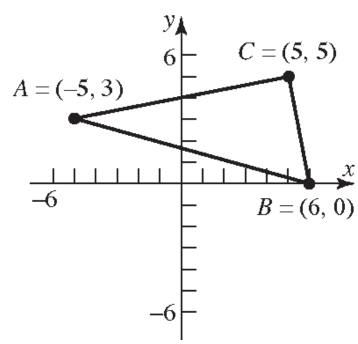
$$[d(A,B)]^2 + [d(B,C)]^2 = [d(A,C)]^2$$
$$\left(10\sqrt{2}\right)^2 + \left(10\sqrt{2}\right)^2 = (20)^2$$
$$200 + 200 = 400$$
$$400 = 400$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A,B)] \cdot [d(B,C)]$$
$$= \frac{1}{2} \cdot 10\sqrt{2} \cdot 10\sqrt{2}$$
$$= \frac{1}{2} \cdot 100 \cdot 2 = 100 \text{ square units}$$

35. $A = (-5,3),\ B = (6,0),\ C = (5,5)$

$$d(A,B) = \sqrt{(6-(-5))^2 + (0-3)^2}$$
$$= \sqrt{11^2 + (-3)^2} = \sqrt{121+9}$$
$$= \sqrt{130}$$
$$d(B,C) = \sqrt{(5-6)^2 + (5-0)^2}$$
$$= \sqrt{(-1)^2 + 5^2} = \sqrt{1+25}$$
$$= \sqrt{26}$$
$$d(A,C) = \sqrt{(5-(-5))^2 + (5-3)^2}$$
$$= \sqrt{10^2 + 2^2} = \sqrt{100+4}$$
$$= \sqrt{104}$$
$$= 2\sqrt{26}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

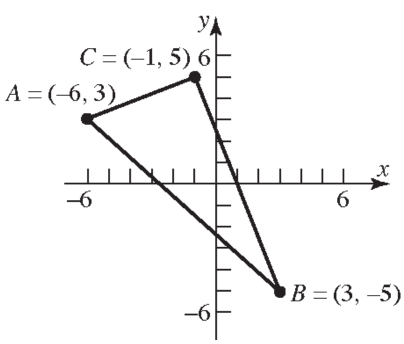
$$[d(A,C)]^2 + [d(B,C)]^2 = [d(A,B)]^2$$
$$\left(\sqrt{104}\right)^2 + \left(\sqrt{26}\right)^2 = \left(\sqrt{130}\right)^2$$
$$104 + 26 = 130$$
$$130 = 130$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A,C)] \cdot [d(B,C)]$$
$$= \frac{1}{2} \cdot \sqrt{104} \cdot \sqrt{26}$$
$$= \frac{1}{2} \cdot 2\sqrt{26} \cdot \sqrt{26}$$
$$= \frac{1}{2} \cdot 2 \cdot 26$$
$$= 26 \text{ square units}$$

36. $A = (-6, 3),\ B = (3, -5),\ C = (-1, 5)$

$$d(A,B) = \sqrt{(3-(-6))^2 + (-5-3)^2}$$
$$= \sqrt{9^2 + (-8)^2} = \sqrt{81+64}$$
$$= \sqrt{145}$$
$$d(B,C) = \sqrt{(-1-3)^2 + (5-(-5))^2}$$
$$= \sqrt{(-4)^2 + 10^2} = \sqrt{16+100}$$
$$= \sqrt{116} = 2\sqrt{29}$$
$$d(A,C) = \sqrt{(-1-(-6))^2 + (5-3)^2}$$
$$= \sqrt{5^2 + 2^2} = \sqrt{25+4}$$
$$= \sqrt{29}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

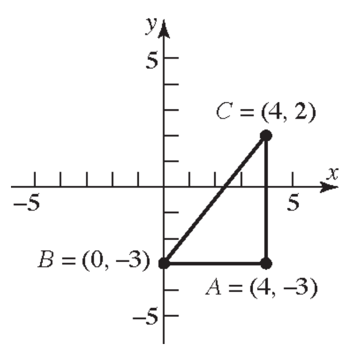
$$[d(A,C)]^2 + [d(B,C)]^2 = [d(A,B)]^2$$
$$\left(\sqrt{29}\right)^2 + \left(2\sqrt{29}\right)^2 = \left(\sqrt{145}\right)^2$$
$$29 + 4 \cdot 29 = 145$$
$$29 + 116 = 145$$
$$145 = 145$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A,C)] \cdot [d(B,C)]$$
$$= \frac{1}{2} \cdot \sqrt{29} \cdot 2\sqrt{29}$$
$$= \frac{1}{2} \cdot 2 \cdot 29$$
$$= 29 \text{ square units}$$

37. $A = (4, -3),\ B = (0, -3),\ C = (4, 2)$

$$d(A,B) = \sqrt{(0-4)^2 + (-3-(-3))^2}$$
$$= \sqrt{(-4)^2 + 0^2} = \sqrt{16+0}$$
$$= \sqrt{16}$$
$$= 4$$
$$d(B,C) = \sqrt{(4-0)^2 + (2-(-3))^2}$$
$$= \sqrt{4^2 + 5^2} = \sqrt{16+25}$$
$$= \sqrt{41}$$
$$d(A,C) = \sqrt{(4-4)^2 + (2-(-3))^2}$$
$$= \sqrt{0^2 + 5^2} = \sqrt{0+25}$$
$$= \sqrt{25}$$
$$= 5$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$[d(A,B)]^2 + [d(A,C)]^2 = [d(B,C)]^2$$
$$4^2 + 5^2 = \left(\sqrt{41}\right)^2$$
$$16 + 25 = 41$$
$$41 = 41$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

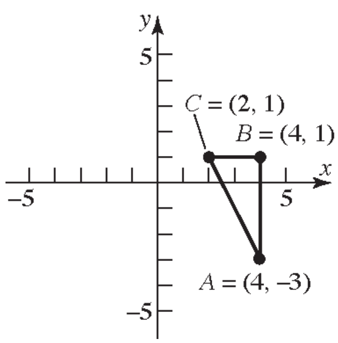
$$\begin{aligned} A &= \frac{1}{2}\cdot[d(A,B)]\cdot[d(A,C)] \\ &= \frac{1}{2}\cdot 4\cdot 5 \\ &= 10 \text{ square units} \end{aligned}$$

38. $A = (4,-3),\ B = (4,1),\ C = (2,1)$

$$\begin{aligned} d(A,B) &= \sqrt{(4-4)^2 + (1-(-3))^2} \\ &= \sqrt{0^2 + 4^2} \\ &= \sqrt{0+16} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

$$\begin{aligned} d(B,C) &= \sqrt{(2-4)^2 + (1-1)^2} \\ &= \sqrt{(-2)^2 + 0^2} = \sqrt{4+0} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

$$\begin{aligned} d(A,C) &= \sqrt{(2-4)^2 + (1-(-3))^2} \\ &= \sqrt{(-2)^2 + 4^2} = \sqrt{4+16} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$[d(A,B)]^2 + [d(B,C)]^2 = [d(A,C)]^2$$
$$4^2 + 2^2 = \left(2\sqrt{5}\right)^2$$
$$16 + 4 = 20$$
$$20 = 20$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$\begin{aligned} A &= \frac{1}{2}\cdot[d(A,B)]\cdot[d(B,C)] \\ &= \frac{1}{2}\cdot 4\cdot 2 \\ &= 4 \text{ square units} \end{aligned}$$

39. The coordinates of the midpoint are:

$$\begin{aligned} (x,y) &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{3+5}{2}, \frac{-4+4}{2}\right) \\ &= \left(\frac{8}{2}, \frac{0}{2}\right) \\ &= (4,0) \end{aligned}$$

40. The coordinates of the midpoint are:

$$\begin{aligned} (x,y) &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{-2+2}{2}, \frac{0+4}{2}\right) \\ &= \left(\frac{0}{2}, \frac{4}{2}\right) \\ &= (0,2) \end{aligned}$$

41. The coordinates of the midpoint are:

$$\begin{aligned} (x,y) &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{-1+8}{2}, \frac{4+0}{2}\right) \\ &= \left(\frac{7}{2}, \frac{4}{2}\right) \\ &= \left(\frac{7}{2}, 2\right) \end{aligned}$$

42. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2+4}{2}, \frac{-3+2}{2}\right) = \left(\frac{6}{2}, \frac{-1}{2}\right) = \left(3, -\frac{1}{2}\right)$$

43. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{7+9}{2}, \frac{-5+1}{2}\right) = \left(\frac{16}{2}, \frac{-4}{2}\right) = (8, -2)$$

44. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-4+2}{2}, \frac{-3+2}{2}\right) = \left(\frac{-2}{2}, \frac{-1}{2}\right) = \left(-1, -\frac{1}{2}\right)$$

45. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$$

46. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{a+0}{2}, \frac{a+0}{2}\right) = \left(\frac{a}{2}, \frac{a}{2}\right)$$

47. The x coordinate would be $2+3=5$ and the y coordinate would be $5-2=3$. Thus the new point would be $(5,3)$.

48. The new x coordinate would be $-1-2=-3$ and the new y coordinate would be $6+4=10$. Thus the new point would be $(-3,10)$

49. **a.** If we use a right triangle to solve the problem, we know the hypotenuse is 13 units in length. One of the legs of the triangle will be 2+3=5. Thus the other leg will be:

$$5^2 + b^2 = 13^2$$
$$25 + b^2 = 169$$
$$b^2 = 144$$
$$b = 12$$

Thus the coordinates will have an y value of $-1-12=-13$ and $-1+12=11$. So the points are $(3,11)$ and $(3,-13)$.

b. Consider points of the form $(3, y)$ that are a distance of 13 units from the point $(-2,-1)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3-(-2))^2 + (-1-y)^2}$$
$$= \sqrt{(5)^2 + (-1-y)^2}$$
$$= \sqrt{25+1+2y+y^2}$$
$$= \sqrt{y^2+2y+26}$$
$$13 = \sqrt{y^2+2y+26}$$
$$13^2 = \left(\sqrt{y^2+2y+26}\right)^2$$
$$169 = y^2+2y+26$$
$$0 = y^2+2y-143$$
$$0 = (y-11)(y+13)$$
$$y-11=0 \quad \text{or} \quad y+13=0$$
$$y=11 \qquad\qquad y=-13$$

Thus, the points $(3,11)$ and $(3,-13)$ are a distance of 13 units from the point $(-2,-1)$.

50. **a.** If we use a right triangle to solve the problem, we know the hypotenuse is 17 units in length. One of the legs of the triangle will be 2+6=8. Thus the other leg will be:

$$8^2 + b^2 = 17^2$$
$$64 + b^2 = 289$$
$$b^2 = 225$$
$$b = 15$$

Thus the coordinates will have an x value of $1-15=-14$ and $1+15=16$. So the points are $(-14,-6)$ and $(16,-6)$.

b. Consider points of the form $(x,-6)$ that are a distance of 17 units from the point $(1,2)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(1-x)^2 + (2-(-6))^2}$$
$$= \sqrt{x^2 - 2x + 1 + (8)^2}$$
$$= \sqrt{x^2 - 2x + 1 + 64}$$
$$= \sqrt{x^2 - 2x + 65}$$
$$17 = \sqrt{x^2 - 2x + 65}$$
$$17^2 = \left(\sqrt{x^2 - 2x + 65}\right)^2$$
$$289 = x^2 - 2x + 65$$
$$0 = x^2 - 2x - 224$$
$$0 = (x+14)(x-16)$$
$$x + 14 = 0 \quad \text{or} \quad x - 16 = 0$$
$$x = -14 \qquad\qquad x = 16$$

Thus, the points $(-14,-6)$ and $(16,-6)$ are a distance of 13 units from the point $(1,2)$.

51. Points on the x-axis have a y-coordinate of 0. Thus, we consider points of the form $(x,0)$ that are a distance of 6 units from the point $(4,-3)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-x)^2 + (-3-0)^2}$$
$$= \sqrt{16 - 8x + x^2 + (-3)^2}$$
$$= \sqrt{16 - 8x + x^2 + 9}$$
$$= \sqrt{x^2 - 8x + 25}$$
$$6 = \sqrt{x^2 - 8x + 25}$$
$$6^2 = \left(\sqrt{x^2 - 8x + 25}\right)^2$$
$$36 = x^2 - 8x + 25$$
$$0 = x^2 - 8x - 11$$
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-11)}}{2(1)}$$
$$= \frac{8 \pm \sqrt{64 + 44}}{2} = \frac{8 \pm \sqrt{108}}{2}$$
$$= \frac{8 \pm 6\sqrt{3}}{2} = 4 \pm 3\sqrt{3}$$
$$x = 4 + 3\sqrt{3} \quad \text{or} \quad x = 4 - 3\sqrt{3}$$

Thus, the points $\left(4+3\sqrt{3},0\right)$ and $\left(4-3\sqrt{3},0\right)$ are on the x-axis and a distance of 6 units from the point $(4,-3)$.

52. Points on the y-axis have an x-coordinate of 0. Thus, we consider points of the form $(0,y)$ that are a distance of 6 units from the point $(4,-3)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-0)^2 + (-3-y)^2}$$
$$= \sqrt{4^2 + 9 + 6y + y^2}$$
$$= \sqrt{16 + 9 + 6y + y^2}$$
$$= \sqrt{y^2 + 6y + 25}$$

$$6 = \sqrt{y^2 + 6y + 25}$$
$$6^2 = \left(\sqrt{y^2 + 6y + 25}\right)^2$$
$$36 = y^2 + 6y + 25$$
$$0 = y^2 + 6y - 11$$
$$y = \frac{(-6) \pm \sqrt{(6)^2 - 4(1)(-11)}}{2(1)}$$
$$= \frac{-6 \pm \sqrt{36 + 44}}{2} = \frac{-6 \pm \sqrt{80}}{2}$$
$$= \frac{-6 \pm 4\sqrt{5}}{2} = -3 \pm 2\sqrt{5}$$
$$y = -3 + 2\sqrt{5} \quad \text{or} \quad y = -3 - 2\sqrt{5}$$

Thus, the points $\left(0, -3 + 2\sqrt{5}\right)$ and $\left(0, -3 - 2\sqrt{5}\right)$ are on the y-axis and a distance of 6 units from the point $\left(4, -3\right)$.

53. **a.** To shift 3 units left and 4 units down, we subtract 3 from the x-coordinate and subtract 4 from the y-coordinate.
$\left(2 - 3, 5 - 4\right) = \left(-1, 1\right)$

b. To shift left 2 units and up 8 units, we subtract 2 from the x-coordinate and add 8 to the y-coordinate.
$\left(2 - 2, 5 + 8\right) = \left(0, 13\right)$

54. Let the coordinates of point B be $\left(x, y\right)$. Using the midpoint formula, we can write

$$\left(2, 3\right) = \left(\frac{-1 + x}{2}, \frac{8 + y}{2}\right).$$

This leads to two equations we can solve.

$$\frac{-1 + x}{2} = 2 \qquad \frac{8 + y}{2} = 3$$
$$-1 + x = 4 \qquad 8 + y = 6$$
$$x = 5 \qquad y = -2$$

Point B has coordinates $\left(5, -2\right)$.

55. $M = \left(x, y\right) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

$P_1 = \left(x_1, y_1\right) = (-3, 6)$ and $(x, y) = (-1, 4)$, so

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$
$$-1 = \frac{-3 + x_2}{2} \qquad 4 = \frac{6 + y_2}{2}$$
$$-2 = -3 + x_2 \qquad 8 = 6 + y_2$$
$$1 = x_2 \qquad 2 = y_2$$

Thus, $P_2 = (1, 2)$.

56. $M = \left(x, y\right) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

$P_2 = \left(x_2, y_2\right) = (7, -2)$ and $(x, y) = (5, -4)$, so

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$
$$5 = \frac{x_1 + 7}{2} \qquad -4 = \frac{y_1 + (-2)}{2}$$
$$10 = x_1 + 7 \qquad -8 = y_1 + (-2)$$
$$3 = x_1 \qquad -6 = y_1$$

Thus, $P_1 = (3, -6)$.

57. The midpoint of AB is: $D = \left(\frac{0 + 6}{2}, \frac{0 + 0}{2}\right) = \left(3, 0\right)$

The midpoint of AC is: $E = \left(\frac{0 + 4}{2}, \frac{0 + 4}{2}\right) = \left(2, 2\right)$

The midpoint of BC is: $F = \left(\frac{6 + 4}{2}, \frac{0 + 4}{2}\right) = \left(5, 2\right)$

$$d(C, D) = \sqrt{\left(0 - 4\right)^2 + (3 - 4)^2}$$
$$= \sqrt{(-4)^2 + (-1)^2} = \sqrt{16 + 1} = \sqrt{17}$$
$$d(B, E) = \sqrt{\left(2 - 6\right)^2 + (2 - 0)^2}$$
$$= \sqrt{(-4)^2 + 2^2} = \sqrt{16 + 4}$$
$$= \sqrt{20} = 2\sqrt{5}$$
$$d(A, F) = \sqrt{(2 - 0)^2 + (5 - 0)^2}$$
$$= \sqrt{2^2 + 5^2} = \sqrt{4 + 25}$$
$$= \sqrt{29}$$

58. Let $P_1 = (0, 0)$, $P_2 = (0, 4)$, $P = (x, y)$

$$d(P_1, P_2) = \sqrt{(0-0)^2 + (4-0)^2} = \sqrt{16} = 4$$

$$d(P_1, P) = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2} = 4$$

$$\rightarrow \quad x^2 + y^2 = 16$$

$$d(P_2, P) = \sqrt{(x-0)^2 + (y-4)^2} = \sqrt{x^2 + (y-4)^2} = 4$$

$$\rightarrow \quad x^2 + (y-4)^2 = 16$$

Therefore,

$$y^2 = (y-4)^2$$
$$y^2 = y^2 - 8y + 16$$
$$8y = 16$$
$$y = 2$$

which gives

$$x^2 + 2^2 = 16$$
$$x^2 = 12$$
$$x = \pm 2\sqrt{3}$$

Two triangles are possible. The third vertex is $\left(-2\sqrt{3}, 2\right)$ or $\left(2\sqrt{3}, 2\right)$.

59.
$$d(P_1, P_2) = \sqrt{(-4-2)^2 + (1-1)^2} = \sqrt{(-6)^2 + 0^2} = \sqrt{36} = 6$$

$$d(P_2, P_3) = \sqrt{\left(-4-(-4)\right)^2 + (-3-1)^2} = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$$

$$d(P_1, P_3) = \sqrt{(-4-2)^2 + (-3-1)^2} = \sqrt{(-6)^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

Since $\left[d(P_1, P_2)\right]^2 + \left[d(P_2, P_3)\right]^2 = \left[d(P_1, P_3)\right]^2$, the triangle is a right triangle.

60.
$$d(P_1, P_2) = \sqrt{\left(6-(-1)\right)^2 + (2-4)^2} = \sqrt{7^2 + (-2)^2} = \sqrt{49+4} = \sqrt{53}$$

$$d(P_2, P_3) = \sqrt{\left(4-6\right)^2 + (-5-2)^2} = \sqrt{(-2)^2 + (-7)^2} = \sqrt{4+49} = \sqrt{53}$$

$$d(P_1, P_3) = \sqrt{\left(4-(-1)\right)^2 + (-5-4)^2} = \sqrt{5^2 + (-9)^2} = \sqrt{25+81} = \sqrt{106}$$

Since $\left[d(P_1, P_2)\right]^2 + \left[d(P_2, P_3)\right]^2 = \left[d(P_1, P_3)\right]^2$, the triangle is a right triangle.

Since $d\left(P_1, P_2\right) = d\left(P_2, P_3\right)$, the triangle is isosceles.

Therefore, the triangle is an isosceles right triangle.

61.
$$d(P_1, P_2) = \sqrt{\left(0-(-2)\right)^2 + \left(7-(-1)\right)^2} = \sqrt{2^2 + 8^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$$

$$d(P_2, P_3) = \sqrt{\left(3-0\right)^2 + (2-7)^2} = \sqrt{3^2 + (-5)^2} = \sqrt{9+25} = \sqrt{34}$$

$$d(P_1, P_3) = \sqrt{\left(3-(-2)\right)^2 + \left(2-(-1)\right)^2} = \sqrt{5^2 + 3^2} = \sqrt{25+9} = \sqrt{34}$$

Since $d(P_2, P_3) = d(P_1, P_3)$, the triangle is isosceles.

Since $\left[d(P_1, P_3)\right]^2 + \left[d(P_2, P_3)\right]^2 = \left[d(P_1, P_2)\right]^2$, the triangle is also a right triangle.

Therefore, the triangle is an isosceles right triangle.

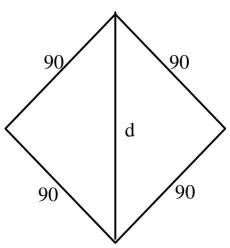
62. $d(P_1,P_2)=\sqrt{(-4-7)^2+(0-2)^2}$
$=\sqrt{(-11)^2+(-2)^2}$
$=\sqrt{121+4}=\sqrt{125}$
$=5\sqrt{5}$

$d(P_2,P_3)=\sqrt{(4-(-4))^2+(6-0)^2}$
$=\sqrt{8^2+6^2}=\sqrt{64+36}$
$=\sqrt{100}$
$=10$

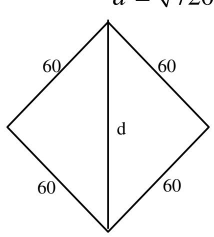
$d(P_1,P_3)=\sqrt{(4-7)^2+(6-2)^2}$
$=\sqrt{(-3)^2+4^2}=\sqrt{9+16}$
$=\sqrt{25}$
$=5$

Since $\left[d(P_1,P_3)\right]^2+\left[d(P_2,P_3)\right]^2=\left[d(P_1,P_2)\right]^2$, the triangle is a right triangle.

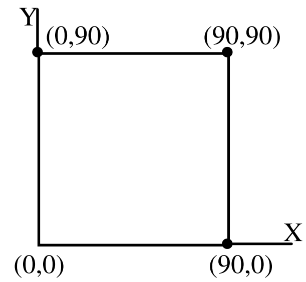
63. Using the Pythagorean Theorem:
$90^2+90^2=d^2$
$8100+8100=d^2$
$16200=d^2$
$d=\sqrt{16200}=90\sqrt{2}\approx 127.28$ feet



64. Using the Pythagorean Theorem:
$60^2+60^2=d^2$
$3600+3600=d^2\to 7200=d^2$
$d=\sqrt{7200}=60\sqrt{2}\approx 84.85$ feet



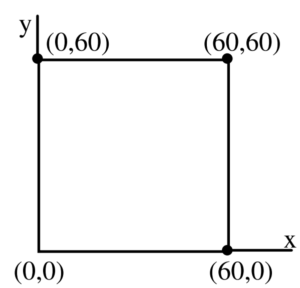
65. **a.** First: (90, 0), Second: (90, 90), Third: (0, 90)



b. Using the distance formula:
$d=\sqrt{(310-90)^2+(15-90)^2}$
$=\sqrt{220^2+(-75)^2}=\sqrt{54025}$
$=5\sqrt{2161}\approx 232.43$ feet

c. Using the distance formula:
$d=\sqrt{(300-0)^2+(300-90)^2}$
$=\sqrt{300^2+210^2}=\sqrt{134100}$
$=30\sqrt{149}\approx 366.20$ feet

66. **a.** First: (60, 0), Second: (60, 60) Third: (0, 60)

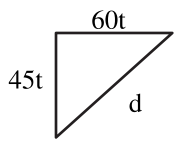


b. Using the distance formula:
$d=\sqrt{(180-60)^2+(20-60)^2}$
$=\sqrt{120^2+(-40)^2}=\sqrt{16000}$
$=40\sqrt{10}\approx 126.49$ feet

c. Using the distance formula:
$d=\sqrt{(220-0)^2+(220-60)^2}$
$=\sqrt{220^2+160^2}=\sqrt{74000}$
$=20\sqrt{185}\approx 272.03$ feet

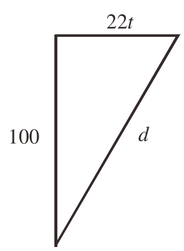
67. The Focus heading east moves a distance $60t$ after t hours. The Tesla heading south moves a distance $40t$ after t hours. Their distance apart after t hours is:

$$\begin{aligned} d &= \sqrt{(60t)^2 + (45t)^2} \\ &= \sqrt{3600t^2 + 2025t^2} \\ &= \sqrt{5625t^2} \\ &= 75t \text{ miles} \end{aligned}$$



68. $\dfrac{15 \text{ miles}}{1 \text{ hr}} \cdot \dfrac{5280 \text{ ft}}{1 \text{ mile}} \cdot \dfrac{1 \text{ hr}}{3600 \text{ sec}} = 22 \text{ ft/sec}$

$$\begin{aligned} d &= \sqrt{100^2 + (22t)^2} \\ &= \sqrt{10000 + 484t^2} \text{ feet} \end{aligned}$$



69. **a.** The shortest side is between $P_1 = (2.6, 1.5)$ and $P_2 = (2.7, 1.7)$. The estimate for the desired intersection point is:

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= \left(\frac{2.6 + 2.7}{2}, \frac{1.5 + 1.7}{2}\right) \\ &= \left(\frac{5.3}{2}, \frac{3.2}{2}\right) \\ &= (2.65, 1.6) \end{aligned}$$

b. Using the distance formula:

$$\begin{aligned} d &= \sqrt{(2.65 - 1.4)^2 + (1.6 - 1.3)^2} \\ &= \sqrt{(1.25)^2 + (0.3)^2} \\ &= \sqrt{1.5625 + 0.09} \\ &= \sqrt{1.6525} \\ &\approx 1.285 \text{ units} \end{aligned}$$

70. Let $P_1 = (2018, 232.89)$ and $P_2 = (2022, 513.98)$. The midpoint is:

$$\begin{aligned} (x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{2018 + 2022}{2}, \frac{232.89 + 513.98}{2}\right) \\ &= \left(\frac{4040}{2}, \frac{746.87}{2}\right) \\ &= (2020, 373.44) \end{aligned}$$

The estimate for 2020 is \$373.44 billion. The estimate net sales of Amazon.com in 2020 is \$12.62 billion off from the reported value of \$386.44 billion.

71. For 2014 we have the ordered pair $(2014, 5645)$ and for 2022 we have the ordered pair $(2022, 7951)$. The midpoint is

$$\begin{aligned} (\text{year}, \$) &= \left(\frac{2014 + 2022}{2}, \frac{5645 + 7951}{2}\right) \\ &= \left(\frac{4036}{2}, \frac{13596}{2}\right) \\ &= (2018, 6798) \end{aligned}$$

Using the midpoint, we estimate the average credit card debt in 2018 to be \$6,798. This is underestimate of the actual value.

72. Let $P_1 = (0,0)$, $P_2 = (a,0)$, and $P_3 = \left(\dfrac{a}{2}, \dfrac{\sqrt{3}\,a}{2}\right)$. Then

$$\begin{aligned} d(P_1, P_2) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(a - 0)^2 + (0 - 0)^2} = \sqrt{a^2} = |a| \\ d(P_2, P_3) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(\frac{a}{2} - a\right)^2 + \left(\frac{\sqrt{3}\,a}{2} - 0\right)^2} \\ &= \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}} = \sqrt{\frac{4a^2}{4}} = \sqrt{a^2} = |a| \end{aligned}$$

$$d(P_1,P_3)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{\left(\frac{a}{2}-0\right)^2+\left(\frac{\sqrt{3}\,a}{2}-0\right)^2}$$
$$=\sqrt{\frac{a^2}{4}+\frac{3a^2}{4}}=\sqrt{\frac{4a^2}{4}}=\sqrt{a^2}=|a|$$

Since the lengths of the three sides are all equal, the triangle is an equilateral triangle.
The midpoints of the saids are

$$M_{P_1P_2}=\left(\frac{0+a}{2},\frac{0+0}{2}\right)=\left(\frac{a}{2},0\right)$$

$$M_{P_2P_3}=\left(\frac{a+\frac{a}{2}}{2},\frac{0+\frac{\sqrt{3}\,a}{2}}{2}\right)=\left(\frac{3a}{4},\frac{\sqrt{3}\,a}{4}\right)$$

$$M_{P_1P_3}=\left(\frac{0+\frac{a}{2}}{2},\frac{0+\frac{\sqrt{3}a}{2}}{2}\right)=\left(\frac{a}{4},\frac{\sqrt{3}\,a}{4}\right)$$

Then,

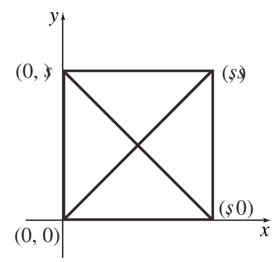
$$d\left(M_{P_1P_2},M_{P_2P_3}\right)=\sqrt{\left(\frac{3a}{4}-\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}\,a}{4}-0\right)^2}$$
$$=\sqrt{\left(\frac{a}{4}\right)^2+\left(\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$=\sqrt{\frac{a^2}{16}+\frac{3a^2}{16}}=\frac{|a|}{2}$$

$$d\left(M_{P_2P_3},M_{P_1P_3}\right)=\sqrt{\left(\frac{3a}{4}-\frac{a}{4}\right)^2+\left(\frac{\sqrt{3}\,a}{4}-\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$=\sqrt{\left(\frac{a}{2}\right)^2+0^2}$$
$$=\sqrt{\frac{a^2}{4}}=\frac{|a|}{2}$$

$$d\left(M_{P_1P_2},M_{P_1P_3}\right)=\sqrt{\left(\frac{a}{4}-\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}\,a}{4}-0\right)^2}$$
$$=\sqrt{\left(-\frac{a}{4}\right)^2+\left(\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$=\sqrt{\frac{a^2}{16}+\frac{3a^2}{16}}=\frac{|a|}{2}$$

Since the lengths of the sides of the triangle formed by the midpoints are all equal, the triangle is equilateral.

73. Let $P_1=(0,0)$, $P_2=(0,s)$, $P_3=(s,0)$, and $P_4=(s,s)$ be the vertices of the square.



The points P_1 and P_4 are endpoints of one diagonal and the points P_2 and P_3 are the endpoints of the other diagonal.

$$M_{P_1P_4}=\left(\frac{0+s}{2},\frac{0+s}{2}\right)=\left(\frac{s}{2},\frac{s}{2}\right)$$
$$M_{P_2P_3}=\left(\frac{0+s}{2},\frac{s+0}{2}\right)=\left(\frac{s}{2},\frac{s}{2}\right)$$

The midpoints of the diagonals are the same. Therefore, the diagonals of a square intersect at their midpoints.

74. Let $P=(a,2a)$. Then

$$\sqrt{(a+5)^2+(2a-1)^2}=\sqrt{(a-4)^2+(2a+4)^2}$$
$$(a+5)^2+(2a-1)^2=(a-4)^2+(2a+4)^2$$
$$5a^2+6a+26=5a^2+8a+32$$
$$6a+26=8a+32$$
$$-2a=6$$
$$a=-3$$

Then $P=(-3,-6)$.

75. Arrange the parallelogram on the coordinate plane so that the vertices are
$P_1=(0,0), P_2=(a,0), P_3=(a+b,c)$ and $P_4=(b,c)$
Then the lengths of the sides are:

$$d(P_1,P_2)=\sqrt{(a-0)^2+(0-0)^2}$$
$$=\sqrt{a^2}=|a|$$

$$d(P_2,P_3)=\sqrt{[(a+b)-a]^2+(c-0)^2}$$
$$=\sqrt{b^2+c^2}$$

$$d(P_3,P_4)=\sqrt{[b-(a+b)]^2+(c-c)^2}$$
$$=\sqrt{a^2}=|a|$$

and

$$d(P_1,P_4)=\sqrt{(b-0)^2+(c-0)^2}$$
$$=\sqrt{b^2+c^2}$$

P_1 and P_3 are the endpoints of one diagonal, and P_2 and P_4 are the endpoints of the other diagonal. The lengths of the diagonals are

$$d(P_1,P_3)=\sqrt{[(a+b)-0]^2+(c-0)^2}$$
$$=\sqrt{a^2+2ab+b^2+c^2}$$

and

$$d(P_2,P_4)=\sqrt{(b-a)^2+(c-0)^2}\ .$$
$$=\sqrt{a^2-2ab+b^2+c^2}$$

Sum of the squares of the sides:

$$a^2+(\sqrt{b^2+c^2})^2+a^2+(\sqrt{b^2+c^2})^2$$
$$=2a^2+2b^2+2c^2$$

Sum of the squares of the diagonals:

$$\left(\sqrt{a^2+2ab+b^2+c^2}\right)^2+\left(\sqrt{a^2-2ab+b^2+c^2}\right)^2$$
$$=2a^2+2b^2+2c^2$$

76. Answers will vary.

Section 1.2

1. $2(x+3)-1=-7$
$$2(x+3)=-6$$
$$x+3=-3$$
$$x=-6$$
The solution set is $\{-6\}$.

2. $x^2-9=0$
$$x^2=9$$
$$x=\pm\sqrt{9}=\pm 3$$
The solution set is $\{-3,3\}$.

3. intercepts

4. $y=0$

5. y-axis

6. 4

7. $(-3,4)$

8. True

9. False; the y-coordinate of a point at which the graph crosses or touches the x-axis is always 0. The x-coordinate of such a point is an x-intercept.

10. False; a graph can be symmetric with respect to both coordinate axes (in such cases it will also be symmetric with respect to the origin).
For example: $x^2+y^2=1$

11. d

12. c

13. $y=x^4-\sqrt{x}$

$0=0^4-\sqrt{0}$ $\quad$ $1=1^4-\sqrt{1}$ $\quad$ $4=(2)^4-\sqrt{2}$

$0=0$ $\quad$ $1\neq 0$ $\quad$ $4\neq 16-\sqrt{2}$

The point (0, 0) is on the graph of the equation.

14. $y=x^3-2\sqrt{x}$

$0=0^3-2\sqrt{0}$ $\quad$ $1=1^3-2\sqrt{1}$ $\quad$ $-1=1^3-2\sqrt{1}$

$0=0$ $\quad$ $1\neq -1$ $\quad$ $-1=-1$

The points (0, 0) and (1, –1) are on the graph of the equation.

15. $y^2=x^2+9$

$3^2=0^2+9$ $\quad$ $0^2=3^2+9$ $\quad$ $0^2=(-3)^2+9$

$9=9$ $\quad$ $0\neq 18$ $\quad$ $0\neq 18$

The point (0, 3) is on the graph of the equation.

16. $y^3 = x+1$

$2^3 = 1+1 \qquad 1^3 = 0+1 \qquad 0^3 = -1+1$

$8 \neq 2 \qquad\quad 1 = 1 \qquad\quad 0 = 0$

The points (0, 1) and (–1, 0) are on the graph of the equation.

17. $x^2 + y^2 = 4$

$0^2 + 2^2 = 4 \qquad (-2)^2 + 2^2 = 4 \qquad \left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2 = 4$

$4 = 4 \qquad\quad 8 \neq 4 \qquad\quad 4 = 4$

(0, 2) and $\left(\sqrt{2}, \sqrt{2}\right)$ are on the graph of the equation.

18. $x^2 + 4y^2 = 4$

$0^2 + 4 \cdot 1^2 = 4 \qquad 2^2 + 4 \cdot 0^2 = 4 \qquad 2^2 + 4\left(\frac{1}{2}\right)^2 = 4$

$4 = 4 \qquad\quad 4 = 4 \qquad\quad 5 \neq 4$

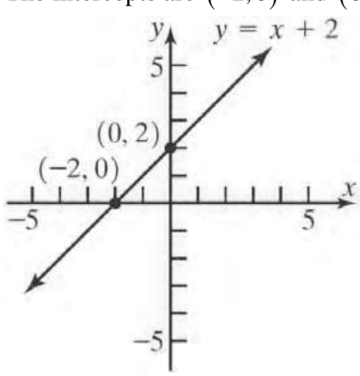
The points (0, 1) and (2, 0) are on the graph of the equation.

19. $y = x+2$

x-intercept: $0 = x+2$; $-2 = x$

y-intercept: $y = 0+2$; $y = 2$

The intercepts are $(-2,0)$ and $(0,2)$.

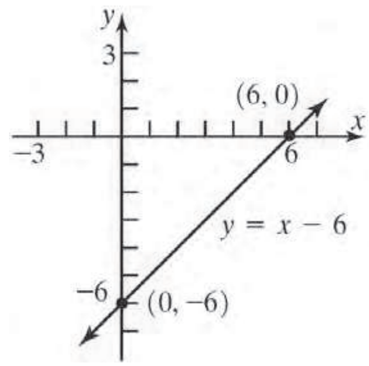


20. $y = x-6$

x-intercept: $0 = x-6$; $6 = x$

y-intercept: $y = 0-6$; $y = -6$

The intercepts are $(6,0)$ and $(0,-6)$.

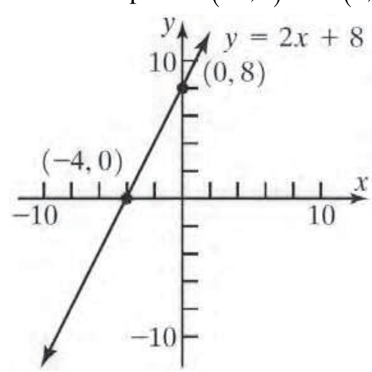


21. $y = 2x+8$

x-intercept: $0 = 2x+8$; $2x = -8$; $x = -4$

y-intercept: $y = 2(0)+8$; $y = 8$

The intercepts are $(-4,0)$ and $(0,8)$.

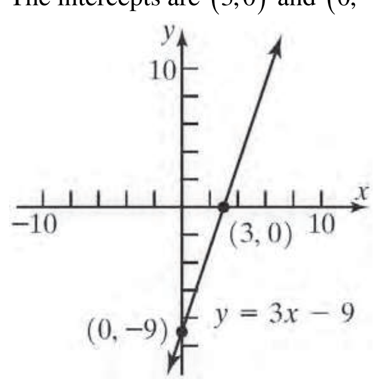


22. $y = 3x-9$

x-intercept: $0 = 3x-9$; $3x = 9$; $x = 3$

y-intercept: $y = 3(0)-9$; $y = -9$

The intercepts are $(3,0)$ and $(0,-9)$.

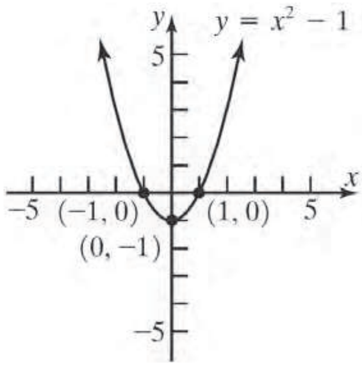


23. $y = x^2 - 1$

x-intercepts: $0 = x^2 - 1$; $x^2 = 1$; $x = \pm 1$

y-intercept: $y = 0^2 - 1$; $y = -1$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,-1)$.

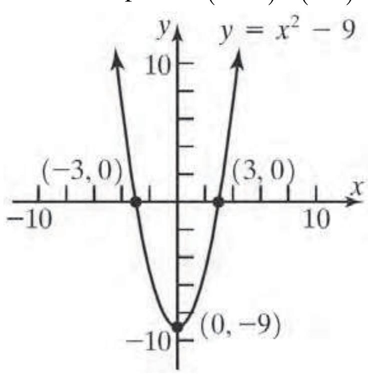


24. $y = x^2 - 9$

x-intercepts: $0 = x^2 - 9$; $x^2 = 9$; $x = \pm 3$

y-intercept: $y = 0^2 - 9$; $y = -9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,-9)$.

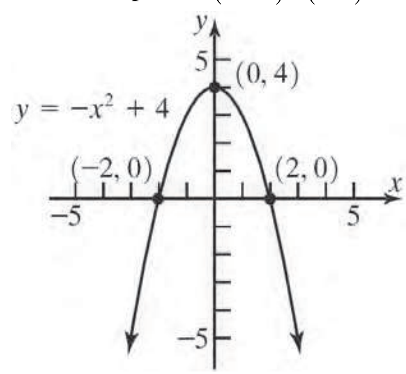


25. $y = -x^2 + 4$

x-intercepts: $0 = -x^2 + 4$; $x^2 = 4$; $x = \pm 2$

y-intercepts: $y = -(0)^2 + 4$; $y = 4$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,4)$.

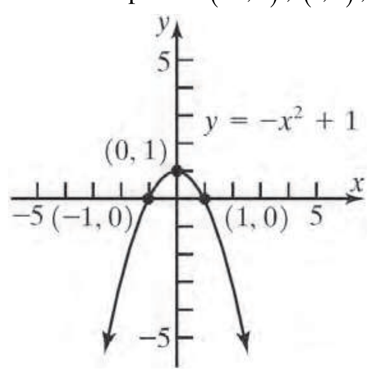


26. $y = -x^2 + 1$

x-intercepts: $0 = -x^2 + 1$; $x^2 = 1$; $x = \pm 1$

y-intercept: $y = -(0)^2 + 1$; $y = 1$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,1)$.

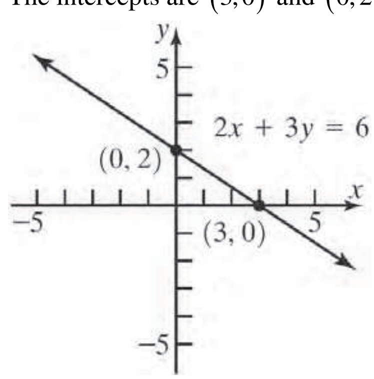


27. $2x + 3y = 6$

x-intercepts: $2x + 3(0) = 6$; $2x = 6$; $x = 3$

y-intercept: $2(0) + 3y = 6$; $3y = 6$; $y = 2$

The intercepts are $(3,0)$ and $(0,2)$.



28. $5x+2y=10$

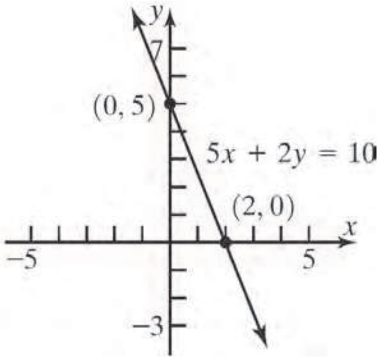
x-intercepts:

$$5x+2(0)=10$$
$$5x=10$$
$$x=2$$

y-intercept:

$$5(0)+2y=10$$
$$2y=10$$
$$y=5$$

The intercepts are $(2,0)$ and $(0,5)$.



29. $9x^2+4y=36$

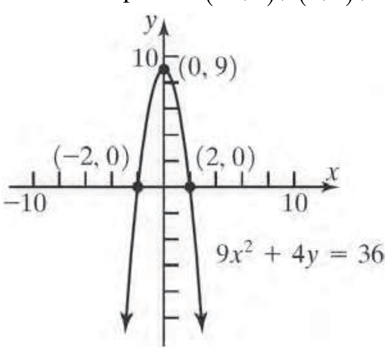
x-intercepts:

$$9x^2+4(0)=36$$
$$9x^2=36$$
$$x^2=4$$
$$x=\pm 2$$

y-intercept:

$$9(0)^2+4y=36$$
$$4y=36$$
$$y=9$$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,9)$.



30. $4x^2+y=4$

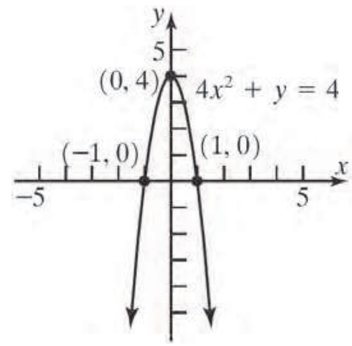
x-intercepts:

$$4x^2+0=4$$
$$4x^2=4$$
$$x^2=1$$
$$x=\pm 1$$

y-intercept:

$$4(0)^2+y=4$$
$$y=4$$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,4)$.



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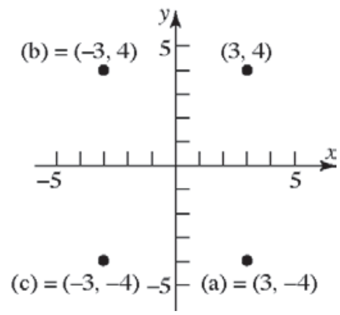

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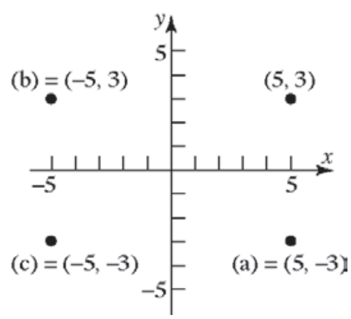

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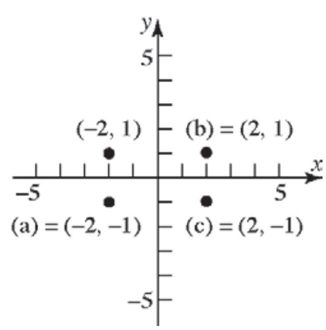

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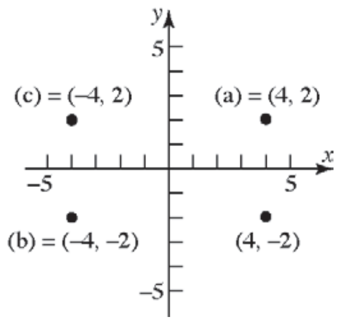

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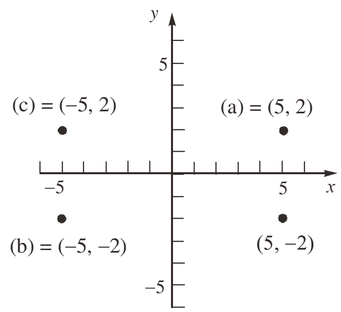

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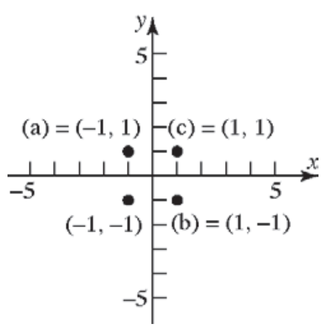

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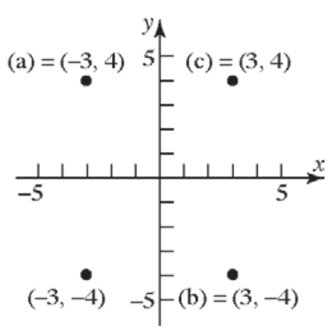

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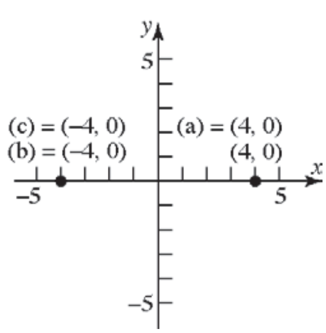

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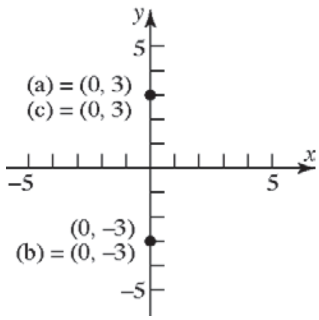

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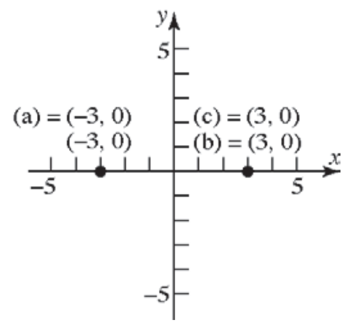


41. **a.** Intercepts: $(-1,0)$ and $(1,0)$

b. Symmetric with respect to the *x*-axis, *y*-axis, and the origin.

42. **a.** Intercepts: $(0,1)$

b. Not symmetric to the *x*-axis, the *y*-axis, nor the origin

43. **a.** Intercepts: $\left(-\frac{\pi}{2},0\right)$, $(0,1)$, and $\left(\frac{\pi}{2},0\right)$

b. Symmetric with respect to the *y*-axis.

44. **a.** Intercepts: $(-2,0)$, $(0,-3)$, and $(2,0)$

b. Symmetric with respect to the *y*-axis.

45. **a.** Intercepts: $(0,0)$

b. Symmetric with respect to the *x*-axis.

46. **a.** Intercepts: $(-2,0)$, $(0,2)$, $(0,-2)$, and $(2,0)$

b. Symmetric with respect to the *x*-axis, *y*-axis, and the origin.

47. **a.** Intercepts: $(-2,0)$, $(0,0)$, and $(2,0)$

b. Symmetric with respect to the origin.

48. **a.** Intercepts: $(-4,0)$, $(0,0)$, and $(4,0)$

b. Symmetric with respect to the origin.

49. **a.** x-intercepts: $[-2,1]$, y-intercept 0

b. Not symmetric to x-axis, y-axis, or origin.

50. **a.** x-intercepts: $[-1,2]$, y-intercept 0

b. Not symmetric to x-axis, y-axis, or origin.

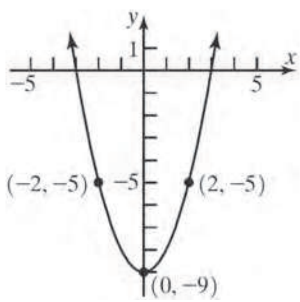
51. **a.** Intercepts: none

b. Symmetric with respect to the origin.

52. **a.** Intercepts: none

b. Symmetric with respect to the x-axis.

53.



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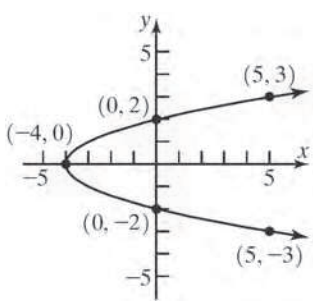

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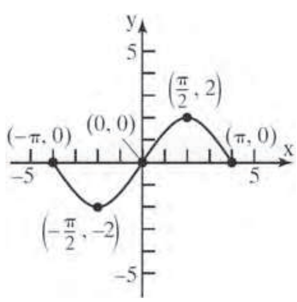
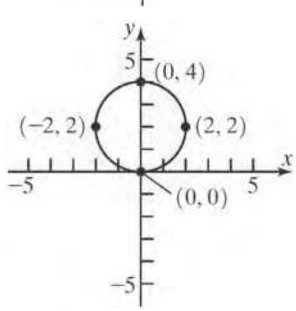


56.



57. $y^2 = x+16$

x-intercepts:
$$0^2 = x+16$$
$$-16 = x$$

y-intercepts:
$$y^2 = 0+16$$
$$y^2 = 16$$
$$y = \pm 4$$

The intercepts are $(-16,0)$, $(0,-4)$ and $(0,4)$.

<u>Test x-axis symmetry</u>: Let $y = -y$
$$(-y)^2 = x+16$$
$$y^2 = x+16 \text{ same}$$

<u>Test y-axis symmetry</u>: Let $x = -x$
$$y^2 = -x+16 \text{ different}$$

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$.
$$(-y)^2 = -x+16$$
$$y^2 = -x+16 \text{ different}$$

Therefore, the graph will have x-axis symmetry.

58. $y^2 = x+9$

x-intercepts:
$$(0)^2 = -x+9$$
$$0 = -x+9$$
$$x = 9$$

y-intercepts:
$$y^2 = 0+9$$
$$y^2 = 9$$
$$y = \pm 3$$

The intercepts are $(-9,0)$, $(0,-3)$ and $(0,3)$.

<u>Test x-axis symmetry</u>: Let $y = -y$
$$(-y)^2 = x+9$$
$$y^2 = x+9 \text{ same}$$

<u>Test y-axis symmetry</u>: Let $x = -x$
$$y^2 = -x+9 \text{ different}$$

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$.
$$(-y)^2 = -x+9$$
$$y^2 = -x+9 \text{ different}$$

Therefore, the graph will have x-axis symmetry.

59. $y = \sqrt[3]{x}$

x-intercepts:
$$0 = \sqrt[3]{x}$$
$$0 = x$$

y-intercepts:
$$y = \sqrt[3]{0} = 0$$

The only intercept is $(0,0)$.

<u>Test x-axis symmetry:</u> Let $y = -y$
$$-y = \sqrt[3]{x} \text{ different}$$

Test y-axis symmetry: Let $x = -x$

$y = \sqrt[3]{-x} = -\sqrt[3]{x}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \sqrt[3]{-x} = -\sqrt[3]{x}$

$y = \sqrt[3]{x}$ same

Therefore, the graph will have origin symmetry.

60. $y = \sqrt[5]{x}$

x-intercepts:

$0 = \sqrt[3]{x}$

$0 = x$

y-intercepts:

$y = \sqrt[5]{0} = 0$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \sqrt[5]{x}$ different

Test y-axis symmetry: Let $x = -x$

$y = \sqrt[5]{-x} = -\sqrt[5]{x}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \sqrt[5]{-x} = -\sqrt[5]{x}$

$y = \sqrt[5]{x}$ same

Therefore, the is symmetric with respect to the origin.

61. $x^2 + y - 9 = 0$

x-intercepts:

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

y-intercepts:

$0^2 + y - 9 = 0$

$y = 9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,9)$.

Test x-axis symmetry: Let $y = -y$

$x^2 - y - 9 = 0$ different

Test y-axis symmetry: Let $x = -x$

$(-x)^2 + y - 9 = 0$

$x^2 + y - 9 = 0$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$(-x)^2 - y - 9 = 0$

$x^2 - y - 9 = 0$ different

Therefore, the graph has y-axis symmetry.

62. $x^2 - y - 4 = 0$

x-intercepts:

$x^2 - 0 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

y-intercept:

$0^2 - y - 4 = 0$

$-y = 4$

$y = -4$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,-4)$.

Test x-axis symmetry: Let $y = -y$

$x^2 - (-y) - 4 = 0$

$x^2 + y - 4 = 0$ different

Test y-axis symmetry: Let $x = -x$

$(-x)^2 - y - 4 = 0$

$x^2 - y - 4 = 0$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$(-x)^2 - (-y) - 4 = 0$

$x^2 + y - 4 = 0$ different

Therefore, the graph has y-axis symmetry.

63. $25x^2 + 4y^2 = 100$

x-intercepts:

$25x^2 + 4(0)^2 = 100$

$25x^2 = 100$

$x^2 = 4$

$x = \pm 2$

y-intercepts:

$25(0)^2 + 4y^2 = 100$

$4y^2 = 25$

$y^2 = 5$

$y = \pm 5$

The intercepts are $(-2,0)$, $(2,0)$, $(0,-5)$, and $(0,5)$.

Test x-axis symmetry: Let $y = -y$

$25x^2 + 4(-y)^2 = 100$

$25x^2 + 4y^2 = 100$ same

Test y-axis symmetry: Let $x = -x$

$25(-x)^2 + 4y^2 = 100$

$25x^2 + 4y^2 = 100$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$25(-x)^2 + 4(-y)^2 = 100$

$25x^2 + 4y^2 = 100$ same

Therefore, the graph has x-axis, y-axis, and origin symmetry.

64. $4x^2 + y^2 = 4$

x-intercepts: y-intercepts:

$4x^2 + 0^2 = 4$ $\quad$ $4(0)^2 + y^2 = 4$

$4x^2 = 4$ $\quad$ $y^2 = 4$

$x^2 = 1$ $\quad$ $y = \pm 2$

$x = \pm 1$

The intercepts are $(-1,0)$, $(1,0)$, $(0,-2)$, and $(0,2)$.

Test x-axis symmetry: Let $y = -y$

$4x^2 + (-y)^2 = 4$

$4x^2 + y^2 = 4$ same

Test y-axis symmetry: Let $x = -x$

$4(-x)^2 + y^2 = 4$

$4x^2 + y^2 = 4$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$4(-x)^2 + (-y)^2 = 4$

$4x^2 + y^2 = 4$ same

Therefore, the graph has x-axis, y-axis, and origin symmetry.

65. $y = x^3 - 64$

x-intercepts: $\quad$ y-intercepts:

$0 = x^3 - 64$ $\quad$ $y = 0^3 - 64$

$x^3 = 64$ $\quad$ $y = -64$

$x = 4$

The intercepts are $(4,0)$ and $(0,-64)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^3 - 64$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^3 - 64$

$y = -x^3 - 64$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^3 - 64$

$y = x^3 + 64$ different

Therefore, the graph has no symmetry.

66. $y = x^4 - 1$

x-intercepts: $\quad$ y-intercepts:

$0 = x^4 - 1$ $\quad$ $y = 0^4 - 1$

$x^4 = 1$ $\quad$ $y = -1$

$x = \pm 1$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,-1)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^4 - 1$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^4 - 1$

$y = x^4 - 1$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^4 - 1$

$-y = x^4 - 1$ different

Therefore, the graph has y-axis symmetry.

67. $y = x^2 - 2x - 8$

x-intercepts: $\quad$ y-intercepts:

$0 = x^2 - 2x - 8$ $\quad$ $y = 0^2 - 2(0) - 8$

$0 = (x-4)(x+2)$ $\quad$ $y = -8$

$x = 4$ or $x = -2$

The intercepts are $(4,0)$, $(-2,0)$, and $(0,-8)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^2 - 2x - 8$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^2 - 2(-x) - 8$

$y = x^2 + 2x - 8$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^2 - 2(-x) - 8$

$-y = x^2 + 2x - 8$ different

Therefore, the graph has no symmetry.

68. $y = x^2 + 4$

x-intercepts: $\quad$ y-intercepts:

$0 = x^2 + 4$ $\quad$ $y = 0^2 + 4$

$x^2 = -4$ $\quad$ $y = 4$

no real solution

The only intercept is $(0,4)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^2 + 4$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^2 + 4$

$y = x^2 + 4$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^2 + 4$

$-y = x^2 + 4$ different

Therefore, the graph has y-axis symmetry.

69. $y = \dfrac{4x}{x^2 + 16}$

x-intercepts:

$0 = \dfrac{4x}{x^2 + 16}$

$4x = 0$

$x = 0$

y-intercepts:

$y = \dfrac{4(0)}{0^2 + 16} = \dfrac{0}{16} = 0$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \dfrac{4x}{x^2 + 16}$ different

Test y-axis symmetry: Let $x = -x$

$y = \dfrac{4(-x)}{(-x)^2 + 16}$

$y = -\dfrac{4x}{x^2 + 16}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \dfrac{4(-x)}{(-x)^2 + 16}$

$-y = -\dfrac{4x}{x^2 + 16}$

$y = \dfrac{4x}{x^2 + 16}$ same

Therefore, the graph has origin symmetry.

70. $y = \dfrac{x^2 - 4}{2x}$

x-intercepts:

$0 = \dfrac{x^2 - 4}{2x}$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

y-intercepts:

$y = \dfrac{0^2 - 4}{2(0)} = \dfrac{-4}{0}$

undefined

The intercepts are $(-2,0)$ and $(2,0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \dfrac{x^2 - 4}{2x}$ different

Test y-axis symmetry: Let $x = -x$

$y = \dfrac{(-x)^2 - 4}{2(-x)}$

$y = -\dfrac{x^2 - 4}{2x}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \dfrac{(-x)^2 - 4}{2(-x)}$

$-y = \dfrac{x^2 - 4}{-2x}$

$y = \dfrac{x^2 - 4}{2x}$ same

Therefore, the graph has origin symmetry.

71. $y = \dfrac{-x^3}{x^2 - 9}$

x-intercepts:

$0 = \dfrac{-x^3}{x^2 - 9}$

$-x^3 = 0$

$x = 0$

y-intercepts:

$y = \dfrac{-0^3}{0^2 - 9} = \dfrac{0}{-9} = 0$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \dfrac{-x^3}{x^2 - 9}$

$y = \dfrac{x^3}{x^2 - 9}$ different

Test y-axis symmetry: Let $x = -x$

$y = \dfrac{-(-x)^3}{(-x)^2 - 9}$

$y = \dfrac{x^3}{x^2 - 9}$ different

Test origin symmetry: Let $x=-x$ and $y=-y$

$$-y=\frac{-(-x)^3}{(-x)^2-9}$$

$$-y=\frac{x^3}{x^2-9}$$

$$y=\frac{-x^3}{x^2-9}\quad\text{same}$$

Therefore, the graph has origin symmetry.

72. $y=\dfrac{x^4+1}{2x^5}$

x-intercepts:

$$0=\frac{x^4+1}{2x^5}$$

$$x^4=-1$$

no real solution

y-intercepts:

$$y=\frac{0^4+1}{2(0)^5}=\frac{1}{0}$$

undefined

There are no intercepts for the graph of this equation.

Test x-axis symmetry: Let $y=-y$

$$-y=\frac{x^4+1}{2x^5}\quad\text{different}$$

Test y-axis symmetry: Let $x=-x$

$$y=\frac{(-x)^4+1}{2(-x)^5}$$

$$y=\frac{x^4+1}{-2x^5}\quad\text{different}$$

Test origin symmetry: Let $x=-x$ and $y=-y$

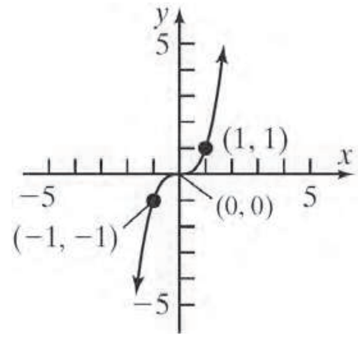
$$-y=\frac{(-x)^4+1}{2(-x)^5}$$

$$-y=\frac{x^4+1}{-2x^5}$$

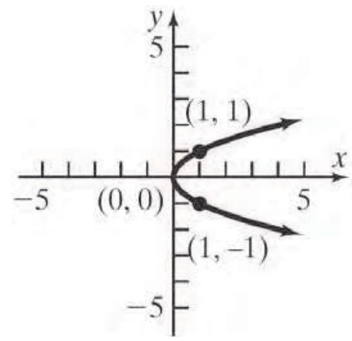
$$y=\frac{x^4+1}{2x^5}\quad\text{same}$$

Therefore, the graph has origin symmetry.

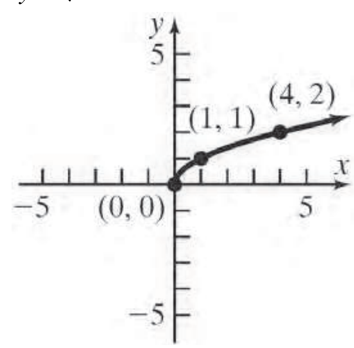
73. $y=x^3$



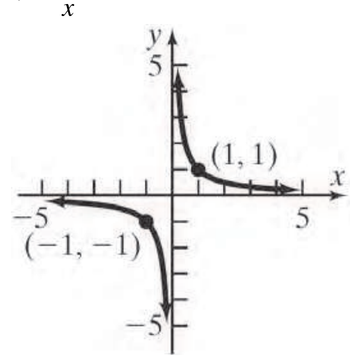
74. $x=y^2$



75. $y=\sqrt{x}$



76. $y=\dfrac{1}{x}$



77. If the point $(a,4)$ is on the graph of $y = x^2 + 3x$, then we have

$$4 = a^2 + 3a$$
$$0 = a^2 + 3a - 4$$
$$0 = (a+4)(a-1)$$
$$a+4=0 \quad \text{or} \quad a-1=0$$
$$a=-4 \qquad\qquad a=1$$

Thus, $a=-4$ or $a=1$.

78. If the point $(a,-5)$ is on the graph of $y = x^2 + 6x$, then we have

$$-5 = a^2 + 6a$$
$$0 = a^2 + 6a + 5$$
$$0 = (a+5)(a+1)$$
$$a+5=0 \quad \text{or} \quad a+1=0$$
$$a=-5 \qquad\qquad a=-1$$

Thus, $a=-5$ or $a=-1$.

79. For a graph with origin symmetry, if the point (a,b) is on the graph, then so is the point $(-a,-b)$. Since the point $(1,2)$ is on the graph of an equation with origin symmetry, the point $(-1,-2)$ must also be on the graph.

80. For a graph with y-axis symmetry, if the point (a,b) is on the graph, then so is the point $(-a,b)$. Since 6 is an x-intercept in this case, the point $(6,0)$ is on the graph of the equation. Due to the y-axis symmetry, the point $(-6,0)$ must also be on the graph. Therefore, -6 is another x-intercept.

81. For a graph with origin symmetry, if the point (a,b) is on the graph, then so is the point $(-a,-b)$. Since -4 is an x-intercept in this case, the point $(-4,0)$ is on the graph of the equation. Due to the origin symmetry, the point $(4,0)$ must also be on the graph. Therefore, 4 is another x-intercept.

82. For a graph with x-axis symmetry, if the point (a,b) is on the graph, then so is the point $(a,-b)$. Since 2 is a y-intercept in this case, the point $(0,2)$ is on the graph of the equation. Due to the x-axis symmetry, the point $(0,-2)$ must also be on the graph. Therefore, -2 is another y-intercept.

83. a. $\left(x^2+y^2-x\right)^2 = x^2+y^2$

x-intercepts:

$$\left(x^2+(0)^2-x\right)^2 = x^2+(0)^2$$
$$\left(x^2-x\right)^2 = x^2$$
$$x^4-2x^3+x^2 = x^2$$
$$x^4-2x^3 = 0$$
$$x^3(x-2) = 0$$
$$x^3=0 \quad \text{or} \quad x-2=0$$
$$x=0 \qquad\qquad x=2$$

y-intercepts:

$$\left((0)^2+y^2-0\right)^2 = (0)^2+y^2$$
$$\left(y^2\right)^2 = y^2$$
$$y^4 = y^2$$
$$y^4-y^2 = 0$$
$$y^2\left(y^2-1\right) = 0$$
$$y^2=0 \quad \text{or} \quad y^2-1=0$$
$$y=0 \qquad\qquad y^2=1$$
$$y=\pm 1$$

The intercepts are $(0,0)$, $(2,0)$, $(0,-1)$, and $(0,1)$.

b. <u>Test x-axis symmetry:</u> Let $y=-y$

$$\left(x^2+(-y)^2-x\right)^2 = x^2+(-y)^2$$
$$\left(x^2+y^2-x\right)^2 = x^2+y^2 \quad \text{same}$$

<u>Test y-axis symmetry:</u> Let $x=-x$

$$\left((-x)^2+y^2-(-x)\right)^2 = (-x)^2+y^2$$
$$\left(x^2+y^2+x\right)^2 = x^2+y^2 \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$\left((-x)^2 + (-y)^2 - (-x)\right)^2 = (-x)^2 + (-y)^2$$

$$\left(x^2 + y^2 + x\right)^2 = x^2 + y^2 \quad \text{different}$$

Thus, the graph will have x-axis symmetry.

84. **a.** $16y^2 = 120x - 225$

y-intercepts:

$$16y^2 = 120(0) - 225$$
$$16y^2 = -225$$
$$y^2 = -\frac{225}{16}$$

no real solution

x-intercepts:

$$16(0)^2 = 120x - 225$$
$$0 = 120x - 225$$
$$-120x = -225$$
$$x = \frac{-225}{-120} = \frac{15}{8}$$

The only intercept is $\left(\frac{15}{8}, 0\right)$.

b. Test x-axis symmetry: Let $y = -y$

$$16(-y)^2 = 120x - 225$$
$$16y^2 = 120x - 225 \quad \text{same}$$

Test y-axis symmetry: Let $x = -x$

$$16y^2 = 120(-x) - 225$$
$$16y^2 = -120x - 225 \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$16(-y)^2 = 120(-x) - 225$$
$$16y^2 = -120x - 225 \quad \text{different}$$

Thus, the graph has x-axis symmetry.

85. Let $y = 0$.

$$(x^2 + 0^2)^2 = a^2(x^2 - 0^2)$$
$$x^4 = a^2(x^2)$$
$$x^4 - a^2x^2 = 0$$
$$x^2(x^2 - a^2) = 0$$
$$x^2 = 0 \quad \text{or} \quad (x^2 - a^2) = 0$$
$$x = 0 \quad \text{or} \quad x^2 = a^2$$
$$x = -a, a$$

Let $x = 0$.

$$(0^2 + y^2)^2 = a^2(0^2 - y^2)$$
$$y^4 = a^2(-y^2)$$
$$y^4 + a^2y^2 = 0$$
$$y^2(y^2 + a^2) = 0$$
$$y = 0$$

(Note that the solutions to $y^2 + a^2 = 0$ are not real)

So the intercepts are are (0,0), (a,0) and (-a,0).
Test x-axis symmetry: Replace y by -y

$$(x^2 + (-y)^2)^2 = a^2(x^2 - (-y)^2)$$
$$(x^2 + y^2)^2 = a^2(x^2 - y^2) \text{ equivalent}$$

Test y-axis symmetry: replace x by -x

$$((-x)^2 + y^2)^2 = a^2((-x)^2 - y^2)$$
$$(x^2 + y^2)^2 = a^2(x^2 - y^2) \text{ equivalent}$$

Test origin symmetry: replace x by -x and y by -y

$$((-x)^2 + (-y)^2)^2 = a^2((-x)^2 - (-y)^2)$$
$$(x^2 + y^2)^2 = a^2(x^2 - y^2) \text{ equivalent}$$

The graph is symmetric by respect to the x-axis, the y-axis, and the origin.

86. Let $y = 0$.

$$(x^2 + 0^2 - ax)^2 = b^2(x^2 + 0^2)$$
$$x^4 - 2ax^3 + a^2x^2 - b^2x^2 = 0$$
$$x^2\left[(x - (a+b)\right]\left[(x - (a-b)\right] = 0$$
$$x = 0 \quad \text{or} \quad x = a + b$$
$$\text{or} \quad x = a - b$$

Let $x = 0$.

$$(0^2 + y^2 - a \cdot 0)^2 = b^2(0^2 + y^2)$$
$$y^4 - b^2y^2 = 0$$
$$y^2(y+b)(y-b) = 0$$
$$y = 0, y = -b, y = b$$

So the intercepts are (0,0), (a-b,0), (a+b,0), (0,-b), (0, b).
Test x-axis symmetry: replace y by -y

$$\left[x^2 + (-y)^2 - ax\right]^2 = b^2\left[x^2 + (-y)^2\right]$$
$$(x^2 + y^2 - ax)^2 = b^2(x^2 + y^2) \text{ Equivalent}$$

<u>Test *y*-axis symmetry:</u> replace *x* by -*x*

$\left[(-x)^2 + y^2 - a(-x)\right]^2 = b^2\left[(-x)^2 + y^2\right]$

$(x^2 + y^2 + ax)^2 = b^2(x^2 + y^2)$ Not equivalent

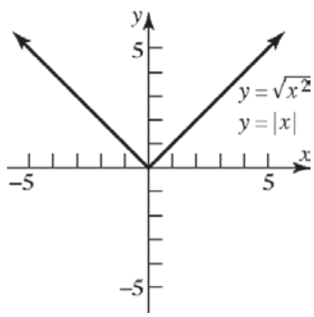
<u>Test origin symmetry</u>: replace *x* by -*x* and *y* by -*y*

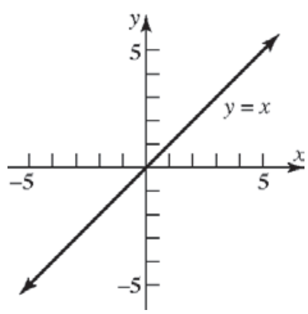
$\left[(-x)^2 + (-y)^2 - a(-x)\right]^2 = b^2\left[(-x)^2 + (-y)^2\right]$

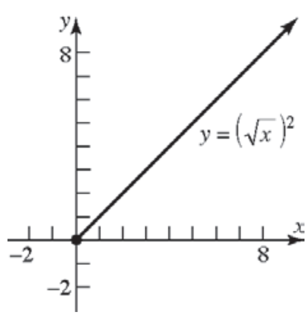
$(x^2 + y^2 + ax)^2 = b^2(x^2 + y^2)$ No equivalent

The graph is symmetric with respect to the *x*-axis only.

87. a.







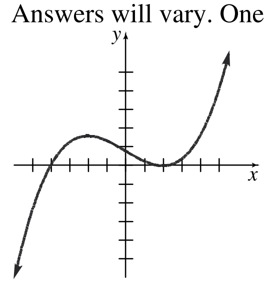
b. Since $\sqrt{x^2} = |x|$ for all x, the graphs of $y = \sqrt{x^2}$ and $y = |x|$ are the same.

c. For $y = \left(\sqrt{x}\right)^2$, the domain of the variable x is $x \geq 0$; for $y = x$, the domain of the variable x is all real numbers. Thus, $\left(\sqrt{x}\right)^2 = x$ only for $x \geq 0$.

d. For $y = \sqrt{x^2}$, the range of the variable y is $y \geq 0$; for $y = x$, the range of the variable y is all real numbers. Also, $\sqrt{x^2} = x$ only if $x \geq 0$. Otherwise, $\sqrt{x^2} = -x$.

88. Answers will vary. A complete graph presents enough of the graph to the viewer so they can "see" the rest of the graph as an obvious continuation of what is shown.

89. Answers will vary. One example:



90. Answers will vary

91. Answers will vary

92. Answers will vary.
Case 1: Graph has *x*-axis and *y*-axis symmetry, show origin symmetry.
(x, y) on graph $\rightarrow (x, -y)$ on graph
(from *x*-axis symmetry)
$(x, -y)$ on graph $\rightarrow (-x, -y)$ on graph
(from *y*-axis symmetry)
Since the point $(-x, -y)$ is also on the graph, the graph has origin symmetry.

Case 2: Graph has *x*-axis and origin symmetry, show *y*-axis symmetry.
(x, y) on graph $\rightarrow (x, -y)$ on graph
(from *x*-axis symmetry)
$(x, -y)$ on graph $\rightarrow (-x, y)$ on graph
(from origin symmetry)
Since the point $(-x, y)$ is also on the graph, the graph has *y*-axis symmetry.

Case 3: Graph has *y*-axis and origin symmetry, show *x*-axis symmetry.

(x, y) on graph $\rightarrow (-x, y)$ on graph
(from y-axis symmetry)
$(-x, y)$ on graph $\rightarrow (x, -y)$ on graph
(from origin symmetry)
Since the point $(x, -y)$ is also on the graph, the graph has x-axis symmetry.

93. Answers may vary. The graph must contain the points $(-2,5)$, $(-1,3)$, and $(0,2)$. For the graph to be symmetric about the y-axis, the graph must also contain the points $(2,5)$ and $(1,3)$ (note that (0, 2) is on the y-axis).

For the graph to also be symmetric with respect to the x-axis, the graph must also contain the points $(-2,-5)$, $(-1,-3)$, $(0,-2)$, $(2,-5)$, and $(1,-3)$. Recall that a graph with two of the symmetries (x-axis, y-axis, origin) will necessarily have the third. Therefore, if the original graph with y-axis symmetry also has x-axis symmetry, then it will also have origin symmetry.

Section 1.3

1. undefined; 0

2. 3; 2
x-intercept: $2x+3(0)=6$
$$2x = 6$$
$$x = 3$$
y-intercept: $2(0)+3y=6$
$$3y = 6$$
$$y = 2$$

3. True

4. False; the slope is $\frac{3}{2}$.
$$2y = 3x+5$$
$$y = \frac{3}{2}x + \frac{5}{2}$$

5. True; $2(1)+(2)\stackrel{?}{=}4$
$$2+2\stackrel{?}{=}4$$
$$4 = 4 \text{ True}$$

6. $m_1 = m_2$; y-intercepts; $m_1 \cdot m_2 = -1$

7. 2

8. $-\frac{1}{2}$

9. c

10. d

11. b

12. d

13. **a.** $\text{Slope} = \frac{1-0}{2-0} = \frac{1}{2}$

b. If x increases by 2 units, y will increase by 1 unit.

14. **a.** $\text{Slope} = \frac{1-0}{-2-0} = -\frac{1}{2}$

b. If x increases by 2 units, y will decrease by 1 unit.

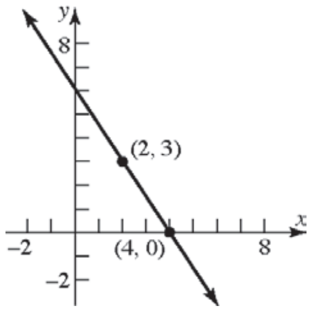
15. **a.** $\text{Slope} = \frac{1-2}{1-(-2)} = -\frac{1}{3}$

b. If x increases by 3 units, y will decrease by 1 unit.

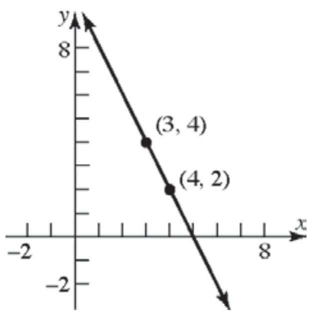
16. **a.** $\text{Slope} = \frac{2-1}{2-(-1)} = \frac{1}{3}$

b. If x increases by 3 units, y will increase by 1 unit.

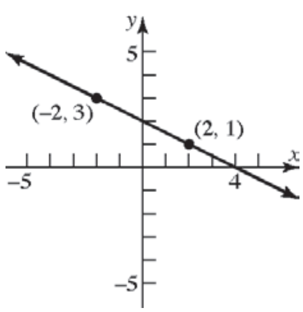
17. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-3}{4-2} = -\dfrac{3}{2}$



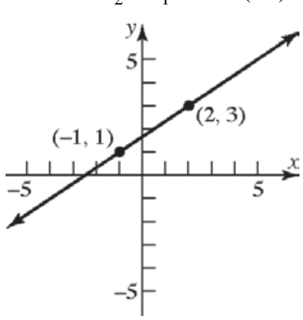
18. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-2}{3-4} = \dfrac{2}{-1} = -2$



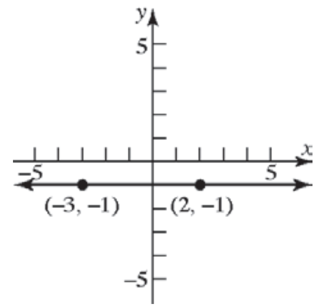
19. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-3}{2-(-2)} = \dfrac{-2}{4} = -\dfrac{1}{2}$



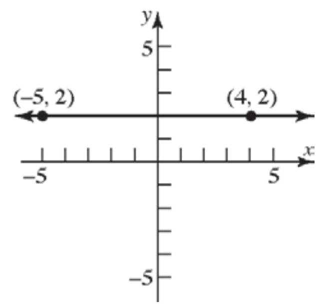
20. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-1}{2-(-1)} = \dfrac{2}{3}$



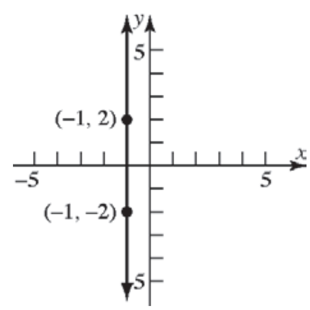
21. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-(-1)}{2-(-3)} = \dfrac{0}{5} = 0$



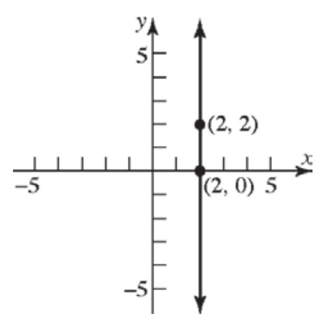
22. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-2}{-5-4} = \dfrac{0}{-9} = 0$



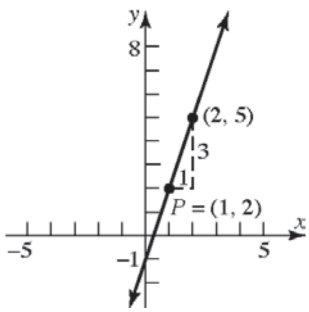
23. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-2}{-1-(-1)} = \dfrac{-4}{0}$ undefined.



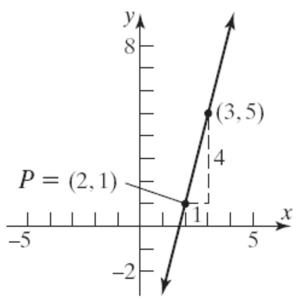
24. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-0}{2-2} = \dfrac{2}{0}$ undefined.



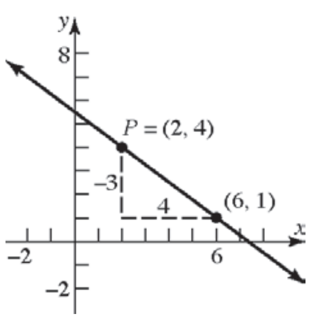
25. $P=(1,2); m=3$



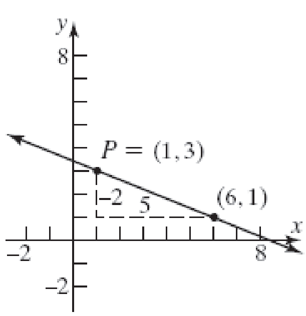
26. $P=(2,1); m=4$



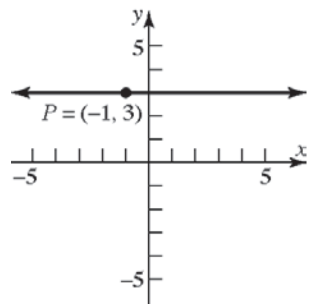
27. $P=(2,4); m=-\dfrac{3}{4}$



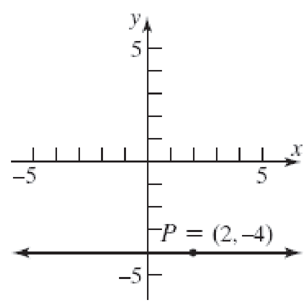
28. $P=(1,3); m=-\dfrac{2}{5}$



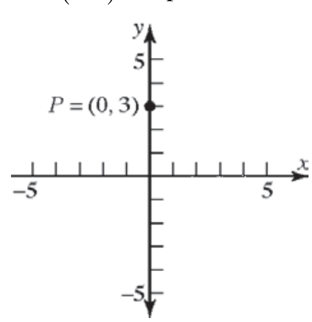
29. $P=(-1,3); m=0$



30. $P=(2,-4); m=0$

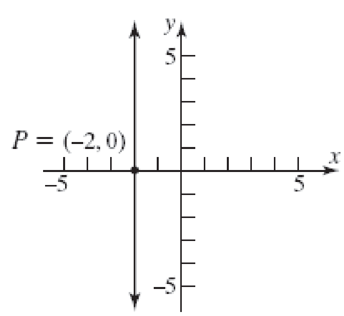


31. $P=(0,3)$; slope undefined



(note: the line is the y-axis)

32. $P=(-2,0)$; slope undefined



33. $P=(1,2); m=3$; $y-2=3(x-1)$

34. $P=(2,1); m=4$ **;** $y-1=4(x-2)$

35. $P=(2,4); m=-\frac{3}{4}$ **;** $y-4=-\frac{3}{4}(x-2)$

36. $P=(1,3); m=-\frac{2}{5}$ **;** $y-3=-\frac{2}{5}(x-1)$

37. $P=(-1,3); m=0$ **;** $y-3=0$

38. $P=(2,-4); m=0$ **;** $y+4=0$

39. Slope $=4=\frac{4}{1}$; point: $(1,2)$

If x increases by 1 unit, then y increases by 4 units.
Answers will vary. Three possible points are:
$x=1+1=2$ and $y=2+4=6$
$(2,6)$
$x=2+1=3$ and $y=6+4=10$
$(3,10)$
$x=3+1=4$ and $y=10+4=14$
$(4,14)$

40. Slope $=2=\frac{2}{1}$; point: $(-2,3)$

If x increases by 1 unit, then y increases by 2 units.
Answers will vary. Three possible points are:
$x=-2+1=-1$ and $y=3+2=5$
$(-1,5)$
$x=-1+1=0$ and $y=5+2=7$
$(0,7)$
$x=0+1=1$ and $y=7+2=9$
$(1,9)$

41. Slope $=-\frac{3}{2}=\frac{-3}{2}$; point: $(2,-4)$

If x increases by 2 units, then y decreases by 3 units.
Answers will vary. Three possible points are:
$x=2+2=4$ and $y=-4-3=-7$
$(4,-7)$
$x=4+2=6$ and $y=-7-3=-10$
$(6,-10)$
$x=6+2=8$ and $y=-10-3=-13$
$(8,-13)$

42. Slope $=\frac{4}{3}$; point: $(-3,2)$

If x increases by 3 units, then y increases by 4 units.
Answers will vary. Three possible points are:
$x=-3+3=0$ and $y=2+4=6$
$(0,6)$
$x=0+3=3$ and $y=6+4=10$
$(3,10)$
$x=3+3=6$ and $y=10+4=14$
$(6,14)$

43. Slope $=-2=\frac{-2}{1}$; point: $(-2,-3)$

If x increases by 1 unit, then y decreases by 2 units.
Answers will vary. Three possible points are:
$x=-2+1=-1$ and $y=-3-2=-5$
$(-1,-5)$
$x=-1+1=0$ and $y=-5-2=-7$
$(0,-7)$
$x=0+1=1$ and $y=-7-2=-9$
$(1,-9)$

44. Slope $=-1=\frac{-1}{1}$; point: $(4,1)$

If x increases by 1 unit, then y decreases by 1 unit.
Answers will vary. Three possible points are:
$x=4+1=5$ and $y=1-1=0$
$(5,0)$
$x=5+1=6$ and $y=0-1=-1$
$(6,-1)$
$x=6+1=7$ and $y=-1-1=-2$
$(7,-2)$

45. (0, 0) and (2, 1) are points on the line.

$$\text{Slope} = \frac{1-0}{2-0} = \frac{1}{2}$$

y-intercept is 0; using $y = mx + b$:

$$\begin{aligned} y &= \frac{1}{2}x + 0 \\ 2y &= x \\ 0 &= x - 2y \end{aligned}$$

$x - 2y = 0$ or $y = \frac{1}{2}x$

46. (0, 0) and (–2, 1) are points on the line.

$$\text{Slope} = \frac{1-0}{-2-0} = \frac{1}{-2} = -\frac{1}{2}$$

y-intercept is 0; using $y = mx + b$:

$$\begin{aligned} y &= -\frac{1}{2}x + 0 \\ 2y &= -x \\ x + 2y &= 0 \end{aligned}$$

$x + 2y = 0$ or $y = -\frac{1}{2}x$

47. (–1, 3) and (1, 1) are points on the line.

$$\text{Slope} = \frac{1-3}{1-(-1)} = \frac{-2}{2} = -1$$

Using $y - y_1 = m(x - x_1)$

$$\begin{aligned} y - 1 &= -1(x-1) \\ y - 1 &= -x + 1 \\ y &= -x + 2 \end{aligned}$$

$x + y = 2$ or $y = -x + 2$

48. (–1, 1) and (2, 2) are points on the line.

$$\text{Slope} = \frac{2-1}{2-(-1)} = \frac{1}{3}$$

Using $y - y_1 = m(x - x_1)$

$$\begin{aligned} y - 1 &= \frac{1}{3}\big(x - (-1)\big) \\ y - 1 &= \frac{1}{3}(x+1) \\ y - 1 &= \frac{1}{3}x + \frac{1}{3} \\ y &= \frac{1}{3}x + \frac{4}{3} \end{aligned}$$

$x - 3y = -4$ or $y = \frac{1}{3}x + \frac{4}{3}$

49. $y - y_1 = m(x - x_1),\ \ m = 2$

$$\begin{aligned} y - 3 &= 2(x-3) \\ y - 3 &= 2x - 6 \\ y &= 2x - 3 \end{aligned}$$

$2x - y = 3$ or $y = 2x - 3$

50. $y - y_1 = m(x - x_1),\ \ m = -1$

$$\begin{aligned} y - 2 &= -1(x-1) \\ y - 2 &= -x + 1 \\ y &= -x + 3 \end{aligned}$$

$x + y = 3$ or $y = -x + 3$

51. $y - y_1 = m(x - x_1),\ \ m = -\frac{1}{2}$

$$\begin{aligned} y - 2 &= -\frac{1}{2}(x-1) \\ y - 2 &= -\frac{1}{2}x + \frac{1}{2} \\ y &= -\frac{1}{2}x + \frac{5}{2} \end{aligned}$$

$x + 2y = 5$ or $y = -\frac{1}{2}x + \frac{5}{2}$

52. $y - y_1 = m(x - x_1),\ \ m = 1$

$$\begin{aligned} y - 1 &= 1(x - (-1)) \\ y - 1 &= x + 1 \\ y &= x + 2 \end{aligned}$$

$x - y = -2$ or $y = x + 2$

53. Slope = 3; containing (–2, 3)

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= 3(x - (-2)) \\ y - 3 &= 3x + 6 \\ y &= 3x + 9 \end{aligned}$$

$3x - y = -9$ or $y = 3x + 9$

54. Slope = 2; containing the point (4, –3)

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-3) &= 2(x-4) \\ y + 3 &= 2x - 8 \\ y &= 2x - 11 \end{aligned}$$

$2x - y = 11$ or $y = 2x - 11$

55. Slope $=\frac{1}{2}$; containing the point (3, 1)

$$y-y_1=m(x-x_1)$$
$$y-1=\frac{1}{2}(x-3)$$
$$y-1=\frac{1}{2}x-\frac{3}{2}$$
$$y=\frac{1}{2}x-\frac{1}{2}$$
$$x-2y=1 \text{ or } y=\frac{1}{2}x-\frac{1}{2}$$

56. Slope $=-\frac{2}{3}$; containing (1, –1)

$$y-y_1=m(x-x_1)$$
$$y-(-1)=-\frac{2}{3}(x-1)$$
$$y+1=-\frac{2}{3}x+\frac{2}{3}$$
$$y=-\frac{2}{3}x-\frac{1}{3}$$
$$2x+3y=-1 \text{ or } y=-\frac{2}{3}x-\frac{1}{3}$$

57. Containing (1, 3) and (–1, 2)

$$m=\frac{2-3}{-1-1}=\frac{-1}{-2}=\frac{1}{2}$$
$$y-y_1=m(x-x_1)$$
$$y-3=\frac{1}{2}(x-1)$$
$$y-3=\frac{1}{2}x-\frac{1}{2}$$
$$y=\frac{1}{2}x+\frac{5}{2}$$
$$x-2y=-5 \text{ or } y=\frac{1}{2}x+\frac{5}{2}$$

58. Containing the points (–3, 4) and (2, 5)

$$m=\frac{5-4}{2-(-3)}=\frac{1}{5}$$
$$y-y_1=m(x-x_1)$$
$$y-5=\frac{1}{5}(x-2)$$
$$y-5=\frac{1}{5}x-\frac{2}{5}$$
$$y=\frac{1}{5}x+\frac{23}{5}$$
$$x-5y=-23 \text{ or } y=\frac{1}{5}x+\frac{23}{5}$$

59. Slope = –3; y-intercept =3

$$y=mx+b$$
$$y=-3x+3$$
$$3x+y=3 \text{ or } y=-3x+3$$

60. Slope = –2; y-intercept = –2

$$y=mx+b$$
$$y=-2x+(-2)$$
$$2x+y=-2 \text{ or } y=-2x-2$$

61. x-intercept = –4; y-intercept = 4
Points are (–4, 0) and (0, 4)

$$m=\frac{4-0}{0-(-4)}=\frac{4}{4}=1$$
$$y=mx+b$$
$$y=1x+4$$
$$y=x+4$$
$$x-y=-4 \text{ or } y=x+4$$

62. x-intercept = 2; y-intercept = –1
Points are (2,0) and (0,–1)

$$m=\frac{-1-0}{0-2}=\frac{-1}{-2}=\frac{1}{2}$$
$$y=mx+b$$
$$y=\frac{1}{2}x-1$$
$$x-2y=2 \text{ or } y=\frac{1}{2}x-1$$

63. Slope undefined; containing the point (2, 4)
This is a vertical line.
$x=2$ No slope-intercept form.

64. Slope undefined; containing the point (3, 8)
This is a vertical line.
$x = 3$ No slope-intercept form.

65. Horizontal lines have slope $m = 0$ and take the form $y = b$. Therefore, the horizontal line passing through the point $(-3, 2)$ is $y = 2$.

66. Vertical lines have an undefined slope and take the form $x = a$. Therefore, the vertical line passing through the point $(4, -5)$ is $x = 4$.

67. Parallel to $y = 2x$; Slope = 2
Containing (–1, 2)

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 2(x - (-1))$$
$$y - 2 = 2x + 2 \rightarrow y = 2x + 4$$
$$2x - y = -4 \text{ or } y = 2x + 4$$

68. Parallel to $y = -3x$; Slope = –3; Containing the point (–1, 2)

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -3(x - (-1))$$
$$y - 2 = -3x - 3 \rightarrow y = -3x - 1$$
$$3x + y = -1 \text{ or } y = -3x - 1$$

69. Parallel to $x - 2y = -5$;

Slope $= \frac{1}{2}$; Containing the point $(0, 0)$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{1}{2}(x - 0) \rightarrow y = \frac{1}{2}x$$
$$x - 2y = 0 \text{ or } y = \frac{1}{2}x$$

70. Parallel to $2x - y = -2$; Slope = 2
Containing the point (0, 0)

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 2(x - 0)$$
$$y = 2x$$
$$2x - y = 0 \text{ or } y = 2x$$

71. Parallel to $x = 5$; Containing (4,2)
This is a vertical line.
$x = 4$ No slope-intercept form.

72. Parallel to $y = 5$; Containing the point (4, 2)
This is a horizontal line. Slope = 0
$y = 2$

73. Perpendicular to $y = \frac{1}{2}x + 4$; Containing (1, –2)
Slope of perpendicular = –2

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -2(x - 1)$$
$$y + 2 = -2x + 2 \rightarrow y = -2x$$
$$2x + y = 0 \text{ or } y = -2x$$

74. Perpendicular to $y = 2x - 3$; Containing the point (1, –2)

Slope of perpendicular $= -\frac{1}{2}$

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -\frac{1}{2}(x - 1)$$
$$y + 2 = -\frac{1}{2}x + \frac{1}{2} \rightarrow y = -\frac{1}{2}x - \frac{3}{2}$$
$$x + 2y = -3 \text{ or } y = -\frac{1}{2}x - \frac{3}{2}$$

75. Perpendicular to $x - 2y = -5$; Containing the point (0, 4)
Slope of perpendicular = –2

$$y = mx + b$$
$$y = -2x + 4$$
$$2x + y = 4 \text{ or } y = -2x + 4$$

76. Perpendicular to $2x + y = 2$; Containing the point (–3, 0)

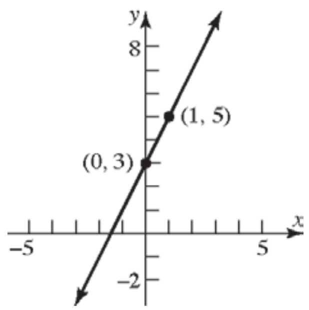
Slope of perpendicular $= \frac{1}{2}$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{1}{2}(x - (-3)) \rightarrow y = \frac{1}{2}x + \frac{3}{2}$$
$$x - 2y = -3 \text{ or } y = \frac{1}{2}x + \frac{3}{2}$$

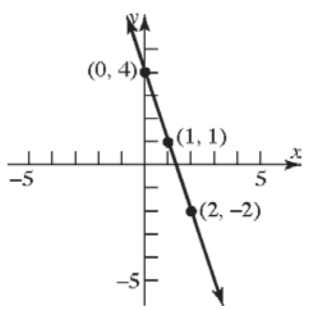
77. Perpendicular to $x = 8$; Containing (3, 4)
Slope of perpendicular = 0 (horizontal line)
$y = 4$

78. Perpendicular to $y = 8$;
Containing the point (3, 4)
Slope of perpendicular is undefined (vertical line). $x = 3$ No slope-intercept form.

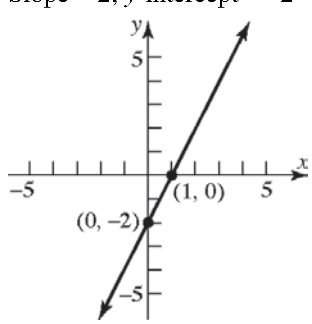
79. $y = 2x + 3$; Slope = 2; y-intercept = 3



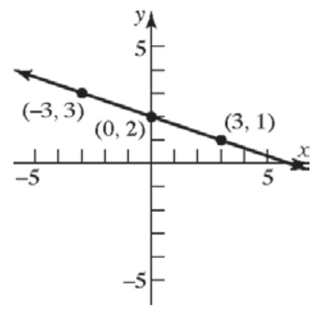
80. $y = -3x + 4$; Slope = –3; y-intercept = 4



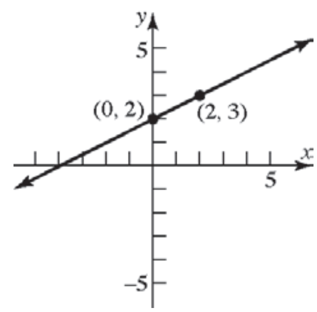
81. $\frac{1}{2}y = x - 1$; $y = 2x - 2$
Slope = 2; y-intercept = –2



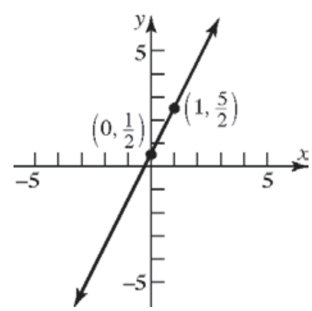
82. $\frac{1}{3}x + y = 2$; $y = -\frac{1}{3}x + 2$
Slope $= -\frac{1}{3}$; y-intercept = 2



83. $y = \frac{1}{2}x + 2$; Slope $= \frac{1}{2}$; y-intercept = 2

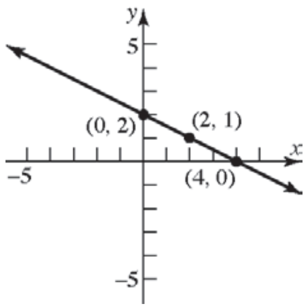


84. $y = 2x + \frac{1}{2}$; Slope = 2; y-intercept $= \frac{1}{2}$



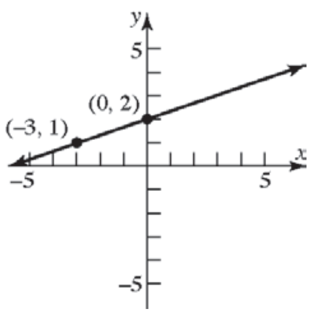
85. $x+2y=4$; $2y=-x+4 \rightarrow y=-\dfrac{1}{2}x+2$

Slope $=-\dfrac{1}{2}$; y-intercept = 2



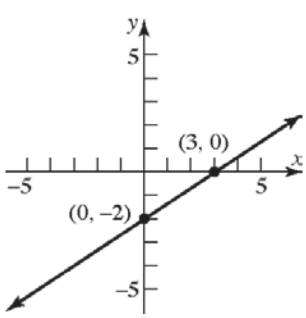
86. $-x+3y=6$; $3y=x+6 \rightarrow y=\dfrac{1}{3}x+2$

Slope $=\dfrac{1}{3}$; y-intercept = 2



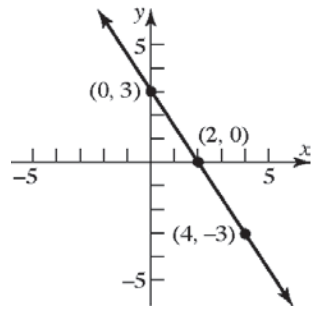
87. $2x-3y=6$; $-3y=-2x+6 \rightarrow y=\dfrac{2}{3}x-2$

Slope $=\dfrac{2}{3}$; y-intercept = –2



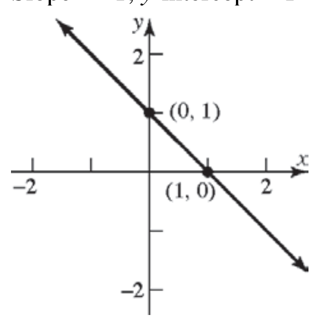
88. $3x+2y=6$; $2y=-3x+6 \rightarrow y=-\dfrac{3}{2}x+3$

Slope $=-\dfrac{3}{2}$; y-intercept = 3



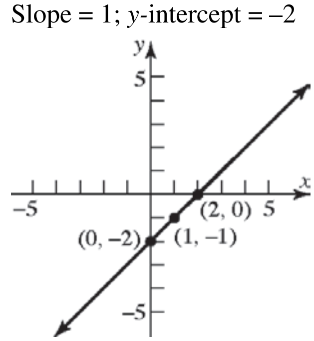
89. $x+y=1$; $y=-x+1$

Slope = –1; y-intercept = 1



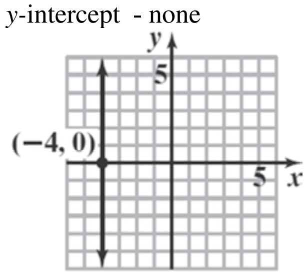
90. $x-y=2$; $y=x-2$

Slope = 1; y-intercept = –2

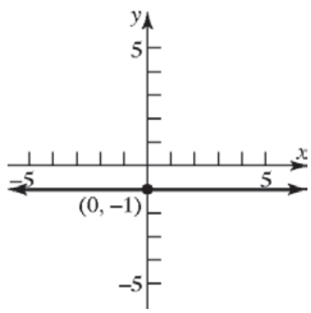


91. $x=-4$; Slope is undefined

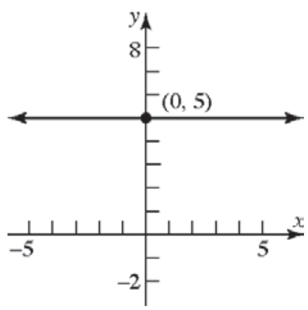
y-intercept - none



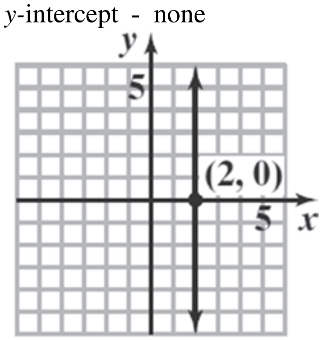
92. $y=-1$; Slope = 0; y-intercept $=-1$



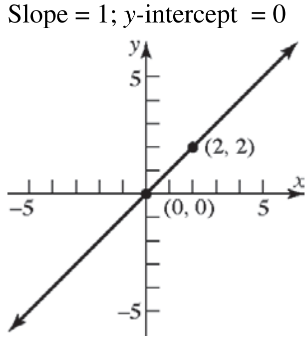
93. $y=5$; Slope = 0; y-intercept = 5



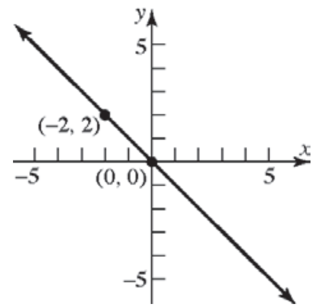
94. $x=2$; Slope is undefined
y-intercept - none



95. $y-x=0$; $y=x$
Slope = 1; y-intercept $=0$

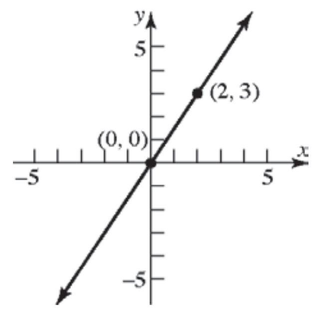


96. $x+y=0$; $y=-x$
Slope $=-1$; y-intercept $=0$



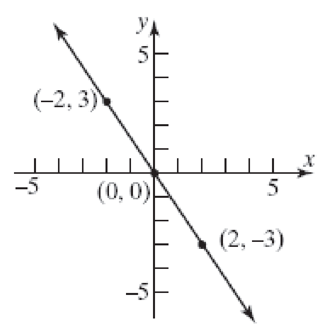
97. $2y-3x=0$; $2y=3x \to y=\frac{3}{2}x$

Slope $=\frac{3}{2}$; y-intercept = 0



98. $3x+2y=0$; $2y=-3x \to y=-\frac{3}{2}x$

Slope $=-\frac{3}{2}$; y-intercept = 0



99. **a.** x-intercept: $2x+3(0)=6$

$$2x=6$$
$$x=3$$

The point $(3,0)$ is on the graph.

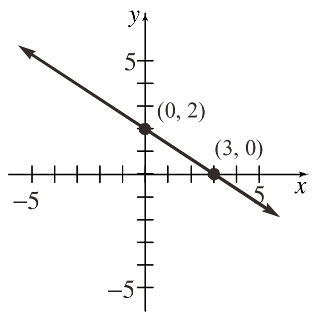
y-intercept: $2(0)+3y=6$

$3y=6$

$y=2$

The point $(0,2)$ is on the graph.

b.



100. a. x-intercept: $3x-2(0)=6$

$3x=6$

$x=2$

The point $(2,0)$ is on the graph.

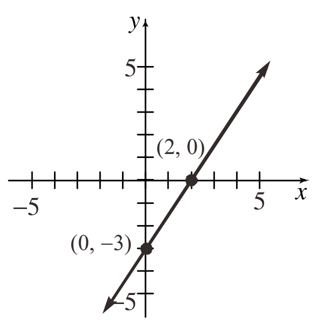
y-intercept: $3(0)-2y=6$

$-2y=6$

$y=-3$

The point $(0,-3)$ is on the graph.

b.



101. a. x-intercept: $-4x+5(0)=40$

$-4x=40$

$x=-10$

The point $(-10,0)$ is on the graph.

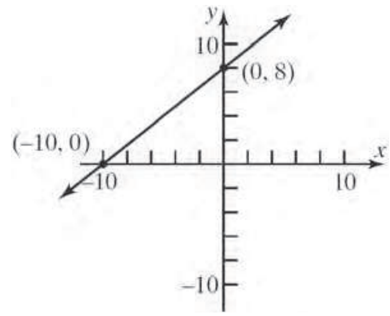
y-intercept: $-4(0)+5y=40$

$5y=40$

$y=8$

The point $(0,8)$ is on the graph.

b.



102. a. x-intercept: $6x-4(0)=24$

$6x=24$

$x=4$

The point $(4,0)$ is on the graph.

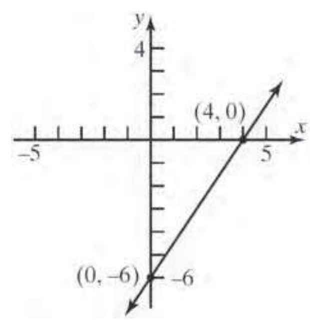
y-intercept: $6(0)-4y=24$

$-4y=24$

$y=-6$

The point $(0,-6)$ is on the graph.

b.



103. a. x-intercept: $7x+2(0)=21$

$7x=21$

$x=3$

The point $(3,0)$ is on the graph.

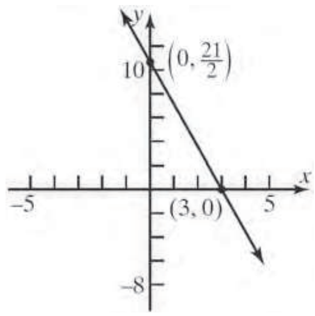
y-intercept: $7(0)+2y=21$

$2y=21$

$y=\frac{21}{2}$

The point $\left(0,\frac{21}{2}\right)$ is on the graph.

b.



104. a. x-intercept: $5x + 3(0) = 18$

$$5x = 18$$

$$x = \frac{18}{5}$$

The point $\left(\frac{18}{5}, 0\right)$ is on the graph.

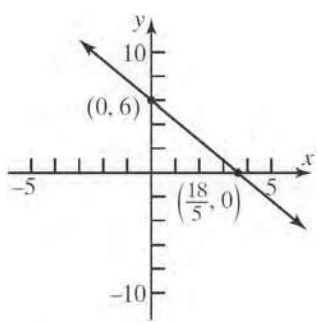
y-intercept: $5(0) + 3y = 18$

$$3y = 18$$

$$y = 6$$

The point $(0, 6)$ is on the graph.

b.



105. a. x-intercept: $\frac{1}{2}x + \frac{1}{3}(0) = 1$

$$\frac{1}{2}x = 1$$

$$x = 2$$

The point $(2, 0)$ is on the graph.

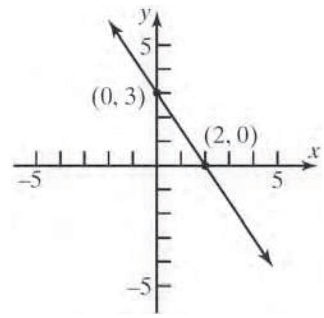
y-intercept: $\frac{1}{2}(0) + \frac{1}{3}y = 1$

$$\frac{1}{3}y = 1$$

$$y = 3$$

The point $(0, 3)$ is on the graph.

b.



106. a. x-intercept: $x - \frac{2}{3}(0) = 4$

$$x = 4$$

The point $(4, 0)$ is on the graph.

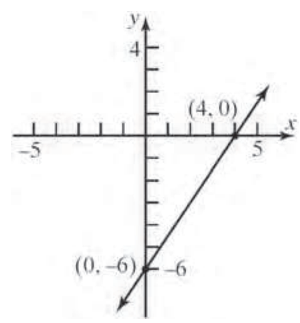
y-intercept: $(0) - \frac{2}{3}y = 4$

$$-\frac{2}{3}y = 4$$

$$y = -6$$

The point $(0, -6)$ is on the graph.

b.



107. a. x-intercept: $0.2x - 0.5(0) = 1$

$$0.2x = 1$$

$$x = 5$$

The point $(5, 0)$ is on the graph.

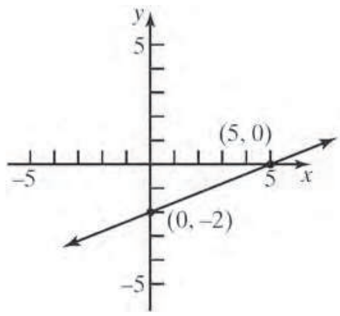
y-intercept: $0.2(0) - 0.5y = 1$

$$-0.5y = 1$$

$$y = -2$$

The point $(0, -2)$ is on the graph.

b.



108. **a.** x-intercept: $-0.3x + 0.4(0) = 1.2$

$$-0.3x = 1.2$$
$$x = -4$$

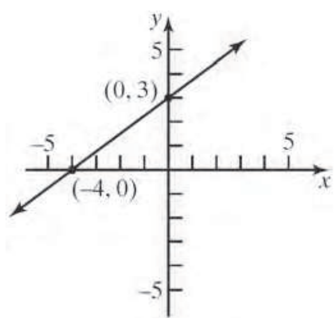
The point $(-4, 0)$ is on the graph.

y-intercept: $-0.3(0) + 0.4y = 1.2$

$$0.4y = 1.2$$
$$y = 3$$

The point $(0, 3)$ is on the graph.

b.



109. The equation of the x-axis is $y = 0$. (The slope is 0 and the y-intercept is 0.)

110. The equation of the y-axis is $x = 0$. (The slope is undefined.)

111. The slopes are the same but the y-intercepts are different. Therefore, the two lines are parallel.

112. The slopes are opposite-reciprocals. That is, their product is -1. Therefore, the lines are perpendicular.

113. The slopes are different and their product does not equal -1. Therefore, the lines are neither parallel nor perpendicular.

114. The slopes are different and their product does not equal -1 (in fact, the signs are the same so the product is positive). Therefore, the lines are neither parallel nor perpendicular.

115. Intercepts: $(0,2)$ and $(-2,0)$. Thus, slope = 1.
$y = x + 2$ or $x - y = -2$

116. Intercepts: $(0,1)$ and $(1,0)$. Thus, slope = –1.
$y = -x + 1$ or $x + y = 1$

117. Intercepts: $(3,0)$ and $(0,1)$. Thus, slope = $-\frac{1}{3}$.

$y = -\frac{1}{3}x + 1$ or $x + 3y = 3$

118. Intercepts: $(0,-1)$ and $(-2,0)$. Thus,

slope = $-\frac{1}{2}$.

$y = -\frac{1}{2}x - 1$ or $x + 2y = -2$

119. $P_1 = (-2,5)$, $P_2 = (1,3)$: $m_1 = \frac{5-3}{-2-1} = \frac{2}{-3} = -\frac{2}{3}$

$P_2 = (1,3)$, $P_3 = (-1,0)$: $m_2 = \frac{3-0}{1-(-1)} = \frac{3}{2}$

Since $m_1 \cdot m_2 = -1$, the line segments $\overline{P_1P_2}$ and $\overline{P_2P_3}$ are perpendicular. Thus, the points P_1, P_2, and P_3 are vertices of a right triangle.

120. $P_1 = (1,-1)$, $P_2 = (4,1)$, $P_3 = (2,2)$, $P_4 = (5,4)$

$m_{12} = \frac{1-(-1)}{4-1} = \frac{2}{3}$; $m_{24} = \frac{4-1}{5-4} = 3$;

$m_{34} = \frac{4-2}{5-2} = \frac{2}{3}$; $m_{13} = \frac{2-(-1)}{2-1} = 3$

Each pair of opposite sides are parallel (same slope) and adjacent sides are not perpendicular. Therefore, the vertices are for a parallelogram.

121. $P_1 = (-1,0)$, $P_2 = (2,3)$, $P_3 = (1,-2)$, $P_4 = (4,1)$

$m_{12} = \frac{3-0}{2-(-1)} = \frac{3}{3} = 1$; $m_{24} = \frac{1-3}{4-2} = -1$;

$m_{34} = \frac{1-(-2)}{4-1} = \frac{3}{3} = 1$; $m_{13} = \frac{-2-0}{1-(-1)} = -1$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). Therefore, the vertices are for a rectangle.

122. $P_1=(0,0)$, $P_2=(1,3)$, $P_3=(4,2)$, $P_4=(3,-1)$

$m_{12}=\dfrac{3-0}{1-0}=3$; $m_{23}=\dfrac{2-3}{4-1}=-\dfrac{1}{3}$;

$m_{34}=\dfrac{-1-2}{3-4}=3$; $m_{14}=\dfrac{-1-0}{3-0}=-\dfrac{1}{3}$

$d_{12}=\sqrt{(1-0)^2+(3-0)^2}=\sqrt{1+9}=\sqrt{10}$

$d_{23}=\sqrt{(4-1)^2+(2-3)^2}=\sqrt{9+1}=\sqrt{10}$

$d_{34}=\sqrt{(3-4)^2+(-1-2)^2}=\sqrt{1+9}=\sqrt{10}$

$d_{14}=\sqrt{(3-0)^2+(-1-0)^2}=\sqrt{9+1}=\sqrt{10}$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). In addition, the length of all four sides is the same. Therefore, the vertices are for a square.

123. Let x = number of miles driven, and let C = cost in dollars.
Total cost = (cost per mile)(number of miles) + fixed cost
$C=0.60x+39$
When $x=110$, $C=(0.60)(110)+39=\$105.00$.
When $x=230$, $C=(0.60)(230)+39=\$177.00$.

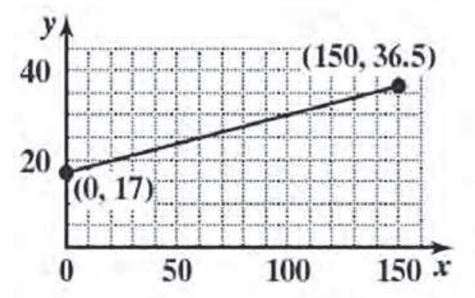
124. Let x = number of pairs of jeans manufactured, and let C = cost in dollars.
Total cost = (cost per pair)(number of pairs) + fixed cost
$C=20x+1200$
When $x=400$, $C=(20)(400)+1200=\$9200$.
When $x=740$, $C=(20)(740)+1200=\$16,000$.

125. Let x = number of miles driven annually, and let C = cost in dollars.
Total cost = (approx cost per mile)(number of miles) + fixed cost
$C=0.28x+6578$

126. Let x = profit in dollars, and let S = salary in dollars.
Weekly salary = (% share of profit)(profit) + weekly pay
$S=0.05x+525$

127. **a.** $C=0.13x+17$; $0\le x\le 1000$

b.



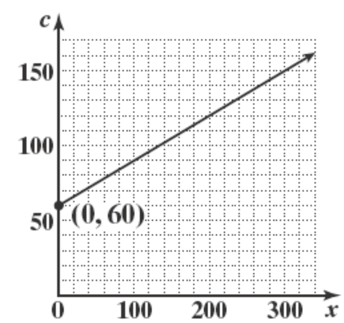
c. For 50 miles,
$C=0.13(50)+17=\$23.50$

d. For 120 miles,
$C=0.13(120)+17=\$32.60$

e. For every 1-mile in distance traveled, the cost will increase 13 cents.

128. **a.** $C=60+0.25x$

b.



c. For 20 minutes,
$C=60+0.25(20)=\$65.00$

d. For 60 minutes,
$C=60+0.25(60)=\$75.00$

e. For every 1-minute increase in international calls the cost will increase by \$0.25 (that is, 25 cents).

129. $(°C, °F) = (0, 32)$; $(°C, °F) = (100, 212)$

$$\text{slope} = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$$

$$°F - 32 = \frac{9}{5}(°C - 0)$$

$$°F - 32 = \frac{9}{5}(°C)$$

$$°C = \frac{5}{9}(°F - 32)$$

If $°F = 70$, then

$$°C = \frac{5}{9}(70-32) = \frac{5}{9}(38)$$

$$°C \approx 21.1°$$

130. **a.** $K = °C + 273$

b. $°C = \frac{5}{9}(°F - 32)$

$$K = \frac{5}{9}(°F - 32) + 273$$

$$K = \frac{5}{9}°F - \frac{160}{9} + 273$$

$$K = \frac{5}{9}°F + \frac{2297}{9}$$

131. **a.** The y-intercept is (0, 30), so $b = 30$. Since the ramp drops 2 inches for every 25 inches of run, the slope is $m = \frac{-2}{25} = -\frac{2}{25}$. Thus, the equation is $y = -\frac{2}{25}x + 30$.

b. Let $y = 0$.

$$0 = -\frac{2}{25}x + 30$$

$$\frac{2}{25}x = 30$$

$$\frac{25}{2}\left(\frac{2}{25}x\right) = \frac{25}{2}(30)$$

$$x = 375$$

The x-intercept is (375, 0). This means that the ramp meets the floor 375 inches (or 31.25 feet) from the base of the platform.

c. No. From part (b), the run is 31.25 feet which exceeds the required maximum of 30 feet.

d. First, design requirements state that the maximum slope is a drop of 1 inch for each 12 inches of run. This means $|m| \le \frac{1}{12}$.

Second, the run is restricted to be no more than 30 feet = 360 inches. For a rise of 30 inches, this means the minimum slope is $\frac{30}{360} = \frac{1}{12}$. That is, $|m| \ge \frac{1}{12}$. Thus, the only possible slope is $|m| = \frac{1}{12}$. The diagram indicates that the slope is negative. Therefore, the only slope that can be used to obtain the 30-inch rise and still meet design requirements is $m = -\frac{1}{12}$. In words, for every 12 inches of run, the ramp must drop *exactly* 1 inch.

132. **a.** Let x represent the percent of internet ad spending. Let y represent the percent of print ad spending. Then the points (0.19, 0.26) and (0.35, 0.16) are on the line.

Thus, $m = \frac{16-26}{35-19} = -\frac{10}{16} = -0.625$. Using the point-slope formula we have

$$y - 26 = -0.625(x - 19)$$

$$y - 26 = -0.625x + 11.875$$

$$y = -0.625x + 37.875$$

b. x-intercept: $0 = -0.625x + 37.875$

$$-37.875 = -0.625x$$

$$60.6 = x$$

y-intercept: $y = -0.625(0) + 37.875$

$$= 37.875$$

The intercepts are (60.6, 0) and (0, 37.875).

c. y-intercept: When Internet ads account for 0% of U.S. advertisement spending, print ads account for 37.875% of the spending.
x-intercept: When Internet ads account for 60.6% of U.S. advertisement spending, print ads account for 0% of the spending.

d. Let $x = 39.2$.

$$y = -0.625(39.2) + 37.875 = 13.4\%$$

133. **a.** Let x = number of boxes to be sold, and A = money, in dollars, spent on advertising. We have the points

$$(x_1, A_1) = (100{,}000,\ 40{,}000);$$

$(x_2, A_2) = (200{,}000, 60{,}000)$

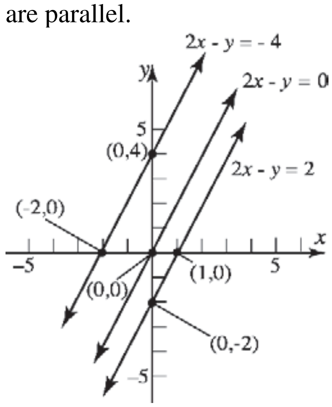
$$\text{slope} = \frac{60{,}000-40{,}000}{200{,}000-100{,}000} = \frac{20{,}000}{100{,}000} = \frac{1}{5}$$

$$A - 40{,}000 = \frac{1}{5}(x - 100{,}000)$$
$$A - 40{,}000 = \frac{1}{5}x - 20{,}000$$
$$A = \frac{1}{5}x + 20{,}000$$

b. If $x = 300{,}000$, then
$$A = \frac{1}{5}(300{,}000) + 20{,}000 = \$80{,}000$$

c. Each additional box sold requires an additional \$0.20 in advertising.

134. $2x - y = C$
Graph the lines:
$2x - y = -4$
$2x - y = 0$
$2x - y = 2$
All the lines have the same slope, 2. The lines are parallel.



135. Put each linear equation in slope/intercept form.

$x + 2y = 5 \qquad 2x - 3y + 4 = 0 \qquad ax + y = 0$
$2y = -x + 5 \qquad -3y = -2x - 4 \qquad y = -ax$
$y = -\frac{1}{2}x + \frac{5}{2} \qquad y = \frac{2}{3}x + \frac{4}{3}$

If the slope of $y = -ax$ equals the slope of either of the other two lines, then no triangle is formed.
So, $-a = -\frac{1}{2} \Rightarrow a = \frac{1}{2}$ and $-a = \frac{2}{3} \Rightarrow a = -\frac{2}{3}$.
Also if all three lines intersect at a single point, then no triangle is formed. So, we find where $y = -\frac{1}{2}x + \frac{5}{2}$ and $y = \frac{2}{3}x + \frac{4}{3}$ intersect.

$$-\frac{1}{2}x + \frac{5}{2} = \frac{2}{3}x + \frac{4}{3}$$
$$-\frac{7}{6}x = -\frac{7}{6}$$
$$x = 1$$
$$-\frac{1}{2}(1) + \frac{5}{2} = 2$$

The two lines intersect at (1, 2). If $y = -ax$ also contains the point (1, 2), then
$2 = -a \cdot 1 \Rightarrow a = -2$.

The three numbers are $\frac{1}{2}, -\frac{2}{3}$, and -2.

136. The slope of the line containing (a,b) and (b,a) is
$$\frac{a-b}{b-a} = -1$$
The slope of the line $y = x$ is 1.

The two lines are perpendicular.

The midpoint of (a,b) and (b,a) is
$$M = \left(\frac{a+b}{2}, \frac{b+a}{2}\right).$$

Since the x and y coordinates of M are equal, M lies on the line $y = x$.

Note: $\frac{a+b}{2} = \frac{b+a}{2}$

137. The three midpoints are
$$\left(\frac{0+a}{2}, \frac{0+0}{2}\right) = \left(\frac{a}{2}, 0\right), \left(\frac{a+b}{2}, \frac{0+c}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$$

and $\left(\frac{0+b}{2}, \frac{0+c}{2}\right) = \left(\frac{b}{2}, \frac{c}{2}\right)$.

Line 1 from (0,0) to $\left(\frac{a+b}{2}, \frac{c}{2}\right)$

$$m = \frac{\frac{c}{2} - 0}{\frac{a+b}{2} - 0} = \frac{c}{a+b};$$

$$y - 0 = \frac{c}{a+b}(x - 0)$$

$$y = \frac{c}{a+b}x_1$$

Line 2 from (a, 0) to $\left(\frac{b}{2}, \frac{c}{2}\right)$

$$m = \frac{\frac{c}{2} - 0}{\frac{b}{2} - a} = \frac{\frac{c}{2}}{\frac{b-2a}{2}} = \frac{c}{b-2a}$$

$$y - 0 = \frac{c}{b-2a}(x - a)$$

$$y = \frac{c}{b-2a}(x - a)$$

Line 3 from $\left(\frac{a}{2}, 0\right)$ to (b, c)

$$m = \frac{c - 0}{b - \frac{a}{2}} = \frac{2c}{2b - a}$$

$$y - 0 = \frac{2c}{2b-a}\left(x - \frac{a}{2}\right)$$

$$y = \frac{2c}{2b-a}\left(x - \frac{a}{2}\right)$$

Find where line 1 and line 2 intersect:

$$\frac{c}{a+b}x = \frac{c}{b-2a}(x - a)$$

$$\frac{b-2a}{a+b}x = x - a$$

$$\frac{b - 2a - a - b}{a+b}x = -a$$

$$\frac{-3a}{a+b}x = -a$$

$$x = \frac{a+b}{3};$$

Substitute into line 1:

$$y = \frac{c}{a+b} \cdot \frac{a+b}{3} = \frac{c}{3}.$$

So, line 1 and line 2 intersect at $\left(\frac{a+b}{3}, \frac{c}{3}\right)$.

Show that line 3 contains the point $\left(\frac{a+b}{3}, \frac{c}{3}\right)$:

$$y = \frac{2c}{2b-a}\left(\frac{a+b}{3} - \frac{a}{2}\right) = \frac{2c}{2b-a} \cdot \frac{2b-a}{6} = \frac{c}{3}$$ So

the three lines intersect at $\left(\frac{a+b}{3}, \frac{c}{3}\right)$.

138. Refer to Figure 47. Assume $m_1 m_2 = -1$. Then

$$\begin{aligned}
[d(A,B)]^2 &= (1-1)^2 + (m_1 - m_2)^2 \\
&= (m_1 - m_2)^2 \\
&= m_1^2 - 2m_1 m_2 + m_2^2 \\
&= m_1^2 - 2(-1) + m_2^2 \\
&= m_1^2 + m_2^2 + 2
\end{aligned}$$

Now,

$$[d(O,B)]^2 = (1-0)^2 + (m_1 - 0)^2 = 1 + m_1^2$$

$$[d(O,A)]^2 = (1-0)^2 + (m_2 - 0)^2 = 1 + m_2^2,$$

So

$$\begin{aligned}
[d(O,B)]^2 + [d(O,A)]^2 &= 1 + m_1^2 + 1 + m_2^2 \\
&= m_1^2 + m_2^2 + 2 = [d(A,B)]^2
\end{aligned}$$

By the converse of the Pythagorean Theorem, ΔAOB is a right triangle with right angle at vertex O. Thus lines OA and OB are perpendicular.

139. (b), (c), (e) and (g)
The line has positive slope and positive *y*-intercept.

140. (a), (c), and (g)
The line has negative slope and positive *y*-intercept.

141. (c)
The equation $x - y = -2$ has slope 1 and *y*-intercept $(0, 2)$. The equation $x - y = 1$ has slope 1 and *y*-intercept $(0, -1)$. Thus, the lines are parallel with positive slopes. One line has a positive *y*-intercept and the other with a negative *y*-intercept.

142. (d)
The equation $y-2x=2$ has slope 2 and y-intercept $(0, 2)$. The equation $x+2y=-1$ has slope $-\frac{1}{2}$ and y-intercept $\left(0,-\frac{1}{2}\right)$. The lines are perpendicular since $2\left(-\frac{1}{2}\right)=-1$. One line has a positive y-intercept and the other with a negative y-intercept.

143 – 145. Answers will vary.

146. No, the equation of a vertical line cannot be written in slope-intercept form because the slope is undefined.

147. No, a line does not need to have both an x-intercept and a y-intercept. Vertical and horizontal lines have only one intercept (unless they are a coordinate axis). Every line must have at least one intercept.

148. Two lines with equal slopes and equal y-intercepts are coinciding lines (i.e. the same).

149. Two lines that have the same x-intercept and y-intercept (assuming the x-intercept is not 0) are the same line since a line is uniquely defined by two distinct points.

150. No. Two lines with the same slope and different x-intercepts are distinct parallel lines and have no points in common.
Assume Line 1 has equation $y=mx+b_1$ and Line 2 has equation $y=mx+b_2$,
Line 1 has x-intercept $-\frac{b_1}{m}$ and y-intercept b_1.
Line 2 has x-intercept $-\frac{b_2}{m}$ and y-intercept b_2.
Assume also that Line 1 and Line 2 have unequal x-intercepts.
If the lines have the same y-intercept, then $b_1=b_2$.

$$b_1=b_2 \Rightarrow \frac{b_1}{m}=\frac{b_2}{m} \Rightarrow -\frac{b_1}{m}=-\frac{b_2}{m}$$

But $-\frac{b_1}{m}=-\frac{b_2}{m} \Rightarrow$ Line 1 and Line 2 have the same x-intercept, which contradicts the original assumption that the lines have unequal x-intercepts. Therefore, Line 1 and Line 2 cannot have the same y-intercept.

151. Yes. Two distinct lines with the same y-intercept, but different slopes, can have the same x-intercept if the x-intercept is $x=0$.
Assume Line 1 has equation $y=m_1x+b$ and Line 2 has equation $y=m_2x+b$,
Line 1 has x-intercept $-\frac{b}{m_1}$ and y-intercept b.
Line 2 has x-intercept $-\frac{b}{m_2}$ and y-intercept b.
Assume also that Line 1 and Line 2 have unequal slopes, that is $m_1 \neq m_2$.
If the lines have the same x-intercept, then $-\frac{b}{m_1}=-\frac{b}{m_2}$.

$$\begin{aligned}
-\frac{b}{m_1}&=-\frac{b}{m_2}\\
-m_2b&=-m_1b\\
-m_2b+m_1b&=0
\end{aligned}$$

$$\begin{aligned}
\text{But } -m_2b+m_1b=0 &\Rightarrow b\left(m_1-m_2\right)=0\\
&\Rightarrow b=0\\
&\text{or } m_1-m_2=0 \Rightarrow m_1=m_2
\end{aligned}$$

Since we are assuming that $m_1 \neq m_2$, the only way that the two lines can have the same x-intercept is if $b=0$.

152. Answers will vary.

153. $m=\frac{y_2-y_1}{x_2-x_1}=\frac{-4-2}{1-(-3)}=\frac{-6}{4}=-\frac{3}{2}$

It appears that the student incorrectly found the slope by switching the direction of one of the subtractions.

Section 1.4

1. add; $\left(\frac{1}{2}\cdot 10\right)^2 = 25$

2. $(x-2)^2 = 9$
$$x-2 = \pm\sqrt{9}$$
$$x-2 = \pm 3$$
$$x = 2 \pm 3$$
$$x = 5 \quad \text{or} \quad x = -1$$
The solution set is $\{-1, 5\}$.

3. False. For example, $x^2 + y^2 + 2x + 2y + 8 = 0$ is not a circle. It has no real solutions.

4. radius

5. True; $r^2 = 9 \rightarrow r = 3$

6. False; the center of the circle $(x+3)^2 + (y-2)^2 = 13$ is $(-3, 2)$.

7. d

8. a

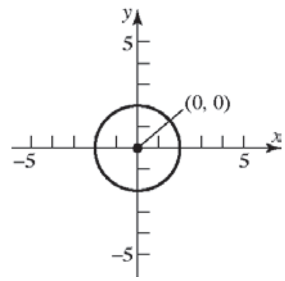
9. Center = (2, 1)
Radius = distance from (0,1) to (2,1)
$$= \sqrt{(2-0)^2 + (1-1)^2} = \sqrt{4} = 2$$
Equation: $(x-2)^2 + (y-1)^2 = 4$

10. Center = (1, 2)
Radius = distance from (1,0) to (1,2)
$$= \sqrt{(1-1)^2 + (2-0)^2} = \sqrt{4} = 2$$
Equation: $(x-1)^2 + (y-2)^2 = 4$

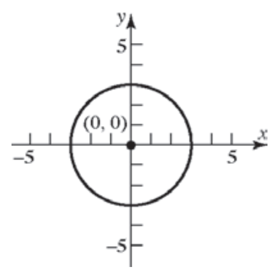
11. Center = midpoint of (1, 2) and (4, 2)
$$= \left(\frac{1+4}{2}, \frac{2+2}{2}\right) = \left(\frac{5}{2}, 2\right)$$
Radius = distance from $\left(\frac{5}{2}, 2\right)$ to (4,2)
$$= \sqrt{\left(4-\frac{5}{2}\right)^2 + (2-2)^2} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$
Equation: $\left(x-\frac{5}{2}\right)^2 + (y-2)^2 = \frac{9}{4}$

12. Center = midpoint of (0, 1) and (2, 3)
$$= \left(\frac{0+2}{2}, \frac{1+3}{2}\right) = (1, 2)$$
Radius = distance from $(1, 2)$ to (2,3)
$$= \sqrt{(2-1)^2 + (3-2)^2} = \sqrt{2}$$
Equation: $(x-1)^2 + (y-2)^2 = 2$

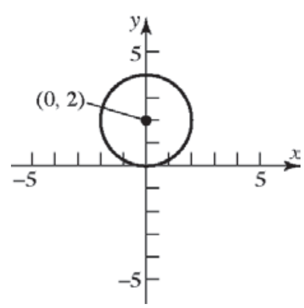
13. $(x-h)^2 + (y-k)^2 = r^2$
$$(x-0)^2 + (y-0)^2 = 2^2$$
$$x^2 + y^2 = 4$$
General form: $x^2 + y^2 - 4 = 0$



14. $(x-h)^2 + (y-k)^2 = r^2$
$$(x-0)^2 + (y-0)^2 = 3^2$$
$$x^2 + y^2 = 9$$
General form: $x^2 + y^2 - 9 = 0$



15. $(x-h)^2 + (y-k)^2 = r^2$
$$(x-0)^2 + (y-2)^2 = 2^2$$
$$x^2 + (y-2)^2 = 4$$
General form: $x^2 + y^2 - 4y + 4 = 4$
$$x^2 + y^2 - 4y = 0$$



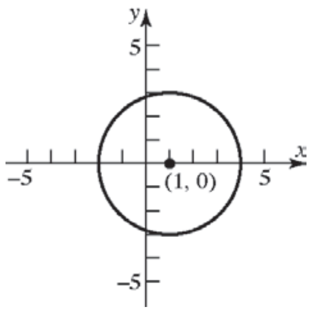
16. $(x-h)^2+(y-k)^2=r^2$

$(x-1)^2+(y-0)^2=3^2$

$(x-1)^2+y^2=9$

General form: $x^2-2x+1+y^2=9$

$x^2+y^2-2x-8=0$



17. $(x-h)^2+(y-k)^2=r^2$

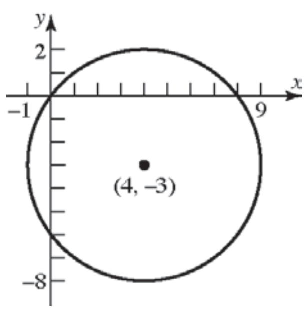
$(x-4)^2+(y-(-3))^2=5^2$

$(x-4)^2+(y+3)^2=25$

General form:

$x^2-8x+16+y^2+6y+9=25$

$x^2+y^2-8x+6y=0$



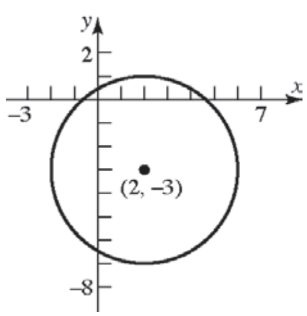
18. $(x-h)^2+(y-k)^2=r^2$

$(x-2)^2+(y-(-3))^2=4^2$

$(x-2)^2+(y+3)^2=16$

General form: $x^2-4x+4+y^2+6y+9=16$

$x^2+y^2-4x+6y-3=0$



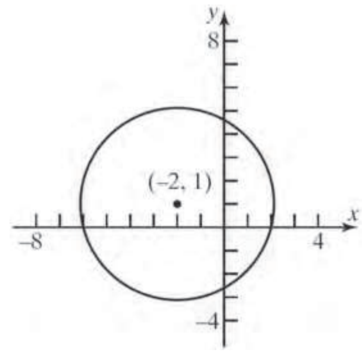
19. $(x-h)^2+(y-k)^2=r^2$

$(x-(-2))^2+(y-1)^2=4^2$

$(x+2)^2+(y-1)^2=16$

General form: $x^2+4x+4+y^2-2y+1=16$

$x^2+y^2+4x-2y-11=0$



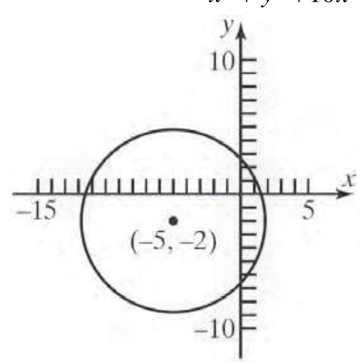
20. $(x-h)^2+(y-k)^2=r^2$

$(x-(-5))^2+(y-(-2))^2=7^2$

$(x+5)^2+(y+2)^2=49$

General form: $x^2+10x+25+y^2+4y+4=49$

$x^2+y^2+10x+4y-20=0$



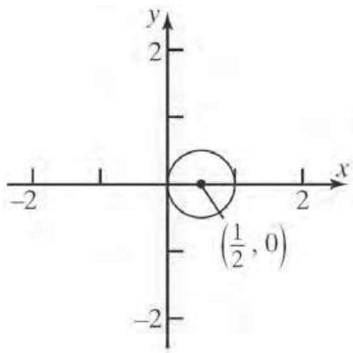
21. $(x-h)^2+(y-k)^2=r^2$

$\left(x-\frac{1}{2}\right)^2+(y-0)^2=\left(\frac{1}{2}\right)^2$

$\left(x-\frac{1}{2}\right)^2+y^2=\frac{1}{4}$

General form: $x^2-x+\frac{1}{4}+y^2=\frac{1}{4}$

$x^2+y^2-x=0$



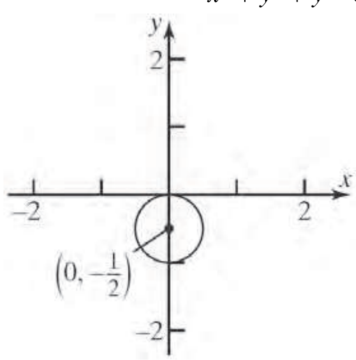
22. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-0)^2 + \left(y-\left(-\frac{1}{2}\right)\right)^2 = \left(\frac{1}{2}\right)^2$$

$$x^2 + \left(y+\frac{1}{2}\right)^2 = \frac{1}{4}$$

General form: $x^2 + y^2 + y + \frac{1}{4} = \frac{1}{4}$

$$x^2 + y^2 + y = 0$$



23. $(x-h)^2 + (y-k)^2 = r^2$

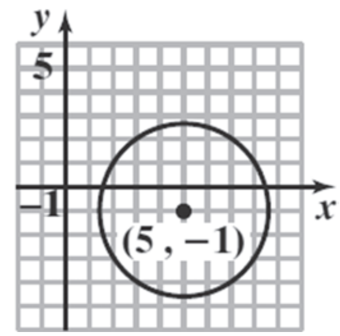
$$(x-5)^2 + (y-(-1))^2 = \left(\sqrt{13}\right)^2$$

$$(x-5)^2 + (y+1)^2 = 13$$

General form:

$$x^2 - 10x + 25 + y^2 + 2y + 1 = 13$$

$$x^2 + y^2 - 10x + 2y + 13 = 0$$



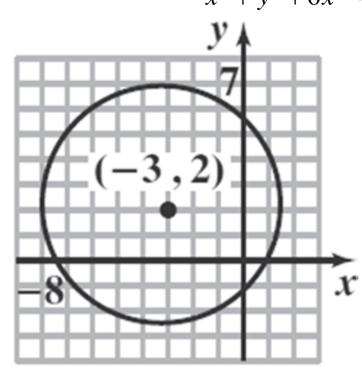
24. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-(-3))^2 + (y-2)^2 = \left(2\sqrt{5}\right)^2$$

$$(x+3)^2 + (y-2)^2 = 20$$

General form: $x^2 + 6x + 9 + y^2 - 4y + 4 = 20$

$$x^2 + y^2 + 6x - 4y - 7 = 0$$

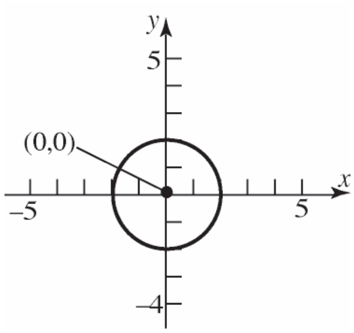


25. $x^2 + y^2 = 4$

$$x^2 + y^2 = 2^2$$

a. Center: $(0,0)$; Radius $= 2$

b.



c. x-intercepts: $x^2 + (0)^2 = 4$

$$x^2 = 4$$
$$x = \pm\sqrt{4} = \pm 2$$

y-intercepts: $(0)^2 + y^2 = 4$

$$y^2 = 4$$
$$y = \pm\sqrt{4} = \pm 2$$

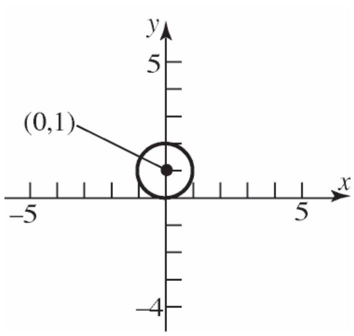
The intercepts are $(-2, 0)$, $(2, 0)$, $(0, -2)$, and $(0, 2)$.

26. $x^2 + (y-1)^2 = 1$

$$x^2 + (y-1)^2 = 1^2$$

a. Center:$(0, 1)$; Radius $= 1$

b.



c. x-intercepts: $x^2 + (0-1)^2 = 1$

$$x^2 + 1 = 1$$
$$x^2 = 0$$
$$x = \pm\sqrt{0} = 0$$

y-intercepts: $(0)^2 + (y-1)^2 = 1$

$$(y-1)^2 = 1$$
$$y - 1 = \pm\sqrt{1}$$
$$y - 1 = \pm 1$$
$$y = 1 \pm 1$$
$$y = 2 \quad \text{or} \quad y = 0$$

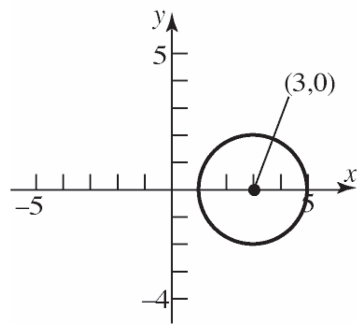
The intercepts are $(0, 0)$ and $(0, 2)$.

27. $2(x-3)^2 + 2y^2 = 8$

$$(x-3)^2 + y^2 = 4$$

a. Center: $(3, 0)$; Radius $= 2$

b.



c. x-intercepts: $(x-3)^2 + (0)^2 = 4$

$$(x-3)^2 = 4$$
$$x - 3 = \pm\sqrt{4}$$
$$x - 3 = \pm 2$$
$$x = 3 \pm 2$$
$$x = 5 \quad \text{or} \quad x = 1$$

y-intercepts: $(0-3)^2 + y^2 = 4$

$$(-3)^2 + y^2 = 4$$
$$9 + y^2 = 4$$
$$y^2 = -5$$

No real solution.

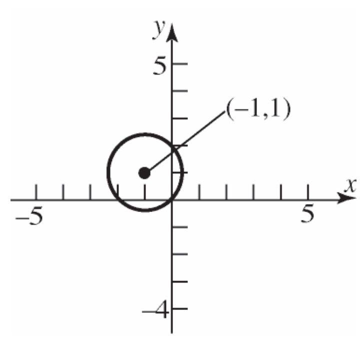
The intercepts are $(1, 0)$ and $(5, 0)$.

28. $3(x+1)^2 + 3(y-1)^2 = 6$

$$(x+1)^2 + (y-1)^2 = 2$$

a. Center: $(-1, 1)$; Radius $= \sqrt{2}$

b.



c. x-intercepts: $(x+1)^2+(0-1)^2=2$

$$(x+1)^2+(-1)^2=2$$
$$(x+1)^2+1=2$$
$$(x+1)^2=1$$
$$x+1=\pm\sqrt{1}$$
$$x+1=\pm 1$$
$$x=-1\pm 1$$
$$x=0 \quad \text{or} \quad x=-2$$

y-intercepts: $(0+1)^2+(y-1)^2=2$

$$(1)^2+(y-1)^2=2$$
$$1+(y-1)^2=2$$
$$(y-1)^2=1$$
$$y-1=\pm\sqrt{1}$$
$$y-1=\pm 1$$
$$y=1\pm 1$$
$$y=2 \quad \text{or} \quad y=0$$

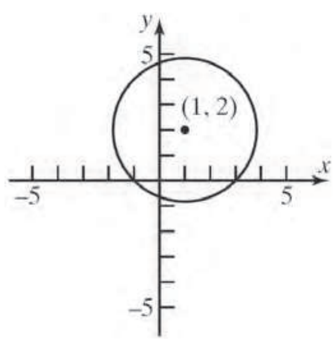
The intercepts are $(-2,0)$, $(0,0)$, and $(0,2)$.

29. $x^2+y^2-2x-4y-4=0$

$$x^2-2x+y^2-4y=4$$
$$(x^2-2x+1)+(y^2-4y+4)=4+1+4$$
$$(x-1)^2+(y-2)^2=3^2$$

a. Center: (1, 2); Radius = 3

b.



c. x-intercepts: $(x-1)^2+(0-2)^2=3^2$

$$(x-1)^2+(-2)^2=3^2$$
$$(x-1)^2+4=9$$
$$(x-1)^2=5$$
$$x-1=\pm\sqrt{5}$$
$$x=1\pm\sqrt{5}$$

y-intercepts: $(0-1)^2+(y-2)^2=3^2$

$$(-1)^2+(y-2)^2=3^2$$
$$1+(y-2)^2=9$$
$$(y-2)^2=8$$
$$y-2=\pm\sqrt{8}$$
$$y-2=\pm 2\sqrt{2}$$
$$y=2\pm 2\sqrt{2}$$

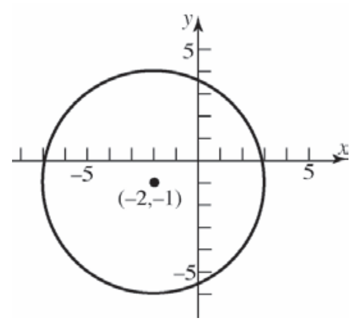
The intercepts are $(1-\sqrt{5},0)$, $(1+\sqrt{5},0)$, $(0,2-2\sqrt{2})$, and $(0,2+2\sqrt{2})$.

30. $x^2+y^2+4x+2y-20=0$

$$x^2+4x+y^2+2y=20$$
$$(x^2+4x+4)+(y^2+2y+1)=20+4+1$$
$$(x+2)^2+(y+1)^2=5^2$$

a. Center: (–2,–1); Radius = 5

b.



c. x-intercepts: $(x+2)^2+(0+1)^2=5^2$

$$(x+2)^2+1=25$$
$$(x+2)^2=24$$
$$x+2=\pm\sqrt{24}$$
$$x+2=\pm 2\sqrt{6}$$
$$x=-2\pm 2\sqrt{6}$$

y-intercepts: $(0+2)^2+(y+1)^2=5^2$

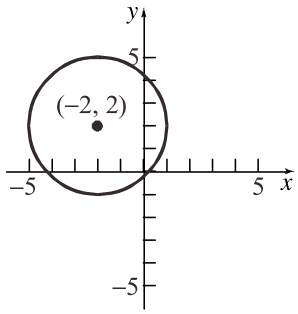
$$4+(y+1)^2=25$$
$$(y+1)^2=21$$
$$y+1=\pm\sqrt{21}$$
$$y=-1\pm\sqrt{21}$$

The intercepts are $(-2-2\sqrt{6},0)$, $(-2+2\sqrt{6},0)$, $(0,-1-\sqrt{21})$, and $(0,-1+\sqrt{21})$.

31.
$$x^2+y^2+4x-4y-1=0$$
$$x^2+4x+y^2-4y=1$$
$$(x^2+4x+4)+(y^2-4y+4)=1+4+4$$
$$(x+2)^2+(y-2)^2=3^2$$

a. Center: (–2, 2); Radius = 3

b.



c. x-intercepts: $(x+2)^2+(0-2)^2=3^2$
$$(x+2)^2+4=9$$
$$(x+2)^2=5$$
$$x+2=\pm\sqrt{5}$$
$$x=-2\pm\sqrt{5}$$

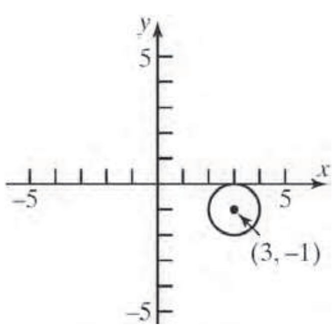
y-intercepts: $(0+2)^2+(y-2)^2=3^2$
$$4+(y-2)^2=9$$
$$(y-2)^2=5$$
$$y-2=\pm\sqrt{5}$$
$$y=2\pm\sqrt{5}$$

The intercepts are $\left(-2-\sqrt{5},0\right)$, $\left(-2+\sqrt{5},0\right)$, $\left(0,2-\sqrt{5}\right)$, and $\left(0,2+\sqrt{5}\right)$.

32.
$$x^2+y^2-6x+2y+9=0$$
$$x^2-6x+y^2+2y=-9$$
$$(x^2-6x+9)+(y^2+2y+1)=-9+9+1$$
$$(x-3)^2+(y+1)^2=1^2$$

a. Center: (3, –1); Radius = 1

b.



c. x-intercepts: $(x-3)^2+(0+1)^2=1^2$
$$(x-3)^2+1=1$$
$$(x-3)^2=0$$
$$x-3=0$$
$$x=3$$

y-intercepts: $(0-3)^2+(y+1)^2=1^2$
$$9+(y+1)^2=1$$
$$(y+1)^2=-8$$

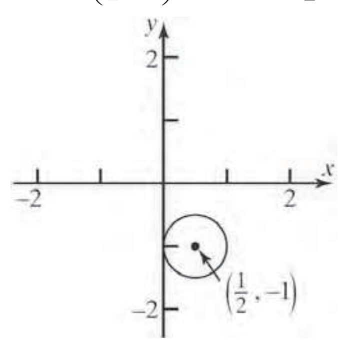
No real solution.

The intercept only intercept is $(3,0)$.

33.
$$x^2+y^2-x+2y+1=0$$
$$x^2-x+y^2+2y=-1$$
$$\left(x^2-x+\frac{1}{4}\right)+(y^2+2y+1)=-1+\frac{1}{4}+1$$
$$\left(x-\frac{1}{2}\right)^2+(y+1)^2=\left(\frac{1}{2}\right)^2$$

a. Center: $\left(\frac{1}{2},-1\right)$; Radius = $\frac{1}{2}$

b.



c. x-intercepts: $\left(x-\frac{1}{2}\right)^2+(0+1)^2=\left(\frac{1}{2}\right)^2$
$$\left(x-\frac{1}{2}\right)^2+1=\frac{1}{4}$$
$$\left(x-\frac{1}{2}\right)^2=-\frac{3}{4}$$

No real solutions

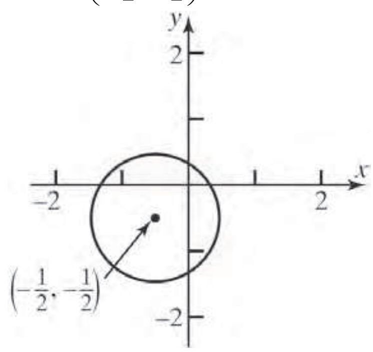
y-intercepts: $\left(0-\frac{1}{2}\right)^2+(y+1)^2=\left(\frac{1}{2}\right)^2$
$$\frac{1}{4}+(y+1)^2=\frac{1}{4}$$
$$(y+1)^2=0$$
$$y+1=0$$
$$y=-1$$

The only intercept is $(0,-1)$.

34.
$$x^2+y^2+x+y-\frac{1}{2}=0$$
$$x^2+x+y^2+y=\frac{1}{2}$$
$$\left(x^2+x+\frac{1}{4}\right)+\left(y^2+y+\frac{1}{4}\right)=\frac{1}{2}+\frac{1}{4}+\frac{1}{4}$$
$$\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=1^2$$

a. Center: $\left(-\frac{1}{2},-\frac{1}{2}\right)$; Radius = 1

b.



c. x-intercepts: $\left(x+\frac{1}{2}\right)^2+\left(0+\frac{1}{2}\right)^2=1^2$
$$\left(x+\frac{1}{2}\right)^2+\frac{1}{4}=1$$
$$\left(x+\frac{1}{2}\right)^2=\frac{3}{4}$$
$$x+\frac{1}{2}=\pm\frac{\sqrt{3}}{2}$$
$$x=\frac{-1\pm\sqrt{3}}{2}$$

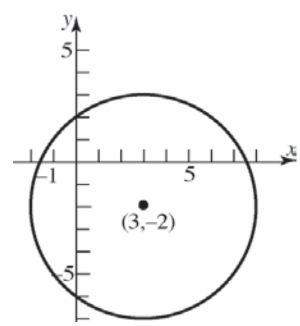
y-intercepts: $\left(0+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=1^2$
$$\frac{1}{4}+\left(y+\frac{1}{2}\right)^2=1$$
$$\left(y+\frac{1}{2}\right)^2=\frac{3}{4}$$
$$y+\frac{1}{2}=\pm\frac{\sqrt{3}}{2}$$
$$y=\frac{-1\pm\sqrt{3}}{2}$$

The intercepts are $\left(\frac{-1-\sqrt{3}}{2},0\right)$, $\left(\frac{-1+\sqrt{3}}{2},0\right)$, $\left(0,\frac{-1-\sqrt{3}}{2}\right)$, and $\left(0,\frac{-1+\sqrt{3}}{2}\right)$.

35.
$$2x^2+2y^2-12x+8y-24=0$$
$$x^2+y^2-6x+4y=12$$
$$x^2-6x+y^2+4y=12$$
$$(x^2-6x+9)+(y^2+4y+4)=12+9+4$$
$$(x-3)^2+(y+2)^2=5^2$$

a. Center: (3,–2); Radius = 5

b.



c. x-intercepts: $(x-3)^2+(0+2)^2=5^2$
$$(x-3)^2+4=25$$
$$(x-3)^2=21$$
$$x-3=\pm\sqrt{21}$$
$$x=3\pm\sqrt{21}$$

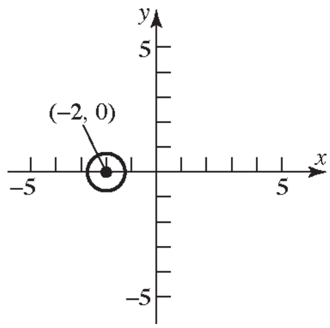
y-intercepts: $(0-3)^2+(y+2)^2=5^2$
$$9+(y+2)^2=25$$
$$(y+2)^2=16$$
$$y+2=\pm 4$$
$$y=-2\pm 4$$
$$y=2 \quad\text{or}\quad y=-6$$

The intercepts are $\left(3-\sqrt{21},0\right)$, $\left(3+\sqrt{21},0\right)$, $(0,-6)$, and $(0,2)$.

36. a.
$$2x^2+2y^2+8x+7=0$$
$$2x^2+8x+2y^2=-7$$
$$x^2+4x+y^2=-\frac{7}{2}$$
$$(x^2+4x+4)+y^2=-\frac{7}{2}+4$$
$$(x+2)^2+y^2=\frac{1}{2}$$
$$(x+2)^2+y^2=\left(\frac{\sqrt{2}}{2}\right)^2$$

Center: (–2, 0); Radius = $\frac{\sqrt{2}}{2}$

b.



c. x-intercepts: $(x+2)^2+(0)^2=\frac{1}{2}$

$$(x+2)^2=\frac{1}{2}$$
$$x+2=\pm\sqrt{\frac{1}{2}}$$
$$x+2=\pm\frac{\sqrt{2}}{2}$$
$$x=-2\pm\frac{\sqrt{2}}{2}$$

y-intercepts: $(0+2)^2+y^2=\frac{1}{2}$

$$4+y^2=\frac{1}{2}$$
$$y^2=-\frac{7}{2}$$

No real solutions.

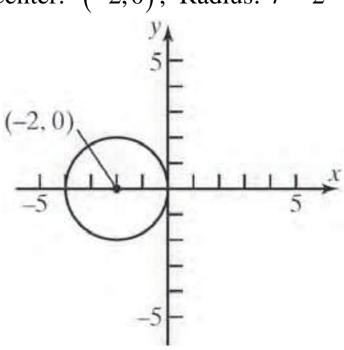
The intercepts are $\left(-2-\frac{\sqrt{2}}{2},0\right)$ and $\left(-2+\frac{\sqrt{2}}{2},0\right)$.

37. $2x^2+8x+2y^2=0$

$$x^2+4x+y^2=0$$
$$x^2+4x+4+y^2=0+4$$
$$(x+2)^2+y^2=2^2$$

a. Center: $(-2,0)$; Radius: $r=2$

b.



c. x-intercepts: $(x+2)^2+(0)^2=2^2$

$$(x+2)^2=4$$
$$(x+2)^2=\pm\sqrt{4}$$
$$x+2=\pm 2$$
$$x=-2\pm 2$$
$$x=0 \quad \text{or} \quad x=-4$$

y-intercepts: $(0+2)^2+y^2=2^2$

$$4+y^2=4$$
$$y^2=0$$
$$y=0$$

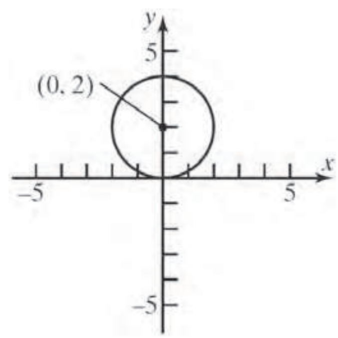
The intercepts are $(-4,0)$ and $(0,0)$.

38. $3x^2+3y^2-12y=0$

$$x^2+y^2-4y=0$$
$$x^2+y^2-4y+4=0+4$$
$$x^2+(y-2)^2=4$$

a. Center: $(0,2)$; Radius: $r=2$

b.



c. x-intercepts: $x^2+(0-2)^2=4$

$$x^2+4=4$$
$$x^2=0$$
$$x=0$$

y-intercepts: $0^2+(y-2)^2=4$

$$(y-2)^2=4$$
$$y-2=\pm\sqrt{4}$$
$$y-2=\pm 2$$
$$y=2\pm 2$$
$$y=4 \quad \text{or} \quad y=0$$

The intercepts are $(0,0)$ and $(0,4)$.

39. Center at (0, 0); containing point (–2, 3).

$$r=\sqrt{(-2-0)^2+(3-0)^2}=\sqrt{4+9}=\sqrt{13}$$

Equation: $(x-0)^2+(y-0)^2=\left(\sqrt{13}\right)^2$

$$x^2+y^2=13$$

40. Center at (1, 0); containing point (–3, 2).

$$r=\sqrt{(-3-1)^2+(2-0)^2}=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$$

Equation: $(x-1)^2+(y-0)^2=\left(\sqrt{20}\right)^2$

$$(x-1)^2+y^2=20$$

41. Endpoints of a diameter are (1, 4) and (–3, 2). The center is at the midpoint of that diameter:

Center: $\left(\frac{1+(-3)}{2},\frac{4+2}{2}\right)=(-1,3)$

Radius: $r=\sqrt{(1-(-1))^2+(4-3)^2}=\sqrt{4+1}=\sqrt{5}$

Equation: $(x-(-1))^2+(y-3)^2=\left(\sqrt{5}\right)^2$

$$(x+1)^2+(y-3)^2=5$$

42. Endpoints of a diameter are (4, 3) and (0, 1). The center is at the midpoint of that diameter:

Center: $\left(\frac{4+0}{2},\frac{3+1}{2}\right)=(2,2)$

Radius: $r=\sqrt{(4-2)^2+(3-2)^2}=\sqrt{4+1}=\sqrt{5}$

Equation: $(x-2)^2+(y-2)^2=\left(\sqrt{5}\right)^2$

$$(x-2)^2+(y-2)^2=5$$

43.

$$C=2\pi r$$
$$16\pi=2\pi r$$
$$r=8$$

$$(x-2)^2+(y-(-4))^2=(8)^2$$
$$(x-2)^2+(y+4)^2=64$$

44.

$$A=\pi r^2$$
$$49\pi=\pi r^2$$
$$r=7$$

$$(x-(-5))^2+(y-6)^2=(7)^2$$
$$(x+5)^2+(y-6)^2=49$$

45. (c); Center: 1; Radius = 2

46. (d) ; Center: $(-3,3)$; Radius = 3

47. (b) ; Center: $(-1,2)$; Radius = 2

48. (a) ; Center: $(-3,3)$; Radius = 3

49. The centers of the circles are: (4,-2) and (-1,5). The slope is $m=\frac{5-(-2)}{-1-4}=\frac{7}{-5}=-\frac{7}{5}$. Use the slope and one point to find the equation of the line.

$$y-(-2)=-\frac{7}{5}(x-4)$$
$$y+2=-\frac{7}{5}x+\frac{28}{5}$$
$$5y+10=-7x+28$$
$$7x+5y=18$$

50. Find the centers of the two circles:

$$x^2+y^2-4x+6y+4=0$$
$$(x^2-4x+4)+(y^2+6y+9)=-4+4+9$$
$$(x-2)^2+(y+3)^2=9$$

Center: $(2,-3)$

$$x^2+y^2+6x+4y+9=0$$
$$(x^2+6x+9)+(y^2+4y+4)=-9+9+4$$
$$(x+3)^2+(y+2)^2=4$$

Center: $(-3,-2)$

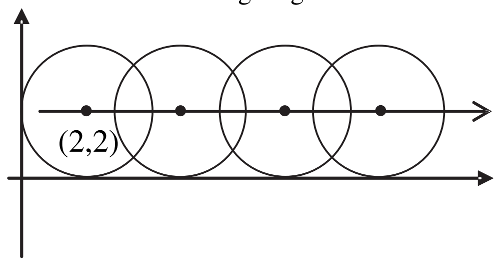
Find the slope of the line containing the centers:

$$m=\frac{-2-(-3)}{-3-2}=-\frac{1}{5}$$

Find the equation of the line containing the centers:

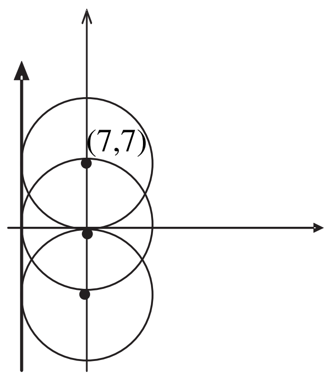
$$y+3=-\frac{1}{5}(x-2)$$
$$5y+15=-x+2$$
$$x+5y=-13$$
$$x+5y+13=0$$

51. Consider the following diagram:



Therefore, the path of the center of the circle has the equation $y = 2$.

52. Consider the following diagram:



Therefore the path of the center of the circle has the equation $x = 7$.

53. Let the upper-right corner of the square be the point (x, y). The circle and the square are both centered about the origin. Because of symmetry, we have that $x = y$ at the upper-right corner of the square. Therefore, we get

$$x^2 + y^2 = 9$$
$$x^2 + x^2 = 9$$
$$2x^2 = 9$$
$$x^2 = \frac{9}{2}$$
$$x = \sqrt{\frac{9}{2}} = \frac{3\sqrt{2}}{2}$$

The length of one side of the square is $2x$. Thus, the area is

$$A = s^2 = \left(2 \cdot \frac{3\sqrt{2}}{2}\right)^2 = \left(3\sqrt{2}\right)^2 = 18 \text{ square units.}$$

54. The area of the shaded region is the area of the circle, less the area of the square. Let the upper-right corner of the square be the point (x, y). The circle and the square are both centered about the origin. Because of symmetry, we have that $x = y$ at the upper-right corner of the square. Therefore, we get

$$x^2 + y^2 = 36$$
$$x^2 + x^2 = 36$$
$$2x^2 = 36$$
$$x^2 = 18$$
$$x = 3\sqrt{2}$$

The length of one side of the square is $2x$. Thus, the area of the square is $\left(2 \cdot 3\sqrt{2}\right)^2 = 72$ square units. From the equation of the circle, we have $r = 6$. The area of the circle is $\pi r^2 = \pi(6)^2 = 36\pi$ square units.

Therefore, the area of the shaded region is $A = 36\pi - 72$ square units.

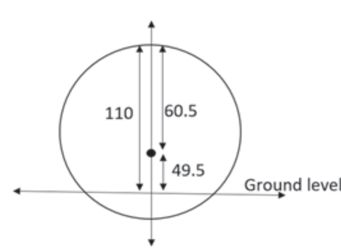
55. The diameter of the Ferris wheel was 250 feet, so the radius was 125 feet. The maximum height was 264 feet, so the center was at a height of $264 - 125 = 139$ feet above the ground. Since the center of the wheel is on the *y*-axis, it is the point (0, 139). Thus, an equation for the wheel is:

$$(x-0)^2 + (y-139)^2 = 125^2$$
$$x^2 + (y-139)^2 = 15{,}625$$

56. The diameter of the wheel is 520 feet, so the radius is 260 feet. The maximum height is 550 feet, so the center of the wheel is at a height of $550 - 260 = 290$ feet above the ground. Since the center of the wheel is on the *y*-axis, it is the point (0, 290). Thus, an equation for the wheel is:

$$(x-0)^2 + (y-290)^2 = 260^2$$
$$x^2 + (y-290)^2 = 67{,}600$$

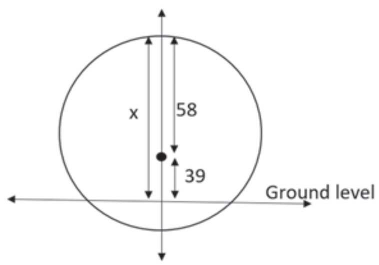
57.



Refer to figure. Since the radius of the building is 60.5 m and the height of the building is 110 m,

then the center of the building is 49.5 m above the ground, so the y-coordinate of the center is 49.5. The equation of the circle is given by
$x^2 + (y - 49.5)^2 = 60.5^2 = 3660.25$

58. Complete the square to find the equation of the circle representing the formula for the building.
$x^2 + y^2 - 78y + 1521 = 1843 + 1521 = 3364$
$x^2 + (y - 39)^2 = 58^2$



Refer to figure. The y coordinate of the center is 39. The radius is 58. Thus the height of the building is 58 + 39 = 97 m.

59. Center at (2, 3); tangent to the x-axis.
$r = 3$
Equation: $(x-2)^2 + (y-3)^2 = 3^2$
$(x-2)^2 + (y-3)^2 = 9$

60. Center at (–3, 1); tangent to the y-axis.
$r = 3$
Equation: $(x+3)^2 + (y-1)^2 = 3^2$
$(x+3)^2 + (y-1)^2 = 9$

61. Center at (–1, 3); tangent to the line $y = 2$.
This means that the circle contains the point (–1, 2), so the radius is $r = 1$.
Equation: $(x+1)^2 + (y-3)^2 = (1)^2$
$(x+1)^2 + (y-3)^2 = 1$

62. Center at (4, –2); tangent to the line $x = 1$.
This means that the circle contains the point (1, –2), so the radius is $r = 3$.
Equation: $(x-4)^2 + (y+2)^2 = (3)^2$
$(x-4)^2 + (y+2)^2 = 9$

63. **a.** Substitute $y = mx + b$ into $x^2 + y^2 = r^2$:

$$x^2 + (mx+b)^2 = r^2$$
$$x^2 + m^2x^2 + 2bmx + b^2 = r^2$$
$$(1+m^2)x^2 + 2bmx + b^2 - r^2 = 0$$

This equation has one solution if and only if the discriminant is zero.

$$(2bm)^2 - 4(1+m^2)(b^2 - r^2) = 0$$
$$4b^2m^2 - 4b^2 + 4r^2 - 4b^2m^2 + 4m^2r^2 = 0$$
$$-4b^2 + 4r^2 + 4m^2r^2 = 0$$
$$-b^2 + r^2 + m^2r^2 = 0$$
$$r^2(1+m^2) = b^2$$

b. From part (a) we know $(1+m^2)x^2 + 2bmx + b^2 - r^2 = 0$. Using the quadratic formula, since the discriminant is zero, we get:

$$x = \frac{-2bm}{2(1+m^2)} = \frac{-bm}{\left(\frac{b^2}{r^2}\right)} = \frac{-bmr^2}{b^2} = \frac{-mr^2}{b}$$

$$y = m\left(\frac{-mr^2}{b}\right) + b$$
$$= \frac{-m^2r^2}{b} + b = \frac{-m^2r^2 + b^2}{b} = \frac{r^2}{b}$$

The point of tangency is $\left(\frac{-mr^2}{b}, \frac{r^2}{b}\right)$.

c. The slope of the tangent line is m.
The slope of the line joining the point of tangency and the center (0,0) is:

$$\frac{\left(\frac{r^2}{b} - 0\right)}{\left(\frac{-mr^2}{b} - 0\right)} = \frac{r^2}{b} \cdot \frac{b}{-mr^2} = -\frac{1}{m}$$

The two lines are perpendicular.

64. Let (h, k) be the center of the circle.
$x - 2y + 4 = 0$
$2y = x + 4$
$y = \frac{1}{2}x + 2$

The slope of the tangent line is $\frac{1}{2}$. The slope from (h, k) to (0, 2) is –2.

$$\frac{2-k}{0-h}=-2$$
$$2-k=2h$$

The other tangent line is $y=2x-7$, and it has slope 2.

The slope from (h, k) to $(3, -1)$ is $-\frac{1}{2}$.

$$\frac{-1-k}{3-h}=-\frac{1}{2}$$
$$2+2k=3-h$$
$$2k=1-h$$
$$h=1-2k$$

Solve the two equations in h and k:

$$2-k=2(1-2k)$$
$$2-k=2-4k$$
$$3k=0$$
$$k=0$$
$$h=1-2(0)=1$$

The center of the circle is (1, 0).

65. The slope of the line containing the center (0,0) and $\left(1, 2\sqrt{2}\right)$ is

$\frac{2\sqrt{2}-0}{1-0}=2\sqrt{2}$ .Then the slope of the tangent line is $\frac{-1}{2\sqrt{2}}=-\frac{\sqrt{2}}{4}$.

So the equation of the tangent line is:

$$y-2\sqrt{2}=-\frac{\sqrt{2}}{4}\left(x-1\right)$$
$$y-2\sqrt{2}=-\frac{\sqrt{2}}{4}x+\frac{\sqrt{2}}{4}$$
$$4y-8\sqrt{2}=-\sqrt{2}\,x+\sqrt{2}$$
$$\sqrt{2}\,x+4y=9\sqrt{2}$$

66. $x^2+y^2-4x+6y+4=0$

$$(x^2-4x+4)+(y^2+6y+9)=-4+4+9$$
$$(x-2)^2+(y+3)^2=9$$

Center: $(2, -3)$

The slope of the line containing the center and $\left(3, 2\sqrt{2}-3\right)$ is $\frac{2\sqrt{2}-3-(-3)}{3-2}=\frac{2\sqrt{2}}{1}=2\sqrt{2}$

Then the slope of the tangent line is:

$$\frac{-1}{2\sqrt{2}}=-\frac{\sqrt{2}}{4}$$

So, the equation of the tangent line is

$$y-\left(2\sqrt{2}-3\right)=-\frac{\sqrt{2}}{4}(x-3)$$
$$y-2\sqrt{2}+3=-\frac{\sqrt{2}}{4}x+\frac{3\sqrt{2}}{4}$$
$$4y-8\sqrt{2}+12=-\sqrt{2}x+3\sqrt{2}$$
$$\sqrt{2}x+4y=11\sqrt{2}-12$$

67. The center of the circle is $\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$ and the radius is $\dfrac{1}{2}\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$. Then the equation of the circle is $\left(x-\dfrac{x_1+x_2}{2}\right)^2+\left(y-\dfrac{y_1+y_2}{2}\right)^2=\dfrac{1}{4}\left[\left(x_1-x_2\right)^2+(y_1-y_2)^2\right]$. Expanding, gives

$$x^2-x(x_1+x_2)+\frac{\left(x_1+x_2\right)^2}{4}+y^2-y(y_1+y_2)+\frac{\left(y_1+y_2\right)^2}{4}=\frac{1}{4}\left[x_1^{\ 2}-2x_1x_2+x_2^{\ 2}+y_1^{\ 2}-2y_1y_2+y_2^{\ 2}\right]$$

$$4x^2-4x_1x-4x_2x+x_1^{\ 2}+2x_1x_2+x_2^{\ 2}+4y^2-4y_1y-4y_2y+y_1^{\ 2}+2y_1y_2+y_2^{\ 2}=x_1^{\ 2}-2x_1x_2+x_2^{\ 2}+y_1^{\ 2}-2y_1y_2+y_2^{\ 2}$$

$$4x^2-4x_1x-4x_2x+4x_1x_2+4y^2-4y_1y-4y_2y+4y_1y_2=0$$

$$x^2-x_1x-x_2x+x_1x_2+y^2-y_1y-y_2y+y_1y_2=0$$

$$x\left(x-x_1\right)-x_2\left(x+x_1\right)+y\left(y-y_1\right)-y_2\left(y+y_1\right)=0$$

$$\left(x-x_1\right)\left(x-x_2\right)+\left(y-y_1\right)\left(y-y_2\right)=0$$

68. Complete the square to get $\left(x+\dfrac{d}{2}\right)^2+\left(y+\dfrac{e}{2}\right)^2=\dfrac{d^2+e^2-4f}{4}$. The slope of the line between the center $\left(-\dfrac{d}{2},-\dfrac{e}{2}\right)$ and the point of tangency $\left(x_0,y_0\right)$ is $m=\dfrac{y_0+\frac{e}{2}}{x_0+\frac{d}{2}}$. So the slope of the tangent line is $m_{\tan}=-\dfrac{x_0+\frac{d}{2}}{y_0+\frac{e}{2}}$.

Therefore, the equation of the tangent line is $y-y_0=-\dfrac{x_0+\frac{d}{2}}{y_0+\frac{e}{2}}(x-x_0)$ which is equivalent to

$$(x-x_0)\left(x_0+\tfrac{d}{2}\right)+\left(y-y_0\right)\left(y_0+\tfrac{e}{2}\right)=0$$

$$x_0x+\frac{d}{2}x-x_0^{\ 2}-\frac{d}{2}x_0+y_0y+\frac{e}{2}y-y_0^{\ 2}-\frac{e}{2}y_0=0$$

$$x_0x+y_0y+\frac{d}{2}x+\frac{e}{2}y-\left(x_0^{\ 2}+y_0^{\ 2}+\frac{d}{2}x_0+\frac{e}{2}y_0\right)=0$$

Because $\left(x_0,y_0\right)$ is on the circle, $x_0^{\ 2}+y_0^{\ 2}+dx_0+ey_0+f=0$, and

$x_0^{\ 2}+y_0^{\ 2}+\dfrac{d}{2}x_0+\dfrac{e}{2}y_0=-\dfrac{d}{2}x_0-\dfrac{e}{2}y_0-f$ Substituting this result gives

$$x_0^{\ 2}+y_0^{\ 2}+\frac{d}{2}x_0+\frac{e}{2}y_0-\left(-\frac{d}{2}x_0-\frac{e}{2}y_0-f\right)=0$$

$$x_0^{\ 2}+y_0^{\ 2}+\frac{d}{2}x_0+\frac{e}{2}y_0+\frac{d}{2}x_0+\frac{e}{2}y_0+f=0$$

$$x_0^{\ 2}+y_0^{\ 2}+d\left(\frac{x+x_0}{2}\right)+e\left(\frac{y+y_0}{2}\right)+f=0$$

69. (b), (c), (e) and (g)
We need $h,k>0$ and $\left(0,0\right)$ on the graph.

70. (b), (e) and (g)
We need $h<0$, $k=0$, and $|h|>r$.

71. Answers will vary.

72. The student has the correct radius, but the signs of the coordinates of the center are incorrect. The student needs to write the equation in the standard form $\left(x-h\right)^2+\left(y-k\right)^2=r^2$.

$$(x+3)^2+(y-2)^2=16$$
$$(x-(-3))^2+(y-2)^2=4^2$$
Thus, $(h,k)=(-3,2)$ and $r=4$.

Chapter 1 Review Exercises

1. $P_1=(0,0)$ and $P_2=(4,2)$

a. $d(P_1,P_2)=\sqrt{(4-0)^2+(2-0)^2}$
$=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$

b. The coordinates of the midpoint are:
$(x,y)=\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$
$=\left(\dfrac{0+4}{2},\dfrac{0+2}{2}\right)=\left(\dfrac{4}{2},\dfrac{2}{2}\right)=(2,1)$

c. slope $=\dfrac{\Delta y}{\Delta x}=\dfrac{2-0}{4-0}=\dfrac{2}{4}=\dfrac{1}{2}$

d. For each run of 2, there is a rise of 1.

2. $P_1=(1,-1)$ and $P_2=(-2,3)$

a. $d(P_1,P_2)=\sqrt{(-2-1)^2+(3-(-1))^2}$
$=\sqrt{9+16}=\sqrt{25}=5$

b. The coordinates of the midpoint are:
$(x,y)=\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$
$=\left(\dfrac{1+(-2)}{2},\dfrac{-1+3}{2}\right)$
$=\left(\dfrac{-1}{2},\dfrac{2}{2}\right)=\left(-\dfrac{1}{2},1\right)$

c. slope $=\dfrac{\Delta y}{\Delta x}=\dfrac{3-(-1)}{-2-1}=\dfrac{4}{-3}=-\dfrac{4}{3}$

d. For each run of 3, there is a rise of -4.

3. $P_1=(4,-4)$ and $P_2=(4,8)$

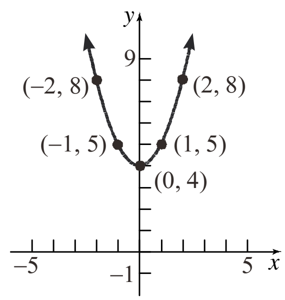
a. $d(P_1,P_2)=\sqrt{(4-4)^2+(8-(-4))^2}$
$=\sqrt{0+144}=\sqrt{144}=12$

b. The coordinates of the midpoint are:
$(x,y)=\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$
$=\left(\dfrac{4+4}{2},\dfrac{-4+8}{2}\right)=\left(\dfrac{8}{2},\dfrac{4}{2}\right)=(4,2)$

c. slope $=\dfrac{\Delta y}{\Delta x}=\dfrac{8-(-4)}{4-4}=\dfrac{12}{0}$, undefined

d. An undefined slope means the points lie on a vertical line. There is no change in x.

4. $y=x^2+4$



5. x-intercepts: $-4, 0, 2$; y-intercepts: $-2, 0, 2$
Intercepts: $(-4,0)$, $(0,0)$, $(2,0)$, $(0,-2)$, $(0,2)$

6. $2x=3y^2$

x-intercepts:	y-intercepts:
$2x=3(0)^2$	$2(0)=3y^2$
$2x=0$	$0=y^2$
$x=0$	$y=0$

The only intercept is $(0,0)$.

<u>Test x-axis symmetry</u>: Let $y=-y$
$2x=3(-y)^2$
$2x=3y^2$ same

<u>Test y-axis symmetry</u>: Let $x=-x$
$2(-x)=3y^2$
$-2x=3y^2$ different

<u>Test origin symmetry</u>: Let $x=-x$ and $y=-y$.
$2(-x)=3(-y)^2$
$-2x=3y^2$ different

Therefore, the graph will have x-axis symmetry.

7. $x^2+4y^2=16$

x-intercepts:
$x^2 + 4(0)^2 = 16$
$x^2 = 16$
$x = \pm 4$

y-intercepts:
$(0)^2 + 4y^2 = 16$
$4y^2 = 16$
$y^2 = 4$
$y = \pm 2$

The intercepts are $(-4, 0)$, $(4, 0)$, $(0, -2)$, and $(0, 2)$.

Test *x*-axis symmetry: Let $y = -y$
$x^2 + 4(-y)^2 = 16$
$x^2 + 4y^2 = 16$ same

Test *y*-axis symmetry: Let $x = -x$
$(-x)^2 + 4y^2 = 16$
$x^2 + 4y^2 = 16$ same

Test origin symmetry: Let $x = -x$ and $y = -y$.
$(-x)^2 + 4(-y)^2 = 16$
$x^2 + 4y^2 = 16$ same

Therefore, the graph will have *x*-axis, *y*-axis, and origin symmetry.

8. $y = x^4 + 2x^2 + 1$

x-intercepts:
$0 = x^4 + 2x^2 + 1$
$0 = (x^2 + 1)(x^2 + 1)$
$x^2 + 1 = 0$
$x^2 = -1$
no real solutions

y-intercepts:
$y = (0)^4 + 2(0)^2 + 1$
$= 1$

The only intercept is (0, 1).

Test *x*-axis symmetry: Let $y = -y$
$-y = x^4 + 2x^2 + 1$
$y = -x^4 - 2x^2 - 1$ different

Test *y*-axis symmetry: Let $x = -x$
$y = (-x)^4 + 2(-x)^2 + 1$
$y = x^4 + 2x^2 + 1$ same

Test origin symmetry: Let $x = -x$ and $y = -y$.
$-y = (-x)^4 + 2(-x)^2 + 1$
$-y = x^4 + 2x^2 + 1$
$y = -x^4 - 2x^2 - 1$ different

Therefore, the graph will have *y*-axis symmetry.

9. $y = x^3 - x$

x-intercepts:
$0 = x^3 - x$
$0 = x(x^2 - 1)$
$0 = x(x + 1)(x - 1)$
$x = 0, x = -1, x = 1$

y-intercepts:
$y = (0)^3 - 0$
$= 0$

The intercepts are $(-1, 0)$, $(0, 0)$, and $(1, 0)$.

Test *x*-axis symmetry: Let $y = -y$
$-y = x^3 - x$
$y = -x^3 + x$ different

Test *y*-axis symmetry: Let $x = -x$
$y = (-x)^3 - (-x)$
$y = -x^3 + x$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.
$-y = (-x)^3 - (-x)$
$-y = -x^3 + x$
$y = x^3 - x$ same

Therefore, the graph will have origin symmetry.

10. $x^2 + x + y^2 + 2y = 0$

x-intercepts: $x^2 + x + (0)^2 + 2(0) = 0$
$x^2 + x = 0$
$x(x + 1) = 0$
$x = 0, x = -1$

y-intercepts: $(0)^2 + 0 + y^2 + 2y = 0$
$y^2 + 2y = 0$
$y(y + 2) = 0$
$y = 0, y = -2$

The intercepts are $(-1, 0)$, $(0, 0)$, and $(0, -2)$.

Test *x*-axis symmetry: Let $y = -y$
$x^2 + x + (-y)^2 + 2(-y) = 0$
$x^2 + x + y^2 - 2y = 0$ different

Test *y*-axis symmetry: Let $x = -x$
$(-x)^2 + (-x) + y^2 + 2y = 0$
$x^2 - x + y^2 + 2y = 0$ different

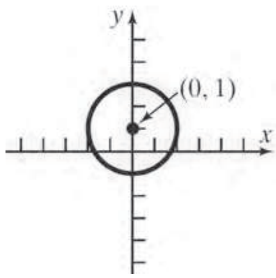
Test origin symmetry: Let $x = -x$ and $y = -y$.
$(-x)^2 + (-x) + (-y)^2 + 2(-y) = 0$
$x^2 - x + y^2 - 2y = 0$ different

The graph has none of the indicated symmetries.

11.
$$(x-h)^2+(y-k)^2=r^2$$
$$\left(x-(-2)\right)^2+\left(y-3\right)^2=4^2$$
$$\left(x+2\right)^2+\left(y-3\right)^2=16$$

12.
$$(x-h)^2+(y-k)^2=r^2$$
$$\left(x-(-1)\right)^2+\left(y-(-2)\right)^2=1^2$$
$$\left(x+1\right)^2+\left(y+2\right)^2=1$$

13.
$$x^2+\left(y-1\right)^2=4$$
$$x^2+\left(y-1\right)^2=2^2$$
Center: (0,1); Radius = 2

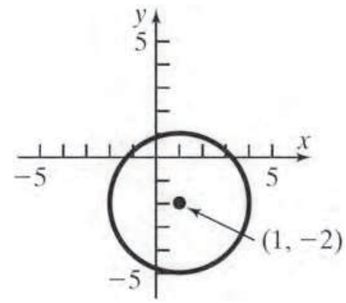


x-intercepts:
$$x^2+\left(0-1\right)^2=4$$
$$x^2+1=4$$
$$x^2=3$$
$$x=\pm\sqrt{3}$$

y-intercepts:
$$0^2+\left(y-1\right)^2=4$$
$$(y-1)^2=4$$
$$y-1=\pm 2$$
$$y=1\pm 2$$
$$y=3 \text{ or } y=-1$$

The intercepts are $\left(-\sqrt{3},0\right)$, $\left(\sqrt{3},0\right)$, $(0,-1)$, and $(0,3)$.

14.
$$x^2+y^2-2x+4y-4=0$$
$$x^2-2x+y^2+4y=4$$
$$\left(x^2-2x+1\right)+\left(y^2+4y+4\right)=4+1+4$$
$$\left(x-1\right)^2+\left(y+2\right)^2=3^2$$
Center: (1, –2) Radius = 3

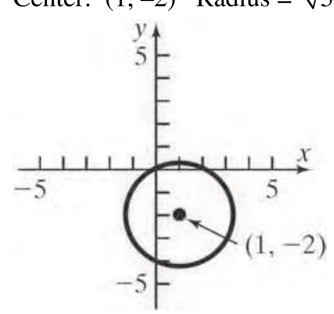


x-intercepts:
$$\left(x-1\right)^2+\left(0+2\right)^2=3^2$$
$$\left(x-1\right)^2+4=9$$
$$\left(x-1\right)^2=5$$
$$x-1=\pm\sqrt{5}$$
$$x=1\pm\sqrt{5}$$

y-intercepts:
$$\left(0-1\right)^2+\left(y+2\right)^2=3^2$$
$$1+\left(y+2\right)^2=9$$
$$\left(y+2\right)^2=8$$
$$y+2=\pm\sqrt{8}$$
$$y+2=\pm 2\sqrt{2}$$
$$y=-2\pm 2\sqrt{2}$$

The intercepts are $\left(1-\sqrt{5},0\right)$, $\left(1+\sqrt{5},0\right)$, $\left(0,-2-2\sqrt{2}\right)$, and $\left(0,-2+2\sqrt{2}\right)$.

15.
$$3x^2+3y^2-6x+12y=0$$
$$x^2+y^2-2x+4y=0$$
$$x^2-2x+y^2+4y=0$$
$$\left(x^2-2x+1\right)+\left(y^2+4y+4\right)=1+4$$
$$\left(x-1\right)^2+\left(y+2\right)^2=\left(\sqrt{5}\right)^2$$
Center: (1, –2) Radius = $\sqrt{5}$



x-intercepts: $(x-1)^2+(0+2)^2=(\sqrt{5})^2$

$$(x-1)^2+4=5$$
$$(x-1)^2=1$$
$$x-1=\pm 1$$
$$x=1\pm 1$$
$$x=2 \quad \text{or} \quad x=0$$

y-intercepts: $(0-1)^2+(y+2)^2=(\sqrt{5})^2$

$$1+(y+2)^2=5$$
$$(y+2)^2=4$$
$$y+2=\pm 2$$
$$y=-2\pm 2$$
$$y=0 \quad \text{or} \quad y=-4$$

The intercepts are $(0,0)$, $(2,0)$, and $(0,-4)$.

16. Slope = –2; containing (3,–1)

$$y-y_1=m(x-x_1)$$
$$y-(-1)=-2(x-3)$$
$$y+1=-2x+6$$
$$y=-2x+5 \quad \text{or} \quad 2x+y=5$$

17. vertical; containing (–3,4)
Vertical lines have equations of the form $x = a$, where a is the x-intercept. Now, a vertical line containing the point (–3, 4) must have an x-intercept of –3, so the equation of the line is $x=-3$. The equation does not have a slope-intercept form.

18. y-intercept = –2; containing (5,–3)
Points are (5,–3) and (0,–2)

$$m=\frac{-2-(-3)}{0-5}=\frac{1}{-5}=-\frac{1}{5}$$
$$y=mx+b$$
$$y=-\frac{1}{5}x-2 \quad \text{or} \quad x+5y=-10$$

19. Containing the points (3,–4) and (2, 1)

$$m=\frac{1-(-4)}{2-3}=\frac{5}{-1}=-5$$
$$y-y_1=m(x-x_1)$$
$$y-(-4)=-5(x-3)$$
$$y+4=-5x+15$$
$$y=-5x+11 \quad \text{or} \quad 5x+y=11$$

20. Parallel to $2x-3y=-4$

$$2x-3y=-4$$
$$-3y=-2x-4$$
$$\frac{-3y}{-3}=\frac{-2x-4}{-3}$$
$$y=\frac{2}{3}x+\frac{4}{3}$$

Slope $=\frac{2}{3}$; containing (–5,3)

$$y-y_1=m(x-x_1)$$
$$y-3=\frac{2}{3}(x-(-5))$$
$$y-3=\frac{2}{3}(x+5)$$
$$y-3=\frac{2}{3}x+\frac{10}{3}$$
$$y=\frac{2}{3}x+\frac{19}{3} \quad \text{or} \quad 2x-3y=-19$$

21. Perpendicular to $x+y=2$

$$x+y=2$$
$$y=-x+2$$

The slope of this line is -1, so the slope of a line perpendicular to it is 1.
Slope = 1; containing (4,–3)

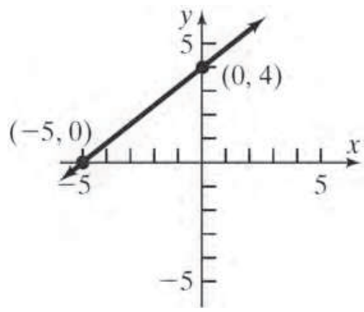
$$y-y_1=m(x-x_1)$$
$$y-(-3)=1(x-4)$$
$$y+3=x-4$$
$$y=x-7 \quad \text{or} \quad x-y=7$$

22.
$$4x-5y=-20$$
$$-5y=-4x-20$$
$$y=\frac{4}{5}x+4$$

slope = $\frac{4}{5}$; y-intercept = 4

x-intercept: Let $y = 0$.

$$4x-5(0)=-20$$
$$4x=-20$$
$$x=-5$$



23. $\frac{1}{2}x-\frac{1}{3}y=-\frac{1}{6}$

$$-\frac{1}{3}y=-\frac{1}{2}x-\frac{1}{6}$$

$$y=\frac{3}{2}x+\frac{1}{2}$$

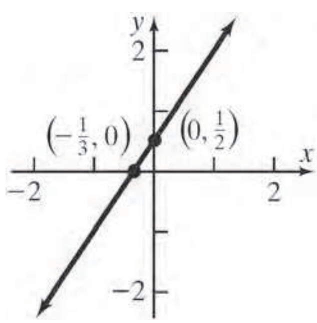
slope $=\frac{3}{2}$; y-intercept $=\frac{1}{2}$

x-intercept: Let $y=0$.

$$\frac{1}{2}x-\frac{1}{3}(0)=-\frac{1}{6}$$

$$\frac{1}{2}x=-\frac{1}{6}$$

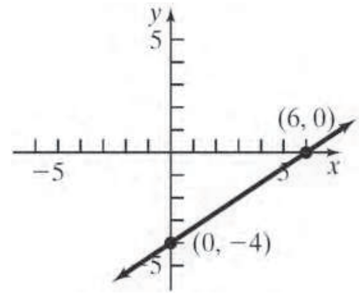
$$x=-\frac{1}{3}$$



24. $2x-3y=12$

x-intercept:	y-intercept:
$2x-3(0)=12$	$2(0)-3y=12$
$2x=12$	$-3y=12$
$x=6$	$y=-4$

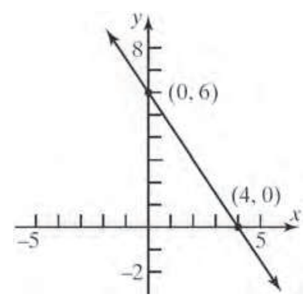
The intercepts are $(6,0)$ and $(0,-4)$.



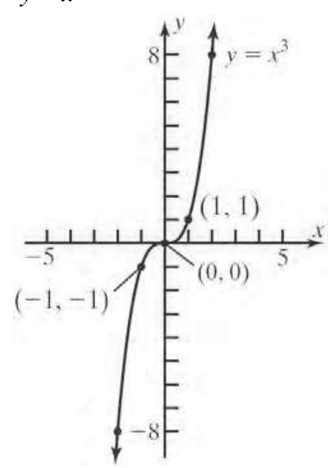
25. $\frac{1}{2}x+\frac{1}{3}y=2$

x-intercept:	y-intercept:
$\frac{1}{2}x+\frac{1}{3}(0)=2$	$\frac{1}{2}(0)+\frac{1}{3}y=2$
$\frac{1}{2}x=2$	$\frac{1}{3}y=2$
$x=4$	$y=6$

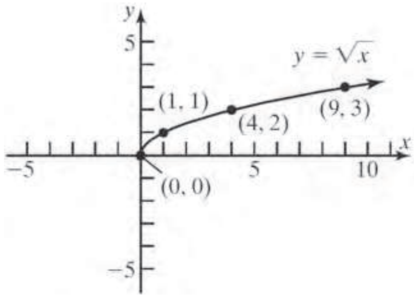
The intercepts are $(4,0)$ and $(0,6)$.



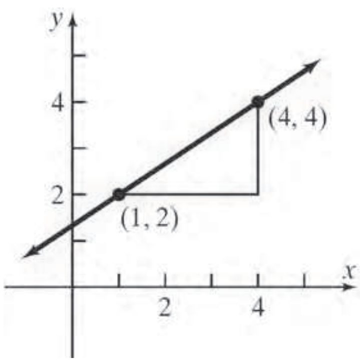
26. $y=x^3$



27. $y=\sqrt{x}$



28. slope = $\frac{2}{3}$, containing the point (1,2)



29. Find the distance between each pair of points.

$d_{A,B}=\sqrt{(1-3)^2+(1-4)^2}=\sqrt{4+9}=\sqrt{13}$

$d_{B,C}=\sqrt{(-2-1)^2+(3-1)^2}=\sqrt{9+4}=\sqrt{13}$

$d_{A,C}=\sqrt{(-2-3)^2+(3-4)^2}=\sqrt{25+1}=\sqrt{26}$

Since *AB* = *BC*, triangle *ABC* is isosceles.

30. Given the points $A=(-2,0)$, $B=(-4,4)$, and $C=(8,5)$.

a. Find the distance between each pair of points.

$$d(A,B)=\sqrt{(-4-(-2))^2+(4-0)^2}=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$$

$$d(B,C)=\sqrt{(8-(-4))^2+(5-4)^2}=\sqrt{144+1}=\sqrt{145}$$

$$d(A,C)=\sqrt{(8-(-2))^2+(5-0)^2}=\sqrt{100+25}=\sqrt{125}=5\sqrt{5}$$

$$[d(A,B)]^2+[d(A,C)]^2=[d(B,C)]^2$$
$$\left(\sqrt{20}\right)^2+\left(\sqrt{125}\right)^2=\left(\sqrt{145}\right)^2$$
$$20+125=145$$
$$145=145$$

The Pythagorean Theorem is satisfied, so this is a right triangle.

b. Find the slopes:

$$m_{AB}=\frac{4-0}{-4-(-2)}=\frac{4}{-2}=-2$$

$$m_{BC}=\frac{5-4}{8-(-4)}=\frac{1}{12}$$

$$m_{AC}=\frac{5-0}{8-(-2)}=\frac{5}{10}=\frac{1}{2}$$

Since $m_{AB}\cdot m_{AC}=-2\cdot\frac{1}{2}=-1$, the sides *AB* and *AC* are perpendicular and the triangle is a right triangle.

31. Endpoints of the diameter are (–3, 2) and (5,–6). The center is at the midpoint of the diameter:

Center: $\left(\frac{-3+5}{2},\frac{2+(-6)}{2}\right)=(1,-2)$

Radius: $r=\sqrt{(1-(-3))^2+(-2-2)^2}=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}$

Equation: $(x-1)^2+(y+2)^2=\left(4\sqrt{2}\right)^2$

$(x-1)^2+(y+2)^2=32$

32. slope of $\overline{AB}=\frac{1-5}{6-2}=-1$

slope of $\overline{AC}=\frac{-1-5}{8-2}=-1$

Therefore, the points lie on a line.

Chapter 1 Test

1. $d(P_1,P_2)=\sqrt{\left(5-(-1)\right)^2+\left(-1-3\right)^2}$
$=\sqrt{6^2+\left(-4\right)^2}$
$=\sqrt{36+16}$
$=\sqrt{52}=2\sqrt{13}$

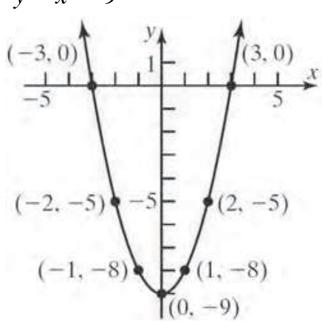
2. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{-1+5}{2},\frac{3+(-1)}{2}\right)$$
$$=\left(\frac{4}{2},\frac{2}{2}\right)$$
$$=\left(2,1\right)$$

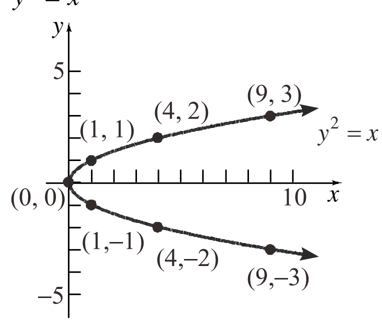
3. a. $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-1-3}{5-(-1)}=\dfrac{-4}{6}=-\dfrac{2}{3}$

b. If x increases by 3 units, y will decrease by 2 units.

4. $y=x^2-9$



5. $y^2=x$



6. $x^2+y=9$

x-intercepts:
$x^2+0=9$
$x^2=9$
$x=\pm 3$

y-intercept:
$(0)^2+y=9$
$y=9$

The intercepts are $\left(-3,0\right)$, $\left(3,0\right)$, and $\left(0,9\right)$.

Test x-axis symmetry: Let $y=-y$
$x^2+\left(-y\right)=9$
$x^2-y=9$ different

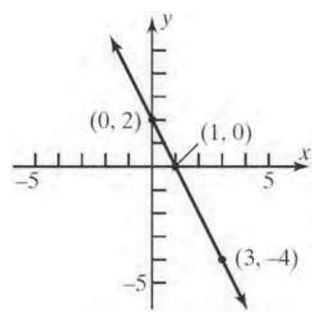
Test y-axis symmetry: Let $x=-x$
$\left(-x\right)^2+y=9$
$x^2+y=9$ same

Test origin symmetry: Let $x=-x$ and $y=-y$
$\left(-x\right)^2+\left(-y\right)=9$
$x^2-y=9$ different

Therefore, the graph will have y-axis symmetry.

7. Slope $=-2$; containing $(3,-4)$

$y-y_1=m(x-x_1)$
$y-(-4)=-2(x-3)$
$y+4=-2x+6$
$y=-2x+2$

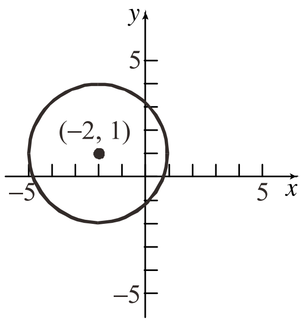


8. $(x-h)^2+(y-k)^2=r^2$
$\left(x-4\right)^2+\left(y-(-3)\right)^2=5^2$
$\left(x-4\right)^2+\left(y+3\right)^2=25$

General form: $\left(x-4\right)^2+\left(y+3\right)^2=25$
$x^2-8x+16+y^2+6y+9=25$
$x^2+y^2-8x+6y=0$

9. $x^2+y^2+4x-2y-4=0$
$x^2+4x+y^2-2y=4$
$(x^2+4x+4)+(y^2-2y+1)=4+4+1$
$(x+2)^2+(y-1)^2=3^2$

Center: (–2, 1); Radius = 3



10. $2x+3y=6$

$$3y=-2x+6$$

$$y=-\frac{2}{3}x+2$$

Parallel line

Any line parallel to $2x+3y=6$ has slope $m=-\frac{2}{3}$. The line contains $(1,-1)$:

$$y-y_1=m(x-x_1)$$

$$y-(-1)=-\frac{2}{3}(x-1)$$

$$y+1=-\frac{2}{3}x+\frac{2}{3}$$

$$y=-\frac{2}{3}x-\frac{1}{3}$$

Perpendicular line

Any line perpendicular to $2x+3y=6$ has slope $m=\frac{3}{2}$. The line contains $(0,3)$:

$$y-y_1=m(x-x_1)$$

$$y-3=\frac{3}{2}(x-0)$$

$$y-3=\frac{3}{2}x$$

$$y=\frac{3}{2}x+3$$

Chapter 1 Project

Internet-based Project

Chapter 2

Functions and Their Graphs

Section 2.1

1. $(-1,3)$

2. $3(-2)^2-5(-2)+\frac{1}{(-2)}=3(4)-5(-2)-\frac{1}{2}$

$$=12+10-\frac{1}{2}$$

$$=\frac{43}{2} \text{ or } 21\tfrac{1}{2} \text{ or } 21.5$$

3. We must not allow the denominator to be 0.
$x+4\neq 0\Rightarrow x\neq -4$; Domain: $\{x|x\neq -4\}$.

4. $3-2x>5$

$-2x>2$

$x<-1$

Solution set: $\{x\mid x<-1\}$ or $(-\infty,-1)$

5. $\sqrt{5}+2$

6. radicals

7. independent; dependent

8. **a**

9. **c**

10. False; $g\neq 0$

11. False; every function is a relation, but not every relation is a function. For example, the relation $x^2+y^2=1$ is not a function.

12. verbally, numerically, graphically, algebraically

13. False; if the domain is not specified, we assume it is the largest set of real numbers for which the value of *f* is a real number.

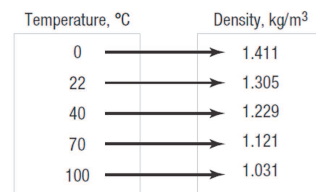
14. False; if x is in the domain of a function *f*, we say that *f* is defined at *x*, or *f(x)* exists.

15. difference quotient

16. explicitly

17. a. Domain: {0,22,40,70,100} in C°
Range: {1.031, 1.121, 1.229, 1.305, 1.411} in kg/m^3

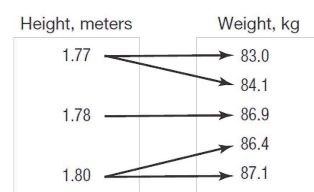
b.



c. {(0, 1.411), (22, 1.305), (40, 1.229), (70, 1.121), (100, 1.031)}

18. a. Domain: {1.80, 1.78, 1.77}
Range: {87.1, 86.9, 92.0, 84.1, 86.4}

b.



c. {(1.80, 87.1), (1.78, 86.9), (1.77, 83.0), (1.77, 84.1), (1.80, 86.4)}

19. Domain: {Elvis, Colleen, Kaleigh, Marissa}
Range: {Jan. 8, Mar. 15, Sept. 17}
Function

20. Domain: {Bob, John, Chuck}
Range: {Beth, Diane, Linda, Marcia}
Not a function

21. Domain: {20, 30, 40}
Range: {200, 300, 350, 425}
Not a function

22. Domain: {Less than 9th grade, 9th-12th grade, High School Graduate, Some College, College Graduate}
Range: {\$18,120, \$23,251, \$36,055, \$45,810, \$67,165}
Function

23. Domain: {-3, 2, 4}
Range: {6, 9, 10}
Not a function

24. Domain: $\{-2, -1, 3, 4\}$
Range: $\{3, 5, 7, 12\}$
Function

25. Domain: $\{1, 2, 3, 4\}$
Range: $\{3\}$
Function

26. Domain: $\{0, 1, 2, 3\}$
Range: $\{-2, 3, 7\}$
Function

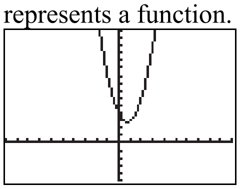
27. Domain: $\{-4, 0, 3\}$
Range: $\{1, 3, 5, 6\}$
Not a function

28. Domain: $\{-4, -3, -2, -1\}$
Range: $\{0, 1, 2, 3, 4\}$
Not a function

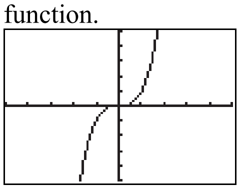
29. Domain: $\{-1, 0, 2, 4\}$
Range: $\{-1, 3, 8\}$
Function

30. Domain: $\{-2, -1, 0, 1\}$
Range: $\{3, 4, 16\}$
Function

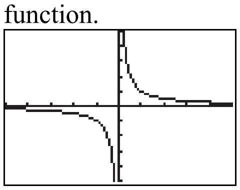
31. Graph $y = 2x^2 - 3x + 4$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



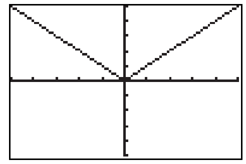
32. Graph $y = x^3$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



33. Graph $y = \frac{1}{x}$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



34. Graph $y = |x|$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



35. $x^2 = 8 - y^2$

Solve for y: $y = \pm\sqrt{8 - x^2}$

For $x = 0$, $y = \pm 2\sqrt{2}$. Thus, $\left(0, -2\sqrt{2}\right)$ and $\left(0, 2\sqrt{2}\right)$ are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

36. $y = \pm\sqrt{1 - 2x}$

For $x = 0$, $y = \pm 1$. Thus, (0, 1) and (0, –1) are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

37. $x = y^2$

Solve for y: $y = \pm\sqrt{x}$

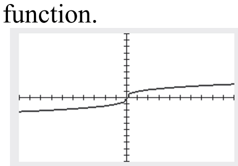
For $x = 1$, $y = \pm 1$. Thus, (1, 1) and (1, –1) are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

38. $x + y^2 = 1$

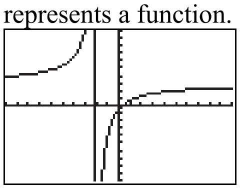
Solve for y: $y = \pm\sqrt{1 - x}$

For $x = 0$, $y = \pm 1$. Thus, (0, 1) and (0, –1) are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

39. Graph $y = \sqrt[3]{x}$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



40. Graph $y = \frac{3x-1}{x+2}$. The graph passes the Vertical-Line Test. Thus, the equation represents a function.



41. $|y| = 2x+3$

Solve for y: $y = 2x+3$ or $y = -(2x+3)$

For $x = 1$, $y = 5$ or $y = -5$. Thus, $(1,5)$ and $(1,-5)$ are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

42. $x^2 - 4y^2 = 1$

Solve for y: $x^2 - 4y^2 = 1$

$$4y^2 = x^2 - 1$$

$$y^2 = \frac{x^2-1}{4}$$

$$y = \frac{\pm\sqrt{x^2-1}}{2}$$

For $x = \sqrt{2}$, $y = \pm\frac{1}{2}$. Thus, $\left(\sqrt{2}, \frac{1}{2}\right)$ and $\left(\sqrt{2}, -\frac{1}{2}\right)$ are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

43. $f(x) = 3x^2 + 2x - 4$

a. $f(0) = 3(0)^2 + 2(0) - 4 = -4$

b. $f(1) = 3(1)^2 + 2(1) - 4 = 3 + 2 - 4 = 1$

c. $f(-1) = 3(-1)^2 + 2(-1) - 4 = 3 - 2 - 4 = -3$

d. $f(-x) = 3(-x)^2 + 2(-x) - 4 = 3x^2 - 2x - 4$

e. $-f(x) = -\left(3x^2 + 2x - 4\right) = -3x^2 - 2x + 4$

f. $$\begin{aligned} f(x+1) &= 3(x+1)^2 + 2(x+1) - 4 \\ &= 3\left(x^2 + 2x + 1\right) + 2x + 2 - 4 \\ &= 3x^2 + 6x + 3 + 2x + 2 - 4 \\ &= 3x^2 + 8x + 1 \end{aligned}$$

g. $f(2x) = 3(2x)^2 + 2(2x) - 4 = 12x^2 + 4x - 4$

h. $$\begin{aligned} f(x+h) &= 3(x+h)^2 + 2(x+h) - 4 \\ &= 3\left(x^2 + 2xh + h^2\right) + 2x + 2h - 4 \\ &= 3x^2 + 6xh + 3h^2 + 2x + 2h - 4 \end{aligned}$$

44. $f(x) = -2x^2 + x - 1$

a. $f(0) = -2(0)^2 + 0 - 1 = -1$

b. $f(1) = -2(1)^2 + 1 - 1 = -2$

c. $f(-1) = -2(-1)^2 + (-1) - 1 = -4$

d. $f(-x) = -2(-x)^2 + (-x) - 1 = -2x^2 - x - 1$

e. $-f(x) = -\left(-2x^2 + x - 1\right) = 2x^2 - x + 1$

f. $$\begin{aligned} f(x+1) &= -2(x+1)^2 + (x+1) - 1 \\ &= -2\left(x^2 + 2x + 1\right) + x + 1 - 1 \\ &= -2x^2 - 4x - 2 + x \\ &= -2x^2 - 3x - 2 \end{aligned}$$

g. $f(2x) = -2(2x)^2 + (2x) - 1 = -8x^2 + 2x - 1$

h. $$\begin{aligned} f(x+h) &= -2(x+h)^2 + (x+h) - 1 \\ &= -2\left(x^2 + 2xh + h^2\right) + x + h - 1 \\ &= -2x^2 - 4xh - 2h^2 + x + h - 1 \end{aligned}$$

45. $f(x) = \frac{x}{x^2+1}$

a. $f(0) = \frac{0}{0^2+1} = \frac{0}{1} = 0$

b. $f(1) = \frac{1}{1^2+1} = \frac{1}{2}$

c. $f(-1) = \frac{-1}{(-1)^2+1} = \frac{-1}{1+1} = -\frac{1}{2}$

d. $f(-x) = \frac{-x}{(-x)^2+1} = \frac{-x}{x^2+1}$

e. $-f(x) = -\left(\frac{x}{x^2+1}\right) = \frac{-x}{x^2+1}$

f. $f(x+1)=\dfrac{x+1}{(x+1)^2+1}$
$=\dfrac{x+1}{x^2+2x+1+1}$
$=\dfrac{x+1}{x^2+2x+2}$

g. $f(2x)=\dfrac{2x}{(2x)^2+1}=\dfrac{2x}{4x^2+1}$

h. $f(x+h)=\dfrac{x+h}{(x+h)^2+1}=\dfrac{x+h}{x^2+2xh+h^2+1}$

46. $f(x)=\dfrac{x^2-1}{x+4}$

a. $f(0)=\dfrac{0^2-1}{0+4}=\dfrac{-1}{4}=-\dfrac{1}{4}$

b. $f(1)=\dfrac{1^2-1}{1+4}=\dfrac{0}{5}=0$

c. $f(-1)=\dfrac{(-1)^2-1}{-1+4}=\dfrac{0}{3}=0$

d. $f(-x)=\dfrac{(-x)^2-1}{-x+4}=\dfrac{x^2-1}{-x+4}$

e. $-f(x)=-\left(\dfrac{x^2-1}{x+4}\right)=\dfrac{-x^2+1}{x+4}$

f. $f(x+1)=\dfrac{(x+1)^2-1}{(x+1)+4}$
$=\dfrac{x^2+2x+1-1}{x+5}=\dfrac{x^2+2x}{x+5}$

g. $f(2x)=\dfrac{(2x)^2-1}{2x+4}=\dfrac{4x^2-1}{2x+4}$

h. $f(x+h)=\dfrac{(x+h)^2-1}{(x+h)+4}=\dfrac{x^2+2xh+h^2-1}{x+h+4}$

47. $f(x)=|x|+4$

a. $f(0)=|0|+4=0+4=4$

b. $f(1)=|1|+4=1+4=5$

c. $f(-1)=|-1|+4=1+4=5$

d. $f(-x)=|-x|+4=|x|+4$

e. $-f(x)=-(|x|+4)=-|x|-4$

f. $f(x+1)=|x+1|+4$

g. $f(2x)=|2x|+4=2|x|+4$

h. $f(x+h)=|x+h|+4$

48. $f(x)=\sqrt{x^2+x}$

a. $f(0)=\sqrt{0^2+0}=\sqrt{0}=0$

b. $f(1)=\sqrt{1^2+1}=\sqrt{2}$

c. $f(-1)=\sqrt{(-1)^2+(-1)}=\sqrt{1-1}=\sqrt{0}=0$

d. $f(-x)=\sqrt{(-x)^2+(-x)}=\sqrt{x^2-x}$

e. $-f(x)=-\left(\sqrt{x^2+x}\right)=-\sqrt{x^2+x}$

f. $f(x+1)=\sqrt{(x+1)^2+(x+1)}$
$=\sqrt{x^2+2x+1+x+1}$
$=\sqrt{x^2+3x+2}$

g. $f(2x)=\sqrt{(2x)^2+2x}=\sqrt{4x^2+2x}$

h. $f(x+h)=\sqrt{(x+h)^2+(x+h)}$
$=\sqrt{x^2+2xh+h^2+x+h}$

49. $f(x)=\dfrac{2x+1}{3x-5}$

a. $f(0)=\dfrac{2(0)+1}{3(0)-5}=\dfrac{0+1}{0-5}=-\dfrac{1}{5}$

b. $f(1)=\dfrac{2(1)+1}{3(1)-5}=\dfrac{2+1}{3-5}=\dfrac{3}{-2}=-\dfrac{3}{2}$

c. $f(-1)=\dfrac{2(-1)+1}{3(-1)-5}=\dfrac{-2+1}{-3-5}=\dfrac{-1}{-8}=\dfrac{1}{8}$

d. $f(-x)=\dfrac{2(-x)+1}{3(-x)-5}=\dfrac{-2x+1}{-3x-5}=\dfrac{2x-1}{3x+5}$

e. $-f(x)=-\left(\dfrac{2x+1}{3x-5}\right)=\dfrac{-2x-1}{3x-5}$

f. $f(x+1)=\dfrac{2(x+1)+1}{3(x+1)-5}=\dfrac{2x+2+1}{3x+3-5}=\dfrac{2x+3}{3x-2}$

g. $f(2x)=\dfrac{2(2x)+1}{3(2x)-5}=\dfrac{4x+1}{6x-5}$

h. $f(x+h)=\dfrac{2(x+h)+1}{3(x+h)-5}=\dfrac{2x+2h+1}{3x+3h-5}$

50. $f(x)=1-\dfrac{1}{(x+2)^2}$

a. $f(0)=1-\dfrac{1}{(0+2)^2}=1-\dfrac{1}{4}=\dfrac{3}{4}$

b. $f(1)=1-\dfrac{1}{(1+2)^2}=1-\dfrac{1}{9}=\dfrac{8}{9}$

c. $f(-1)=1-\dfrac{1}{(-1+2)^2}=1-\dfrac{1}{1}=0$

d. $f(-x)=1-\dfrac{1}{(-x+2)^2}$

e. $-f(x)=-\left(1-\dfrac{1}{(x+2)^2}\right)=\dfrac{1}{(x+2)^2}-1$

f. $f(x+1)=1-\dfrac{1}{(x+1+2)^2}=1-\dfrac{1}{(x+3)^2}$

g. $f(2x)=1-\dfrac{1}{(2x+2)^2}=1-\dfrac{1}{4(x+1)^2}$

h. $f(x+h)=1-\dfrac{1}{(x+h+2)^2}$

51. $f(x)=-5x+4$

Domain: $\{x \mid x \text{ is any real number}\}$

52. $f(x)=x^2+2$

Domain: $\{x \mid x \text{ is any real number}\}$

53. $f(x)=\dfrac{x+1}{2x^2+8}$

Domain: $\{x \mid x \text{ is any real number}\}$

54. $f(x)=\dfrac{x^2}{x^2+1}$

Domain: $\{x \mid x \text{ is any real number}\}$

55. $g(x)=\dfrac{x}{x^2-16}$

$x^2-16\neq 0$

$x^2\neq 16 \Rightarrow x\neq \pm 4$

Domain: $\{x \mid x\neq -4,\, x\neq 4\}$

56. $h(x)=\dfrac{2x}{x^2-4}$

$x^2-4\neq 0$

$x^2\neq 4 \Rightarrow x\neq \pm 2$

Domain: $\{x \mid x\neq -2,\, x\neq 2\}$

57. $F(x)=\dfrac{x-2}{x^3+x}$

$x^3+x\neq 0$

$x(x^2+1)\neq 0$

$x\neq 0,\quad x^2\neq -1$

Domain: $\{x \mid x\neq 0\}$

58. $G(x)=\dfrac{x+4}{x^3-4x}$

$x^3-4x\neq 0$

$x(x^2-4)\neq 0$

$x\neq 0,\quad x^2\neq 4$

$x\neq 0,\quad x\neq \pm 2$

Domain: $\{x \mid x\neq -2,\, x\neq 0,\, x\neq 2\}$

59. $h(x)=\sqrt{3x-12}$

$3x-12\geq 0$

$3x\geq 12$

$x\geq 4$

Domain: $\{x \mid x\geq 4\}$

60. $G(x)=\sqrt{1-x}$

$1-x\ge 0$

$-x\ge -1$

$x\le 1$

Domain: $\{x \mid x\le 1\}$

61. $p(x)=\dfrac{x}{|2x+3|-1}$

$|2x+3|-1=0$

$|2x+3|=1$

$2x+3=-1$ or $2x+3=1$

$2x=-4 \qquad 2x=-2$

$x=-2 \qquad x=-1$

Domain: $\{x \mid x\ne -2,\ x\ne -1\}$

62. $p(x)=\dfrac{x-1}{|3x-1|-4}$

$|3x-1|-4=0$

$|3x-1|=4$

$3x-1=-4$ or $3x-1=4$

$3x=-3 \qquad 3x=5$

$x=-1 \qquad x=\dfrac{5}{3}$

Domain: $\left\{x \,\middle|\, x\ne -1,\ x\ne \dfrac{5}{3}\right\}$

63. $f(x)=\dfrac{x}{\sqrt{x-4}}$

$x-4>0$

$x>4$

Domain: $\{x \mid x>4\}$

64. $q(x)=\frac{-x}{\sqrt{-x-2}}$

$-x-2>0$

$-x>2$

$x<-2$

Domain: $\{x \mid x<-2\}$

65. $P(t)=\dfrac{\sqrt{t-4}}{3t-21}$

$t-4\ge 0$

$t\ge 4$

Also $3t-21\ne 0$

$3t-21\ne 0$

$3t\ne 21$

$t\ne 7$

Domain: $\{t \mid t\ge 4, t\ne 7\}$

66. $h(z)=\dfrac{\sqrt{z+3}}{z-2}$

$z+3\ge 0$

$z\ge -3$

Also $z-2\ne 0$

$z\ne 2$

Domain: $\{z \mid z\ge -3, z\ne 2\}$

67. $f(x)=\sqrt[3]{5x-4}$

Domain: $\{x \mid x \text{ is any real number}\}$.

68. $g(t)=-t^2+\sqrt[3]{t^2+7t}$

Domain: $\{t \mid t \text{ is any real number}\}$.

69. $M(t)=\sqrt[5]{\dfrac{t+1}{t^2-5t-14}}$

$t^2-5t-14=0$

$(t+2)(t-7)=0$

$t+2=0$ or $t-7=0$

$t=-2 \qquad t=7$

Domain: $\{t \mid t\ne -2,\ x\ne 7\}$

70. $N(p)=\sqrt[5]{\dfrac{p}{2p^2-98}}$

$2p^2-98=0$

$2(p^2-49)=0$

$2(p+7)(p-7)=0$

$p+7=0$ or $p-7=0$

$p=-7 \qquad p=7$

Domain: $\{p \mid p\ne -7,\ x\ne 7\}$

71. $f(x)=3x+4 \qquad g(x)=2x-3$

a. $(f+g)(x)=3x+4+2x-3=5x+1$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x) = (3x+4)-(2x-3)$
$= 3x+4-2x+3$
$= x+7$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f \cdot g)(x) = (3x+4)(2x-3)$
$= 6x^2-9x+8x-12$
$= 6x^2-x-12$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{3x+4}{2x-3}$

$2x-3 \neq 0 \Rightarrow 2x \neq 3 \Rightarrow x \neq \dfrac{3}{2}$

Domain: $\left\{x \,\middle|\, x \neq \dfrac{3}{2}\right\}$.

e. $(f+g)(3) = 5(3)+1 = 15+1 = 16$

f. $(f-g)(4) = 4+7 = 11$

g. $(f \cdot g)(2) = 6(2)^2-2-12 = 24-2-12 = 10$

h. $\left(\dfrac{f}{g}\right)(1) = \dfrac{3(1)+4}{2(1)-3} = \dfrac{3+4}{2-3} = \dfrac{7}{-1} = -7$

72. $f(x) = 2x+1 \qquad g(x) = 3x-2$

a. $(f+g)(x) = 2x+1+3x-2 = 5x-1$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x) = (2x+1)-(3x-2)$
$= 2x+1-3x+2$
$= -x+3$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f \cdot g)(x) = (2x+1)(3x-2)$
$= 6x^2-4x+3x-2$
$= 6x^2-x-2$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x+1}{3x-2}$

$3x-2 \neq 0$

$3x \neq 2 \Rightarrow x \neq \dfrac{2}{3}$

Domain: $\left\{x \,\middle|\, x \neq \dfrac{2}{3}\right\}$.

e. $(f+g)(3) = 5(3)-1 = 15-1 = 14$

f. $(f-g)(4) = -4+3 = -1$

g. $(f \cdot g)(2) = 6(2)^2-2-2$
$= 6(4)-2-2$
$= 24-2-2 = 20$

h. $\left(\dfrac{f}{g}\right)(1) = \dfrac{2(1)+1}{3(1)-2} = \dfrac{2+1}{3-2} = \dfrac{3}{1} = 3$

73. $f(x) = x-1 \qquad g(x) = 2x^2$

a. $(f+g)(x) = x-1+2x^2 = 2x^2+x-1$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x) = (x-1)-(2x^2)$
$= x-1-2x^2$
$= -2x^2+x-1$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f \cdot g)(x) = (x-1)(2x^2) = 2x^3-2x^2$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{x-1}{2x^2}$

Domain: $\{x \mid x \neq 0\}$.

e. $(f+g)(3) = 2(3)^2+3-1$
$= 2(9)+3-1$
$= 18+3-1 = 20$

f. $(f-g)(4) = -2(4)^2+4-1$
$= -2(16)+4-1$
$= -32+4-1 = -29$

g. $(f \cdot g)(2) = 2(2)^3-2(2)^2$
$= 2(8)-2(4)$
$= 16-8 = 8$

h. $\left(\dfrac{f}{g}\right)(1) = \dfrac{1-1}{2(1)^2} = \dfrac{0}{2(1)} = \dfrac{0}{2} = 0$

74. $f(x) = 2x^2+3 \qquad g(x) = 4x^3+1$

a. $(f+g)(x) = 2x^2+3+4x^3+1$
$= 4x^3+2x^2+4$
Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x) = \left(2x^2+3\right)-\left(4x^3+1\right)$
$= 2x^2+3-4x^3-1$
$= -4x^3+2x^2+2$
Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f \cdot g)(x) = \left(2x^2+3\right)\left(4x^3+1\right)$
$= 8x^5+12x^3+2x^2+3$
Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x^2+3}{4x^3+1}$
$4x^3+1 \neq 0$
$4x^3 \neq -1$
$x^3 \neq -\dfrac{1}{4} \Rightarrow x \neq \sqrt[3]{-\dfrac{1}{4}} = -\dfrac{\sqrt[3]{2}}{2}$
Domain: $\left\{x \,\middle|\, x \neq -\dfrac{\sqrt[3]{2}}{2}\right\}$.

e. $(f+g)(3) = 4(3)^3+2(3)^2+4$
$= 4(27)+2(9)+4$
$= 108+18+4 = 130$

f. $(f-g)(4) = -4(4)^3+2(4)^2+2$
$= -4(64)+2(16)+2$
$= -256+32+2 = -222$

g. $(f \cdot g)(2) = 8(2)^5+12(2)^3+2(2)^2+3$
$= 8(32)+12(8)+2(4)+3$
$= 256+96+8+3 = 363$

h. $\left(\dfrac{f}{g}\right)(1) = \dfrac{2(1)^2+3}{4(1)^3+1} = \dfrac{2(1)+3}{4(1)+1} = \dfrac{2+3}{4+1} = \dfrac{5}{5} = 1$

75. $f(x) = \sqrt{x} \qquad g(x) = 3x-5$

a. $(f+g)(x) = \sqrt{x}+3x-5$
Domain: $\{x \mid x \geq 0\}$.

b. $(f-g)(x) = \sqrt{x}-(3x-5) = \sqrt{x}-3x+5$
Domain: $\{x \mid x \geq 0\}$.

c. $(f \cdot g)(x) = \sqrt{x}(3x-5) = 3x\sqrt{x}-5\sqrt{x}$
Domain: $\{x \mid x \geq 0\}$.

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x}}{3x-5}$
$x \geq 0$ and $3x-5 \neq 0$
$3x \neq 5 \Rightarrow x \neq \dfrac{5}{3}$
Domain: $\left\{x \,\middle|\, x \geq 0 \text{ and } x \neq \dfrac{5}{3}\right\}$.

e. $(f+g)(3) = \sqrt{3}+3(3)-5$
$= \sqrt{3}+9-5 = \sqrt{3}+4$

f. $(f-g)(4) = \sqrt{4}-3(4)+5$
$= 2-12+5 = -5$

g. $(f \cdot g)(2) = 3(2)\sqrt{2}-5\sqrt{2}$
$= 6\sqrt{2}-5\sqrt{2} = \sqrt{2}$

h. $\left(\dfrac{f}{g}\right)(1) = \dfrac{\sqrt{1}}{3(1)-5} = \dfrac{1}{3-5} = \dfrac{1}{-2} = -\dfrac{1}{2}$

76. $f(x) = |x| \qquad g(x) = x$

a. $(f+g)(x) = |x|+x$
Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x) = |x|-x$
Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f \cdot g)(x) = |x| \cdot x = x|x|$
Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{|x|}{x}$
Domain: $\{x \mid x \neq 0\}$.

e. $(f+g)(3) = |3|+3 = 3+3 = 6$

f. $(f-g)(4) = |4|-4 = 4-4 = 0$

g. $(f \cdot g)(2) = 2|2| = 2 \cdot 2 = 4$

h. $\left(\dfrac{f}{g}\right)(1) = \dfrac{|1|}{1} = \dfrac{1}{1} = 1$

77. $f(x) = 1 + \dfrac{1}{x} \qquad g(x) = \dfrac{1}{x}$

a. $(f+g)(x) = 1 + \dfrac{1}{x} + \dfrac{1}{x} = 1 + \dfrac{2}{x}$

Domain: $\{x \mid x \neq 0\}$.

b. $(f-g)(x) = 1 + \dfrac{1}{x} - \dfrac{1}{x} = 1$

Domain: $\{x \mid x \neq 0\}$.

c. $(f \cdot g)(x) = \left(1 + \dfrac{1}{x}\right)\dfrac{1}{x} = \dfrac{1}{x} + \dfrac{1}{x^2}$

Domain: $\{x \mid x \neq 0\}$.

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{1+\dfrac{1}{x}}{\dfrac{1}{x}} = \dfrac{\dfrac{x+1}{x}}{\dfrac{1}{x}} = \dfrac{x+1}{x} \cdot \dfrac{x}{1} = x+1$

Domain: $\{x \mid x \neq 0\}$.

e. $(f+g)(3) = 1 + \dfrac{2}{3} = \dfrac{5}{3}$

f. $(f-g)(4) = 1$

g. $(f \cdot g)(2) = \dfrac{1}{2} + \dfrac{1}{(2)^2} = \dfrac{1}{2} + \dfrac{1}{4} = \dfrac{3}{4}$

h. $\left(\dfrac{f}{g}\right)(1) = 1 + 1 = 2$

78. $f(x) = \sqrt{x-1} \qquad g(x) = \sqrt{4-x}$

a. $(f+g)(x) = \sqrt{x-1} + \sqrt{4-x}$

$x - 1 \geq 0$ and $4 - x \geq 0$

$x \geq 1$ and $-x \geq -4$

$x \leq 4$

Domain: $\{x \mid 1 \leq x \leq 4\}$.

b. $(f-g)(x) = \sqrt{x-1} - \sqrt{4-x}$

$x - 1 \geq 0$ and $4 - x \geq 0$

$x \geq 1$ and $-x \geq -4$

$x \leq 4$

Domain: $\{x \mid 1 \leq x \leq 4\}$.

c. $(f \cdot g)(x) = \left(\sqrt{x-1}\right)\left(\sqrt{4-x}\right)$

$= \sqrt{-x^2 + 5x - 4}$

$x - 1 \geq 0$ and $4 - x \geq 0$

$x \geq 1$ and $-x \geq -4$

$x \leq 4$

Domain: $\{x \mid 1 \leq x \leq 4\}$.

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x-1}}{\sqrt{4-x}} = \sqrt{\dfrac{x-1}{4-x}}$

$x - 1 \geq 0$ and $4 - x > 0$

$x \geq 1$ and $-x > -4$

$x < 4$

Domain: $\{x \mid 1 \leq x < 4\}$.

e. $(f+g)(3) = \sqrt{3-1} + \sqrt{4-3}$

$= \sqrt{2} + \sqrt{1} = \sqrt{2} + 1$

f. $(f-g)(4) = \sqrt{4-1} - \sqrt{4-4}$

$= \sqrt{3} - \sqrt{0} = \sqrt{3} - 0 = \sqrt{3}$

g. $(f \cdot g)(2) = \sqrt{-(2)^2 + 5(2) - 4}$

$= \sqrt{-4 + 10 - 4} = \sqrt{2}$

h. $\left(\dfrac{f}{g}\right)(1) = \sqrt{\dfrac{1-1}{4-1}} = \sqrt{\dfrac{0}{3}} = \sqrt{0} = 0$

79. $f(x) = \dfrac{2x+3}{3x-2} \qquad g(x) = \dfrac{4x}{3x-2}$

a. $(f+g)(x) = \dfrac{2x+3}{3x-2} + \dfrac{4x}{3x-2}$

$= \dfrac{2x+3+4x}{3x-2} = \dfrac{6x+3}{3x-2}$

$3x - 2 \neq 0$

$3x \neq 2 \Rightarrow x \neq \frac{2}{3}$

Domain: $\left\{x \,\middle|\, x \neq \frac{2}{3}\right\}$.

b. $(f-g)(x)=\dfrac{2x+3}{3x-2}-\dfrac{4x}{3x-2}$
$=\dfrac{2x+3-4x}{3x-2}=\dfrac{-2x+3}{3x-2}$

$3x-2\neq 0$

$3x\neq 2 \Rightarrow x\neq \dfrac{2}{3}$

Domain: $\left\{x \,\middle|\, x\neq \dfrac{2}{3}\right\}$.

c. $(f\cdot g)(x)=\left(\dfrac{2x+3}{3x-2}\right)\left(\dfrac{4x}{3x-2}\right)=\dfrac{8x^2+12x}{(3x-2)^2}$

$3x-2\neq 0$

$3x\neq 2 \Rightarrow x\neq \dfrac{2}{3}$

Domain: $\left\{x \,\middle|\, x\neq \dfrac{2}{3}\right\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{\frac{2x+3}{3x-2}}{\frac{4x}{3x-2}}=\dfrac{2x+3}{3x-2}\cdot\dfrac{3x-2}{4x}=\dfrac{2x+3}{4x}$

$3x-2\neq 0$ and $x\neq 0$

$3x\neq 2$

$x\neq \dfrac{2}{3}$

Domain: $\left\{x \,\middle|\, x\neq \dfrac{2}{3} \text{ and } x\neq 0\right\}$.

e. $(f+g)(3)=\dfrac{6(3)+3}{3(3)-2}=\dfrac{18+3}{9-2}=\dfrac{21}{7}=3$

f. $(f-g)(4)=\dfrac{-2(4)+3}{3(4)-2}=\dfrac{-8+3}{12-2}=\dfrac{-5}{10}=-\dfrac{1}{2}$

g. $(f\cdot g)(2)=\dfrac{8(2)^2+12(2)}{(3(2)-2)^2}$
$=\dfrac{8(4)+24}{(6-2)^2}=\dfrac{32+24}{(4)^2}=\dfrac{56}{16}=\dfrac{7}{2}$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{2(1)+3}{4(1)}=\dfrac{2+3}{4}=\dfrac{5}{4}$

80. $f(x)=\sqrt{x+1}$ $\quad g(x)=\dfrac{2}{x}$

a. $(f+g)(x)=\sqrt{x+1}+\dfrac{2}{x}$

$x+1\geq 0$ and $x\neq 0$

$x\geq -1$

Domain: $\{x \mid x\geq -1, \text{ and } x\neq 0\}$.

b. $(f-g)(x)=\sqrt{x+1}-\dfrac{2}{x}$

$x+1\geq 0$ and $x\neq 0$

$x\geq -1$

Domain: $\{x \mid x\geq -1, \text{ and } x\neq 0\}$.

c. $(f\cdot g)(x)=\sqrt{x+1}\cdot\dfrac{2}{x}=\dfrac{2\sqrt{x+1}}{x}$

$x+1\geq 0$ and $x\neq 0$

$x\geq -1$

Domain: $\{x \mid x\geq -1, \text{ and } x\neq 0\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{x+1}}{\frac{2}{x}}=\dfrac{x\sqrt{x+1}}{2}$

$x+1\geq 0$ and $x\neq 0$

$x\geq -1$

Domain: $\{x \mid x\geq -1, \text{ and } x\neq 0\}$.

e. $(f+g)(3)=\sqrt{3+1}+\dfrac{2}{3}=\sqrt{4}+\dfrac{2}{3}=2+\dfrac{2}{3}=\dfrac{8}{3}$

f. $(f-g)(4)=\sqrt{4+1}-\dfrac{2}{4}=\sqrt{5}-\dfrac{1}{2}$

g. $(f\cdot g)(2)=\dfrac{2\sqrt{2+1}}{2}=\dfrac{2\sqrt{3}}{2}=\sqrt{3}$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{1\sqrt{1+1}}{2}=\dfrac{\sqrt{2}}{2}$

81. $f(x)=3x+1$ $\quad (f+g)(x)=6-\dfrac{1}{2}x$

$6-\dfrac{1}{2}x=3x+1+g(x)$

$5-\dfrac{7}{2}x=g(x)$

$g(x)=5-\dfrac{7}{2}x$

82. $f(x)=\dfrac{1}{x}$ $\quad \left(\dfrac{f}{g}\right)(x)=\dfrac{x+1}{x^2-x}$

$$\frac{x+1}{x^2-x}=\frac{\frac{1}{x}}{g(x)}$$

$$g(x)=\frac{\frac{1}{x}}{\frac{x+1}{x^2-x}}=\frac{1}{x}\cdot\frac{x^2-x}{x+1}$$

$$=\frac{1}{x}\cdot\frac{x(x-1)}{x+1}=\frac{x-1}{x+1}$$

83. $f(x)=4x+3$

$$\frac{f(x+h)-f(x)}{h}=\frac{4(x+h)+3-(4x+3)}{h}$$

$$=\frac{4x+4h+3-4x-3}{h}$$

$$=\frac{4h}{h}=4$$

84. $f(x)=-3x+1$

$$\frac{f(x+h)-f(x)}{h}=\frac{-3(x+h)+1-(-3x+1)}{h}$$

$$=\frac{-3x-3h+1+3x-1}{h}$$

$$=\frac{-3h}{h}=-3$$

85. $f(x)=x^2-4$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{(x+h)^2-4-(x^2-4)}{h}$$

$$=\frac{x^2+2xh+h^2-4-x^2+4}{h}$$

$$=\frac{2xh+h^2}{h}$$

$$=2x+h$$

86. $f(x)=3x^2+2$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{3(x+h)^2+2-(3x^2+2)}{h}$$

$$=\frac{3x^2+6xh+3h^2+2-3x^2-2}{h}$$

$$=\frac{6xh+3h^2}{h}$$

$$=6x+3h$$

87. $f(x)=x^2-x+4$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{(x+h)^2-(x+h)+4-(x^2-x+4)}{h}$$

$$=\frac{x^2+2xh+h^2-x-h+4-x^2+x-4}{h}$$

$$=\frac{2xh+h^2-h}{h}$$

$$=2x+h-1$$

88. $f(x)=3x^2-2x+6$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{\left[3(x+h)^2-2(x+h)+6\right]-\left[3x^2-2x+6\right]}{h}$$

$$=\frac{3\left(x^2+2xh+h^2\right)-2x-2h+6-3x^2+2x-6}{h}$$

$$=\frac{3x^2+6xh+3h^2-2h-3x^2}{h}=\frac{6xh+3h^2-2h}{h}$$

$$=6x+3h-2$$

89. $f(x)=\dfrac{5}{4x-3}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{5}{4(x+h)-3}-\frac{5}{4x-3}}{h}$$
$$=\frac{\frac{5(4x-3)-5\left(4(x+h)-3\right)}{\left(4(x+h)-3\right)(4x-3)}}{h}$$
$$=\left(\frac{20x+15-20x-15-20h}{\left(4(x+h)-3\right)(4x-3)}\right)\left(\frac{1}{h}\right)$$
$$=\left(\frac{-20h}{\left(4(x+h)-3\right)(4x-3)}\right)\left(\frac{1}{h}\right)$$
$$=\frac{-20}{\left(4(x+h)-3\right)(4x-3)}$$

90. $f(x)=\dfrac{1}{x+3}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{x+h+3}-\frac{1}{x+3}}{h}$$
$$=\frac{\frac{x+3-(x+3+h)}{(x+h+3)(x+3)}}{h}$$
$$=\left(\frac{x+3-x-3-h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right)$$
$$=\left(\frac{-h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right)$$
$$=\frac{-1}{(x+h+3)(x+3)}$$

91. $f(x)=\dfrac{2x}{x+3}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{2(x+h)}{x+h+3}-\frac{2x}{x+3}}{h}$$
$$=\frac{\frac{2(x+h)(x+3)-2x(x+3+h)}{(x+h+3)(x+3)}}{h}$$
$$=\frac{\frac{2x^2+6x+2hx+6h-2x^2-6x-2xh}{(x+h+3)(x+3)}}{h}$$
$$=\left(\frac{6h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right)$$
$$=\frac{6}{(x+h+3)(x+3)}$$

92. $f(x)=\dfrac{5x}{x-4}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{5(x+h)}{x+h-4}-\frac{5x}{x-4}}{h}$$
$$=\frac{\frac{5(x+h)(x-4)-5x(x-4+h)}{(x+h-4)(x-4)}}{h}$$
$$=\frac{\frac{5x^2-20x+5hx-20h-5x^2+20x-5xh}{(x+h-4)(x-4)}}{h}$$
$$=\left(\frac{-20h}{(x+h-4)(x-4)}\right)\left(\frac{1}{h}\right)$$
$$=-\frac{20}{(x+h-4)(x-4)}$$

93. $f\left(x\right)=\sqrt{x-2}$

$$\frac{f\left(x+h\right)-f\left(x\right)}{h}$$
$$=\frac{\sqrt{x+h-2}-\sqrt{x-2}}{h}$$
$$=\frac{\sqrt{x+h-2}-\sqrt{x-2}}{h}\cdot\frac{\sqrt{x+h-2}+\sqrt{x-2}}{\sqrt{x+h-2}+\sqrt{x-2}}$$
$$=\frac{x+h-2-x+2}{h\left(\sqrt{x+h-2}+\sqrt{x-2}\right)}$$
$$=\frac{h}{h\left(\sqrt{x+h-2}+\sqrt{x-2}\right)}$$
$$=\frac{1}{\sqrt{x+h-2}+\sqrt{x-2}}$$

94. $f(x)=\sqrt{x+1}$

$$\frac{f\left(x+h\right)-f\left(x\right)}{h}$$
$$=\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}$$
$$=\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}\cdot\frac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}$$
$$=\frac{x+h+1-(x+1)}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}=\frac{h}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$
$$=\frac{1}{\sqrt{x+h+1}+\sqrt{x+1}}$$

95. $f\left(x\right)=\dfrac{1}{x^2}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\dfrac{1}{\left(x+h\right)^2}-\dfrac{1}{x^2}}{h}$$
$$=\frac{\dfrac{x^2-\left(x+h\right)^2}{x^2\left(x+h\right)^2}}{h}$$
$$=\frac{\dfrac{x-\left(x^2+2xh+h^2\right)}{x^2\left(x+h\right)^2}}{h}$$
$$=\left(\frac{1}{h}\right)\frac{-2xh-h^2}{x^2\left(x+h\right)^2}$$
$$=\left(\frac{1}{h}\right)\frac{h\left(-2x-h\right)}{x^2\left(x+h\right)^2}$$
$$=\frac{-2x-h}{x^2\left(x+h\right)^2}=\frac{-\left(2x+h\right)}{x^2\left(x+h\right)^2}$$

96. $f\left(x\right)=\dfrac{1}{x^2+1}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\dfrac{1}{\left(x+h\right)^2+1}-\dfrac{1}{x^2+1}}{h}$$
$$=\frac{\dfrac{x^2+1-\left(\left(x+h\right)^2+1\right)}{(x^2+1)(\left(x+h\right)^2+1)}}{h}$$
$$=\frac{\dfrac{x^2+1-\left(x^2+2xh+h^2\right)-1}{(x^2+1)(\left(x+h\right)^2+1)}}{h}$$
$$=\left(\frac{1}{h}\right)\frac{-2xh-h^2}{(x^2+1)(\left(x+h\right)^2+1)}$$
$$=\left(\frac{1}{h}\right)\frac{h\left(-2x-h\right)}{(x^2+1)(\left(x+h\right)^2+1)}$$
$$=\frac{-2x-h}{(x^2+1)(\left(x+h\right)^2+1)}$$
$$=\frac{-\left(2x+h\right)}{(x^2+1)(\left(x+h\right)^2+1)}$$

97. $f(x)=\sqrt{4-x^2}$

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\sqrt{4-(x+h)^2}-\sqrt{4-x^2}}{h}$$
$$=\frac{\sqrt{4-(x+h)^2}-\sqrt{4-x^2}}{h}\cdot\frac{\sqrt{4-(x+h)^2}+\sqrt{4-x^2}}{\sqrt{4-(x+h)^2}+\sqrt{4-x^2}}$$
$$=\frac{4-(x+h)^2-(4-x^2)}{h\left(\sqrt{4-(x+h)^2}-\sqrt{4-x^2}\right)}$$
$$=\frac{4-(x^2+2xh+h^2)-(4-x^2)}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$
$$=\frac{-2xh-h^2}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$
$$=\frac{-2x-h}{\sqrt{4-(x+h)^2}-\sqrt{4-x^2}}$$
$$=\frac{-(2x+h)}{\sqrt{4-(x+h)^2}-\sqrt{4-x^2}}$$

98. $f(x)=\dfrac{1}{\sqrt{x+2}}$

$$\frac{f(x+h)-f(x)}{h}=$$
$$\frac{\dfrac{1}{\sqrt{x+h+2}}-\dfrac{1}{\sqrt{x+2}}}{h}=$$
$$\frac{\sqrt{x+2}-\sqrt{x+h+2}}{h\sqrt{x+2}\sqrt{x+h+2}}=$$
$$\frac{\sqrt{x+2}-\sqrt{x+h+2}}{h\sqrt{x+2}\sqrt{x+h+2}}\cdot\frac{\sqrt{x+2}+\sqrt{x+h+2}}{\sqrt{x+2}+\sqrt{x+h+2}}=$$
$$\frac{x+2-(x+h+2)}{h(x+2)\sqrt{x+h+2}+(x+h+2)\sqrt{x+2}}=$$
$$\frac{x+2-x-h-2}{h(x+2)\sqrt{x+h+2}+(x+h+2)\sqrt{x+2}}=$$
$$\frac{-h}{h(x+2)\sqrt{x+h+2}+(x+h+2)\sqrt{x+2}}=$$
$$-\frac{1}{(x+2)\sqrt{x+h+2}+(x+h+2)\sqrt{x+2}}$$

99.
$$11=x^2-2x+3$$
$$0=x^2-2x-8$$
$$0=(x-4)(x+2)$$
$$x-4=0 \quad \text{or} \quad x+2=0$$
$$x=4 \quad \text{or} \quad x=-2$$

The solution set is: $\{-2,4\}$

100.
$$-\frac{7}{16}=\frac{5}{6}x-\frac{3}{4}$$
$$-\frac{7}{16}+\frac{3}{4}=\frac{5}{6}x$$
$$\frac{5}{6}x=-\frac{7}{16}+\frac{12}{16}$$
$$\frac{5}{6}x=\frac{5}{16}$$
$$x=\frac{5}{16}\cdot\frac{6}{5}=\frac{3}{8}$$

The solution set is: $\left\{\dfrac{3}{8}\right\}$

101. $f(x)=2x^3+Ax^2+4x-5$ and $f(2)=5$
$$f(2)=2(2)^3+A(2)^2+4(2)-5$$
$$5=16+4A+8-5$$
$$5=4A+19$$
$$-14=4A$$
$$A=\frac{-14}{4}=-\frac{7}{2}$$

102. $f(x)=3x^2-Bx+4$ and $f(-1)=12$:
$$f(-1)=3(-1)^2-B(-1)+4$$
$$12=3+B+4$$
$$B=5$$

103. $f(x)=\dfrac{3x+8}{2x-A}$ and $f(0)=2$
$$f(0)=\frac{3(0)+8}{2(0)-A}$$
$$2=\frac{8}{-A}$$
$$-2A=8$$
$$A=-4$$

104. $f(x)=\dfrac{2x-B}{3x+4}$ and $f(2)=\dfrac{1}{2}$

$$f(2)=\frac{2(2)-B}{3(2)+4}$$
$$\frac{1}{2}=\frac{4-B}{10}$$
$$5=4-B$$
$$B=-1$$

105. Let x represent the length of the rectangle.

Then, $\dfrac{x}{2}$ represents the width of the rectangle since the length is twice the width. The function for the area is: $A(x)=x\cdot\dfrac{x}{2}=\dfrac{x^2}{2}=\dfrac{1}{2}x^2$

106. Let x represent the length of one of the two equal sides. The function for the area is:
$A(x)=\dfrac{1}{2}\cdot x\cdot x=\dfrac{1}{2}x^2$

107. Let x represent the number of hours worked. The function for the gross salary is:
$G(x)=16x$

108. Let x represent the number of items sold. The function for the gross salary is:
$G(x)=10x+100$

109. a. $H(1)=20-4.9(1)^2$
$=20-4.9=15.1$ meters

$H(1.1)=20-4.9(1.1)^2$
$=20-4.9(1.21)$
$=20-5.929=14.071$ meters

$H(1.2)=20-4.9(1.2)^2$
$=20-4.9(1.44)$
$=20-7.056=12.944$ meters

b. $H(x)=15$:
$15=20-4.9x^2$
$-5=-4.9x^2$
$x^2\approx 1.0204$
$x\approx 1.01$ seconds

$H(x)=10$:
$10=20-4.9x^2$
$-10=-4.9x^2$
$x^2\approx 2.0408$
$x\approx 1.43$ seconds

$H(x)=5$:
$5=20-4.9x^2$
$-15=-4.9x^2$
$x^2\approx 3.0612$
$x\approx 1.75$ seconds

c. $H(x)=0$
$0=20-4.9x^2$
$-20=-4.9x^2$
$x^2\approx 4.0816$
$x\approx 2.02$ seconds

110. a. $H(1)=20-13(1)^2=20-13=7$ meters

$H(1.1)=20-13(1.1)^2=20-13(1.21)$
$=20-15.73=4.27$ meters

$H(1.2)=20-13(1.2)^2=20-13(1.44)$
$=20-18.72=1.28$ meters

b. $H(x)=15$
$15=20-13x^2$
$-5=-13x^2$
$x^2\approx 0.3846$
$x\approx 0.62$ seconds

$H(x)=10$
$10=20-13x^2$
$-10=-13x^2$
$x^2\approx 0.7692$
$x\approx 0.88$ seconds

$$H(x)=5$$
$$5=20-13x^2$$
$$-15=-13x^2$$
$$x^2\approx 1.1538$$
$$x\approx 1.07\text{ seconds}$$

c. $H(x)=0$
$$0=20-13x^2$$
$$-20=-13x^2$$
$$x^2\approx 1.5385$$
$$x\approx 1.24\text{ seconds}$$

111. $C(x)=100+\dfrac{x}{10}+\dfrac{36{,}000}{x}$

a. $C(500)=100+\dfrac{500}{10}+\dfrac{36{,}000}{500}$
$=100+50+72$
$=\$222$

b. $C(450)=100+\dfrac{450}{10}+\dfrac{36{,}000}{450}$
$=100+45+80$
$=\$225$

c. $C(600)=100+\dfrac{600}{10}+\dfrac{36{,}000}{600}$
$=100+60+60$
$=\$220$

d. $C(400)=100+\dfrac{400}{10}+\dfrac{36{,}000}{400}$
$=100+40+90$
$=\$230$

112. $A(x)=4x\sqrt{1-x^2}$

a. $A\left(\dfrac{1}{3}\right)=4\cdot\dfrac{1}{3}\sqrt{1-\left(\dfrac{1}{3}\right)^2}=\dfrac{4}{3}\sqrt{\dfrac{8}{9}}=\dfrac{4}{3}\cdot\dfrac{2\sqrt{2}}{3}$
$=\dfrac{8\sqrt{2}}{9}\approx 1.26\text{ ft}^2$

b. $A\left(\dfrac{1}{2}\right)=4\cdot\dfrac{1}{2}\sqrt{1-\left(\dfrac{1}{2}\right)^2}=2\sqrt{\dfrac{3}{4}}=2\cdot\dfrac{\sqrt{3}}{2}$
$=\sqrt{3}\approx 1.73\text{ ft}^2$

c. $A\left(\dfrac{2}{3}\right)=4\cdot\dfrac{2}{3}\sqrt{1-\left(\dfrac{2}{3}\right)^2}=\dfrac{8}{3}\sqrt{\dfrac{5}{9}}=\dfrac{8}{3}\cdot\dfrac{\sqrt{5}}{3}$
$=\dfrac{8\sqrt{5}}{9}\approx 1.99\text{ ft}^2$

113. $R(x)=\left(\dfrac{L}{P}\right)(x)=\dfrac{L(x)}{P(x)}$

114. $T(x)=(V+P)(x)=V(x)+P(x)$

115. $H(x)=(P\cdot I)(x)=P(x)\cdot I(x)$

116. $N(x)=(I-T)(x)=I(x)-T(x)$

117. a. $P(x)=R(x)-C(x)$
$=\left(-1.2x^2+220x\right)-\left(0.05x^3-2x^2+65x+500\right)$
$=-1.2x^2+220x-0.05x^3+2x^2-65x-500$
$=-0.05x^3+0.8x^2+155x-500$

b. $P(15)=-0.05(15)^3+0.8(15)^2+155(15)-500$
$=-168.75+180+2325-500$
$=\$1836.25$

c. When 15 hundred smartphones are sold, the profit is \$1836.25.

118. a. P is the dependent variable; a is the independent variable

b. $P(20)=0.027(20)^2-6.530(20)+363.804$
$=10.8-130.6+363.804$
$=244.004$
In 2015 there are 244.004 million people who are 20 years of age or older.

c. $P(0)=0.027(0)^2-6.530(0)+363.804$
$=363.804$
In 2015 there are 363.804 million people.

119. a. $R(v)=2.2v;B(v)=0.05v^2+0.4\text{v}-15$
$D(v)=R(v)+B(v)$
$=2.2v+0.05v^2+0.4\text{v}-15$
$=0.05v^2+2.6v-15$

b. $D(60)=0.05(60)^2+2.6(60)-15$
$=180+156-15$
$=321$

c. The car will need 321 feet to stop once the impediment is observed.

120. a. $h(x)=2x$

$h(a+b)=2(a+b)=2a+2b$

$=h(a)+h(b)$

$h(x)=2x$ has the property.

b. $g(x)=x^2$

$g(a+b)=(a+b)^2=a^2+2ab+b^2$

Since

$a^2+2ab+b^2 \neq a^2+b^2=g(a)+g(b)$,

$g(x)=x^2$ does not have the property.

c. $F(x)=5x-2$

$F(a+b)=5(a+b)-2=5a+5b-2$

Since

$5a+5b-2 \neq 5a-2+5b-2=F(a)+F(b)$,

$F(x)=5x-2$ does not have the property.

d. $G(x)=\frac{1}{x}$

$G(a+b)=\frac{1}{a+b} \neq \frac{1}{a}+\frac{1}{b}=G(a)+G(b)$

$G(x)=\frac{1}{x}$ does not have the property.

---

121. $$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}=$$

$$=\frac{(x+h)^{1/3}-x^{1/3}}{h}$$

$$=\frac{(x+h)^{1/3}-x^{1/3}}{h}\cdot\frac{(x+h)^{2/3}+x^{1/3}(x+h)^{1/3}+x^{2/3}}{(x+h)^{2/3}+x^{1/3}(x+h)^{1/3}+x^{2/3}}$$

$$=\frac{h}{(x+h)^{2/3}+x^{1/3}(x+h)^{1/3}+x^{2/3}}$$

$$=\frac{x+h-x}{h\left[(x+h)^{2/3}+x^{1/3}(x+h)^{1/3}+x^{2/3}\right]}=\frac{h}{h\left[(x+h)^{2/3}+x^{1/3}(x+h)^{1/3}+x^{2/3}\right]}$$

$$=\frac{1}{(x+h)^{2/3}+x^{1/3}(x+h)^{1/3}+x^{2/3}}$$

122. $f\left(\frac{x+4}{5x-4}\right)=3x^2-2$

Solve $\frac{x+4}{5x-4}=1$.

$\frac{x+4}{5x-4}=1$

$x+4=5x-4$

$x=2$

Therefore, $f(1)=3(2)^2-2=10$

123. We need $\frac{x^2+1}{7-|3x-1|}\geq 0$. Since $x^2+1>0$ for all real numbers *x*, we need $7-|3x-1|>0$.

$7-|3x-1|>0$

$|3x-1|<7$

$-7<3x-1<7$

$-2<x<\frac{8}{3}$

The domain of *f* is $\left\{x \mid -2<x<\frac{8}{3}\right\}$, or $\left(-2,\frac{8}{3}\right)$ in interval notation.

124. No. The domain of *f* is $\{x \mid x \text{ is any real number}\}$, but the domain of g is $\{x \mid x \neq -1\}$.

125. $\dfrac{3x - x^3}{(\textit{your age})}$

126. Answers will vary.

127. $(x+12)^2 + y^2 = 16$

x-intercept (y=0):

$(x+12)^2 + 0^2 = 16$

$(x+12)^2 = 16$

$(x+12) = \pm 4$

$x = -12 \pm 4$

$x = -16, x = -8$

$(-16, 0), (-8, 0)$

y**-**intercept (x=0):

$(0+12)^2 + y^2 = 16$

$(12)^2 + y^2 = 16$

$y^2 = 16 - 144 = -128$

There are no real solutions so there are no y-intercepts.

Symmetry: $(x+12)^2 + (-y)^2 = 16$

$(x+12)^2 + y^2 = 16$

This shows x-axis symmetry.

128. $y = 3x^2 - 8\sqrt{x}$

$y = 3(-1)^2 - 8\sqrt{-1}$

There is no solution so (-1,-5) is NOT a solution.

$y = 3x^2 - 8\sqrt{x}$

$y = 3(4)^2 - 8\sqrt{4}$

$= 48 - 16 = 32$

So (4,32) is a solution.

$y = 3x^2 - 8\sqrt{x}$

$y = 3(9)^2 - 8\sqrt{9}$

$= 243 - 24 = 219 \neq 171$

So (9,171) is NOT a solution.

129. Let x represent the amount of the 7% fat hamburger added.

% fat	tot. amt.	amt. of fat
20%	12	$(0.20)(12)$
7%	x	$(0.07)(x)$
15%	$12+x$	$(0.15)(12+x)$

$$(0.20)(12) + (0.07)(x) = (0.15)(12+x)$$

$$2.4 + 0.07x = 1.8 + 0.15x$$

$$0.6 = .08x$$

$$x = 7.5$$

7.5 lbs. of the 7% fat hamburger must be added, producing 19.5 lbs. of the 15% fat hamburger.

130. $x^3 - 9x = 2x^2 - 18$

$x^3 - 2x^2 - 9x + 18 = 0$

$(x^3 - 2x^2) - (9x - 18) = 0$

$x^2(x-2) - 9(x-2) = 0$

$(x^2 - 9)(x-2) = 0$

$(x-3)(x+3)(x-2) = 0$

$(x-3) = 0$ or $(x+3) = 0$ or $(x-2) = 0$

$x = 3, x = -3, x = 2$

The solution set is: $\{3, -3, 2\}$

131. $a + bx = ac + d$

$a - ac = d - bx$

$a(1-c) = d - bx$

$a = \dfrac{d - bx}{1 - c}$

132. $r = kd^2$

$0.4 = k(0.6)^2$

$\dfrac{10}{9} = k$

Thus,

$r = \dfrac{10}{9}(1.5)^2$

$= 2.5 \text{ kg} \cdot m^2$

133. $3x-10y=12$

$$-10y=-3x+12$$

$$y=\frac{3}{10}x-\frac{6}{5}$$

The slope of the line is $\frac{3}{10}$. The slope of a perpendicular line would be $-\frac{10}{3}$.

134. $\frac{(4x^2-7)\cdot 3-(3x+5)\cdot 8x}{(4x^2-7)^2}=$

$$\frac{12x^2-21-(24x^2+40x)}{(4x^2-7)^2}=$$

$$\frac{12x^2-21-24x^2-40x}{(4x^2-7)^2}=\frac{-12x^2-40x-21}{(4x^2-7)^2}$$

$$=-\frac{12x^2+40x+21}{(4x^2-7)^2}$$

135. Add the powers of x to obtain a degree of 7.

Section 2.2

1. $x^2+4y^2=16$

x-intercepts:

$$x^2+4(0)^2=16$$

$$x^2=16$$

$$x=\pm 4\Rightarrow(-4,0),(4,0)$$

y-intercepts:

$$(0)^2+4y^2=16$$

$$4y^2=16$$

$$y^2=4$$

$$y=\pm 2\Rightarrow(0,-2),(0,2)$$

2. False; $x=2y-2$

$$-2=2y-2$$

$$0=2y$$

$$0=y$$

The point $(-2,0)$ is on the graph.

3. vertical

4. $f(5)=-3$

5. $f(x)=ax^2+4$

$$a(-1)^2+4=2\Rightarrow a=-2$$

6. False. The graph must pass the Vertical-Line Test in order to be the graph of a function.

7. False; e.g. $y=\frac{1}{x}$.

8. True

9. c

10. a

11. **a.** $f(0)=3$ since $(0,3)$ is on the graph.
$f(-6)=-3$ since $(-6,-3)$ is on the graph.

b. $f(6)=0$ since $(6, 0)$ is on the graph.
$f(11)=1$ since $(11, 1)$ is on the graph.

c. $f(3)$ is positive since $f(3)\approx 3.7$.

d. $f(-4)$ is negative since $f(-4)\approx -1$.

e. $f(x)=0$ when $x=-3$, $x=6$, and $x=10$.

f. $f(x)>0$ when $-3<x<6$, and $10<x\le 11$.

g. The domain of f is $\{x|-6\le x\le 11\}$ or $[-6,11]$.

h. The range of f is $\{y|-3\le y\le 4\}$ or $[-3,4]$.

i. The x-intercepts are -3, 6, and 10.

j. The y-intercept is 3.

k. The line $y=\frac{1}{2}$ intersects the graph 3 times.

l. The line $x=5$ intersects the graph 1 time.

m. $f(x)=3$ when $x=0$ and $x=4$.

n. $f(x)=-2$ when $x=-5$ and $x=8$.

12. **a.** $f(0)=0$ since $(0,0)$ is on the graph.
$f(6)=0$ since $(6,0)$ is on the graph.

b. $f(2) = -2$ since $(2, -2)$ is on the graph.
$f(-2) = 1$ since $(-2, 1)$ is on the graph.

c. $f(3)$ is negative since $f(3) \approx -1$.

d. $f(-1)$ is positive since $f(-1) \approx 1.0$.

e. $f(x) = 0$ when $x = 0$, $x = 4$, and $x = 6$.

f. $f(x) < 0$ when $0 < x < 4$.

g. The domain of f is $\{x \mid -4 \le x \le 6\}$ or $[-4, 6]$.

h. The range of f is $\{y \mid -2 \le y \le 3\}$ or $[-2, 3]$.

i. The x-intercepts are 0, 4, and 6.

j. The y-intercept is 0.

k. The line $y = -1$ intersects the graph 2 times.

l. The line $x = 1$ intersects the graph 1 time.

m. $f(x) = 3$ when $x = 5$.

n. $f(x) = -2$ when $x = 2$.

13. Not a function since vertical lines will intersect the graph in more than one point.

a. Domain: $\{x \mid x \le -1 \text{ or } x \ge 1\}$;
Range: $\{y \mid y \text{ is any real number}\}$

b. Intercepts: $(-1,0),(1,0)$

c. Symmetry about the x-axis, y-axis and the origin

14. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;
Range: $\{y \mid y > 0\}$

b. Intercepts: $(0,1)$

c. None

15. Function

a. Domain: $\{x \mid -\pi \le x \le \pi\}$;
Range: $\{y \mid -1 \le y \le 1\}$

b. Intercepts: $\left(-\frac{\pi}{2}, 0\right)$, $\left(\frac{\pi}{2}, 0\right)$, $(0,1)$

c. Symmetry about y-axis.

16. Function

a. Domain: $\{x \mid -\pi \le x \le \pi\}$;
Range: $\{y \mid -1 \le y \le 1\}$

b. Intercepts: $(-\pi, 0)$, $(\pi, 0)$, $(0, 0)$

c. Symmetry about the origin.

17. Not a function since vertical lines will intersect the graph in more than one point.

a. Domain: $\{x \mid x \le 0\}$;
Range: $\{y \mid y \text{ is any real number}\}$

b. Intercepts: $(0,0)$

c. Symmetry about the x-axis

18. Not a function since vertical lines will intersect the graph in more than one point.

a. Domain: $\{x \mid -2 \le x \le 2\}$;
Range: $\{y \mid -2 \le y \le 2\}$

b. Intercepts: $(-2,0)(2,0)(0,-2)(0,2)$

c. Symmetry about the x-axis, y-axis and the origin

19. Function

a. Domain: $\{x \mid 0 < x < 3\}$;
Range: $\{y \mid y < 2\}$

b. Intercepts: $(1, 0)$

c. None

20. Function

a. Domain: $\{x \mid 0 \le x < 4\}$;
Range: $\{y \mid 0 \le y < 3\}$

b. Intercepts: $(0, 0)$

c. None

21. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;
Range: $\{y \mid y \le 2\}$

b. Intercepts: $(-3, 0)$, $(3, 0)$, $(0,2)$

c. Symmetry about y-axis.

22. Function

a. Domain: $\{x \mid x \geq -3\}$;

Range: $\{y \mid y \geq 0\}$

b. Intercepts: (–3, 0), (2,0), (0,2)

c. None

23. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y \geq -3\}$

b. Intercepts: (1, 0), (3,0), (0,9)

c. None

24. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y \leq 5\}$

b. Intercepts: (–1, 0), (2,0), (0,4)

c. None

25. $f(x) = 3x^2 + x - 2$

a. $f(1) = 3(1)^2 + (1) - 2 = 2$

The point $(-1, 2)$ is on the graph of *f*.

b. $f(-2) = 3(-2)^2 + (-2) - 2 = 8$

The point $(-2, 8)$ is on the graph of *f*.

c. Solve for x:

$-2 = 3x^2 + x - 2$

$0 = 3x^2 + x$

$0 = x(3x+1) \Rightarrow x = 0, x = -\frac{1}{3}$

(0, –2) and $\left(-\frac{1}{3}, -2\right)$ are on the graph of f.

d. The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

e. x-intercepts:

$f(x)=0 \Rightarrow 3x^2 + x - 2 = 0$

$(3x-2)(x+1) = 0 \Rightarrow x = \dfrac{2}{3}, x = -1$

f. y-intercept: $f(0)=3(0)^2 + 0 - 2 = -2$

26. $f(x) = -3x^2 + 5x$

a. $f(-1) = -3(-1)^2 + 5(-1) = -8 \neq 2$

The point $(-1, 2)$ is not on the graph of *f*.

b. $f(-2) = -3(-2)^2 + 5(-2) = -22$

The point $(-2, -22)$ is on the graph of *f*.

c. Solve for x:

$-2 = -3x^2 + 5x \Rightarrow 3x^2 - 5x - 2 = 0$

$(3x+1)(x-2) = 0 \Rightarrow x = -\frac{1}{3}, x = 2$

(2, –2) and $\left(-\frac{1}{3}, -2\right)$ on the graph of f.

d. The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

e. x-intercepts:

$f(x)=0 \Rightarrow -3x^2 + 5x = 0$

$x(-3x+5) = 0 \Rightarrow x = 0, x = \frac{5}{3}$

f. y-intercept:

$f(0) = -3(0)^2 + 5(0) = 0$

27. $f(x) = \dfrac{x+2}{x-6}$

a. $f(3) = \dfrac{3+2}{3-6} = -\dfrac{5}{3} \neq 14$

The point $(3, 14)$ is not on the graph of *f*.

b. $f(4) = \dfrac{4+2}{4-6} = \dfrac{6}{-2} = -3$

The point $(4, -3)$ is on the graph of *f*.

c. Solve for x:

$2 = \dfrac{x+2}{x-6}$

$2x - 12 = x + 2$

$x = 14$

(14, 2) is a point on the graph of f.

d. The domain of *f* is $\{x \mid x \neq 6\}$.

e. x-intercepts:

$f(x)=0 \Rightarrow \dfrac{x+2}{x-6} = 0$

$x + 2 = 0 \Rightarrow x = -2$

f. y-intercept: $f(0)=\dfrac{0+2}{0-6}=-\dfrac{1}{3}$

28. $f(x)=\dfrac{x^2+2}{x+4}$

a. $f(1)=\dfrac{1^2+2}{1+4}=\dfrac{3}{5}$

The point $\left(1,\dfrac{3}{5}\right)$ is on the graph of *f*.

b. $f(0)=\dfrac{0^2+2}{0+4}=\dfrac{2}{4}=\dfrac{1}{2}$

The point $\left(0,\dfrac{1}{2}\right)$ is on the graph of *f*.

c. Solve for x:

$$\frac{1}{2}=\frac{x^2+2}{x+4}\Rightarrow x+4=2x^2+4$$
$$0=2x^2-x$$
$$x(2x-1)=0\Rightarrow x=0 \text{ or } x=\frac{1}{2}$$

$\left(0,\dfrac{1}{2}\right)$ and $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$ are on the graph of f.

d. The domain of *f* is $\{x \mid x\neq -4\}$.

e. x-intercepts:

$$f(x)=0\Rightarrow\frac{x^2+2}{x+4}=0\Rightarrow x^2+2=0$$

This is impossible, so there are no x-intercepts.

f. y-intercept:

$$f(0)=\frac{0^2+2}{0+4}=\frac{2}{4}=\frac{1}{2}$$

29. $f(x)=\dfrac{12x^4}{x^2+1}$

a. $f(-1)=\dfrac{12(-1)^4}{(-1)^2+1}=\dfrac{12}{2}=6$

The point $(-1,1)$ is on the graph of *f*.

b. $f(3)=\dfrac{12(3)^4}{(3)^2+1}=\dfrac{972}{10}=\dfrac{486}{5}$

The point $\left(3,\dfrac{486}{5}\right)$ is on the graph of *f*.

c. Solve for x:

$$1=\frac{12x^4}{x^2+1}$$
$$x^2+1=12x^4$$
$$12x^4-x^2-1=0$$
$$(3x^2-1)(4x^2+1)=0$$
$$3x^2-1=0\Rightarrow x=\pm\frac{1}{\sqrt{3}}=\pm\frac{\sqrt{3}}{3}$$

$\left(-\dfrac{\sqrt{3}}{3},1\right),\left(\dfrac{\sqrt{3}}{3},1\right)$ are on the graph of f.

d. The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

e. x-intercept:

$$f(x)=0\Rightarrow\frac{12x^4}{x^2+1}=0$$
$$12x^4=0\Rightarrow x=0$$

f. y-intercept:

$$f(0)=\frac{12(0)^4}{0^2+1}=\frac{0}{0+1}=0$$

30. $f(x)=\dfrac{2x}{x-2}$

a. $f\left(\dfrac{1}{2}\right)=\dfrac{2\left(\frac{1}{2}\right)}{\frac{1}{2}-2}=\dfrac{1}{-\frac{3}{2}}=-\dfrac{2}{3}$

The point $\left(\dfrac{1}{2},-\dfrac{2}{3}\right)$ is on the graph of *f*.

b. $f(4)=\dfrac{2(4)}{4-2}=\dfrac{8}{2}=4$

The point $(4,4)$ is on the graph of *f*.

c. Solve for x:

$$1=\frac{2x}{x-2}\Rightarrow x-2=2x\Rightarrow -2=x$$

$(-2,1)$ is a point on the graph of f.

d. The domain of *f* is $\{x \mid x\neq 2\}$.

e. x-intercept:

$$f(x)=0\Rightarrow\frac{2x}{x-2}=0\Rightarrow 2x=0$$
$$\Rightarrow x=0$$

f. y-intercept: $f(0)=\dfrac{0}{0-2}=0$

31. **a.** $(f+g)(2) = f(2)+g(2) = 2+1 = 3$

b. $(f+g)(4) = f(4)+g(4) = 1+(-3) = -2$

c. $(f-g)(6) = f(6)-g(6) = 0-1 = -1$

d. $(g-f)(6) = g(6)-f(6) = 1-0 = 1$

e. $(f \cdot g)(2) = f(2)\cdot g(2) = 2(1) = 2$

f. $\left(\dfrac{f}{g}\right)(4) = \dfrac{f(4)}{g(4)} = \dfrac{1}{-3} = -\dfrac{1}{3}$

32. $h(x) = -\dfrac{136x^2}{v^2} + 2.7x + 3.5$

a. We want $h(15) = 10$.

$$-\frac{136(15)^2}{v^2} + 2.7(15) + 3.5 = 10$$
$$-\frac{30,600}{v^2} = -34$$
$$v^2 = 900$$
$$v = 30 \text{ ft/sec}$$

The ball needs to be thrown with an initial velocity of 30 feet per second.

b. $h(x) = -\dfrac{126x^2}{30^2} + 2.7x + 3.5$

which simplifies to

$h(x) = -\dfrac{34}{225}x^2 + 2.7x + 3.5$

c. Using the velocity from part (b),

$h(9) = -\dfrac{34}{225}(9)^2 + 2.7(9) + 3.5 = 15.56$ ft

The ball will be 15.56 feet above the floor when it has traveled 9 feet in front of the foul line.

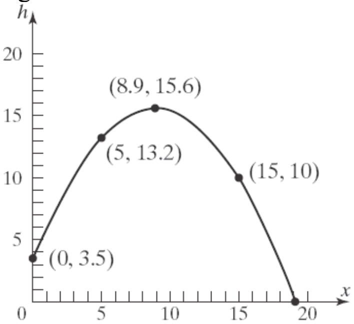
d. Select several values for x and use these to find the corresponding values for h. Use the results to form ordered pairs (x,h). Plot the points and connect with a smooth curve.

$h(0) = -\dfrac{34}{225}(0)^2 + 2.7(0) + 3.5 = 3.5$ ft

$h(5) = -\dfrac{34}{225}(5)^2 + 2.7(5) + 3.5 \approx 13.2$ ft

$h(15) = -\dfrac{24}{225}(15)^2 + 2.7(15) + 3.5 \approx 10$ ft

Thus, some points on the graph are $(0,3.5)$, $(5,13.2)$, and $(15,10)$. The complete graph is given below.



33. $h(x) = -\dfrac{44x^2}{v^2} + x + 6$

a. $h(8) = -\dfrac{44(8)^2}{28^2} + (8) + 6$

$= -\dfrac{2816}{784} + 14$

≈ 10.4 feet

b. $h(12) = -\dfrac{44(12)^2}{28^2} + (12) + 6$

$= -\dfrac{6336}{784} + 18$

≈ 9.9 feet

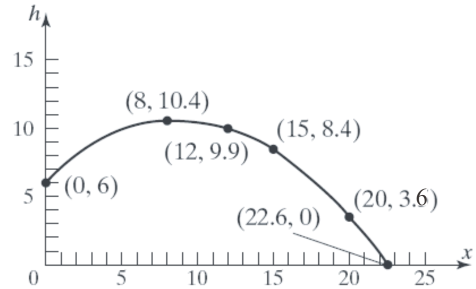
c. From part (a) we know the point $(8,10.4)$ is on the graph and from part (b) we know the point $(12,9.9)$ is on the graph. We could evaluate the function at several more values of x (e.g. $x=0$, $x=15$, and $x=20$) to obtain additional points.

$h(0) = -\dfrac{44(0)^2}{28^2} + (0) + 6 = 6$

$h(15) = -\dfrac{44(15)^2}{28^2} + (15) + 6 \approx 8.4$

$h(20) = -\dfrac{44(20)^2}{28^2} + (20) + 6 \approx 3.6$

Some additional points are $(0,6)$, $(15,8.4)$ and $(20,3.6)$. The complete graph is given below.



d. $h(15) = -\dfrac{44(15)^2}{28^2} + (15) + 6 \approx 8.4$ feet

No; when the ball is 15 feet in front of the foul line, it will be below the hoop. Therefore it cannot go through the hoop.

In order for the ball to pass through the hoop, we need to have $h(15) = 10$.

$$10 = -\frac{44(15)^2}{v^2} + (15) + 6$$

$$-11 = -\frac{44(15)^2}{v^2}$$

$$v^2 = 4(225)$$

$$v^2 = 900$$

$$v = 30 \text{ ft/sec}$$

The ball must be shot with an initial velocity of 30 feet per second in order to go through the hoop.

34. $A(x) = 4x\sqrt{1-x^2}$

a. Domain of $A(x) = 4x\sqrt{1-x^2}$; we know that x must be greater than or equal to zero, since x represents a length. We also need $1-x^2 \geq 0$, since this expression occurs under a square root. In fact, to avoid Area = 0, we require $x > 0$ and $1-x^2 > 0$.

Solve: $1-x^2 > 0$

$(1+x)(1-x) > 0$

Case1: $1+x > 0$ and $1-x > 0$

$x > -1$ and $x < 1$

(i.e. $-1 < x < 1$)

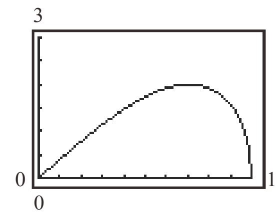
Case2: $1+x < 0$ and $1-x < 0$

$x < -1$ and $x > 1$

(which is impossible)

Therefore the domain of A is $\{x \mid 0 < x < 1\}$.

b. Graphing $A(x) = 4x\sqrt{1-x^2}$



c. When $x = 0.7$ feet, the cross-sectional area is maximized at approximately 1.9996 square feet. Therefore, the length of the base of the beam should be 1.4 feet in order to maximize the cross-sectional area.

X	Y1
.3	1.1447
.4	1.4664
.5	1.7321
.6	1.92
.7	1.9996
.8	1.92
.9	1.5692

X=.7

35. $h(x) = \dfrac{-32x^2}{130^2} + x$

a. $h(100) = \dfrac{-32(100)^2}{130^2} + 100$

$= \dfrac{-320{,}000}{16{,}900} + 100 \approx 81.07$ feet

b. $h(300) = \dfrac{-32(300)^2}{130^2} + 300$

$= \dfrac{-2{,}880{,}000}{16{,}900} + 300 \approx 129.59$ feet

c. $h(500) = \dfrac{-32(500)^2}{130^2} + 500$

$= \dfrac{-8{,}000{,}000}{16{,}900} + 500 \approx 26.63$ feet

The ball is about 26.63 feet high after it has traveled 500 feet.

d. Solving $h(x) = \dfrac{-32x^2}{130^2} + x = 0$

$$\frac{-32x^2}{130^2} + x = 0$$

$$x\left(\frac{-32x}{130^2} + 1\right) = 0$$

$$x = 0 \quad \text{or} \quad \frac{-32x}{130^2} + 1 = 0$$

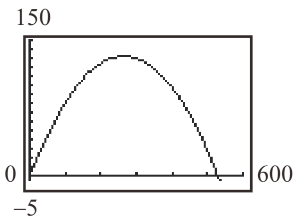
$$1 = \frac{32x}{130^2}$$

$$130^2 = 32x$$

$$x = \frac{130^2}{32} = 528.13 \text{ feet}$$

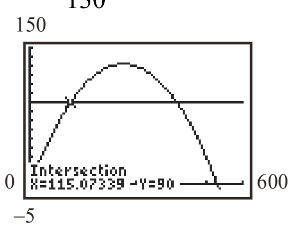
Therefore, the golf ball travels 528.13 feet.

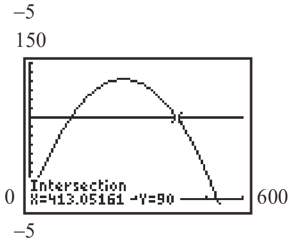
e. $y_1 = \dfrac{-32x^2}{130^2} + x$



f. Use INTERSECT on the graphs of

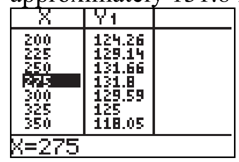
$y_1 = \dfrac{-32x^2}{130^2} + x$ and $y_2 = 90$.



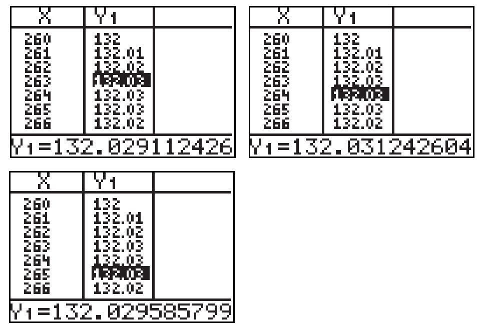


The ball reaches a height of 90 feet twice. The first time is when the ball has traveled approximately 115.07 feet, and the second time is when the ball has traveled about 413.05 feet.

g. The ball travels approximately 275 feet before it reaches its maximum height of approximately 131.8 feet.



h. The ball travels approximately 264 feet before it reaches its maximum height of approximately 132.03 feet.



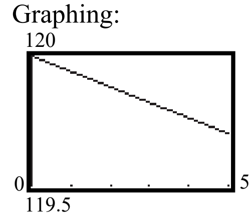
36. $W(h) = m\left(\dfrac{4000}{4000+h}\right)^2$

a. $h = 14110 \text{ feet} \approx 2.67 \text{ miles}$;

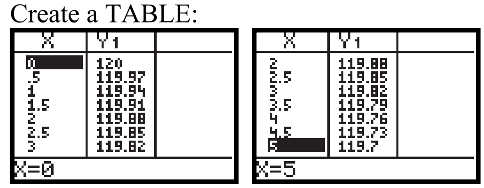
$$W(2.67) = 120\left(\frac{4000}{4000+2.67}\right)^2 \approx 119.84$$

On Pike's Peak, Amy will weigh about 119.84 pounds.

b. Graphing:



c. Create a TABLE:



The weight W will vary from 120 pounds to about 119.7 pounds.

d. By refining the table, Amy will weigh 119.95 lbs at a height of about 0.83 miles

(4382 feet).

X	Y_1	
.5	119.97	
.6	119.96	
.7	119.96	
.8	119.95	
.9	119.95	
1	119.94	
1.1	119.93	

X=.8

X	Y_1	
.8	119.95	
.81	119.95	
.82	119.95	
.83	119.95	
.84	119.95	
.85	119.95	
.86	119.95	

Y_1=119.950215496

e. Yes, 4382 feet is reasonable.

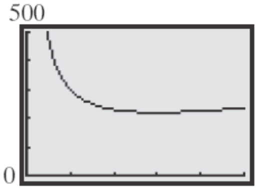
37. $C(x) = 100 + \dfrac{x}{10} + \dfrac{36000}{x}$

a. $C(480) = 100 + \dfrac{480}{10} + \dfrac{36000}{480}$
$= \$223$

$C(600) = 100 + \dfrac{600}{10} + \dfrac{36000}{600}$
$= \$220$

b. $\{x \mid x > 0\}$

c. Graphing:



d. TblStart = 0; ΔTbl = 50

X	Y_1	
0	ERROR	
50	825	
100	470	
150	355	
200	300	
250	269	
300	250	

Y_1=100+X/10+360…

e. The cost per passenger is minimized to about \$220 when the ground speed is roughly 600 miles per hour.

X	Y_1	
450	225	
500	222	
550	220.45	
600	220	
650	220.38	
700	221.43	
750	223	

X=600

38. **a.** $C(0) = 5000$

This represents the fixed overhead costs. That is, the company will incur costs of \$5000 per day even if no computers are manufactured.

b. $C(10) = 19{,}000$

It costs the company \$19,000 to produce 10 computers in a day.

c. $C(50) = 51{,}000$

It costs the company \$51,000 to produce 50 computers in a day.

d. The domain is $\{q \mid 0 \le q \le 80\}$. This indicates that production capacity is limited to 80 computers in a day.

e. The graph is curved down and rises slowly at first. As production increases, the graph rises more quickly and changes to being curved up.

f. The inflection point is where the graph changes from being curved down to being curved up.

39. **a.** $C(0) = \$50$

It costs \$50 if you use 0 gigabytes.

b. $C(5) = \$50$

It costs \$50 if you use 5 gigabytes.

c. $C(15) = \$150$

It costs \$90 if you use 15 gigabytes.

d. The domain is $\{g \mid 0 \le g \le 30\}$. This indicates that there are at most 30 gigabytes in a month.

e. The graph is flat at first and then rises in a straight line.

40. $g(-2) = 5 = f(-2) = 4$

Since $f(-2) = (-2)^2 - 4(-2) + c$
$= 12 + c$

we have

$$\frac{12+c}{3} - 4 = 5$$
$$\frac{12+c}{3} = 9$$
$$12 + c = 27$$
$$c = 15$$
$$f(3) = 3^2 - 4 \cdot 3 + 15 = 12$$

41. $g(5) = 5^2 + n = 25 + n$

$f(g(5)) = f(25+n) = \sqrt{25+n} + 2 = 4$

so, $\sqrt{25+n} = 2$

$25 + n = 4$

$n = -21$

$g(n) = n^2 + n = (-21)^2 + (-21) = 420$.

42. Answers will vary. From a graph, the domain can be found by visually locating the x-values for which the graph is defined. The range can be found in a similar fashion by visually locating the y-values for which the function is defined.

If an equation is given, the domain can be found by locating any restricted values and removing them from the set of real numbers. The range can be found by using known properties of the graph of the equation, or estimated by means of a table of values.

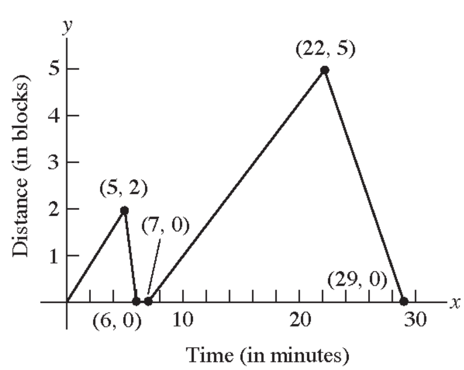
43. The graph of a function can have any number of x-intercepts. The graph of a function can have at most one y-intercept (otherwise the graph would fail the Vertical-Line Test).

44. Yes, the graph of a single point is the graph of a function since it would pass the Vertical-Line Test. The equation of such a function would be something like the following: $f(x) = 2$, where $x = 7$.

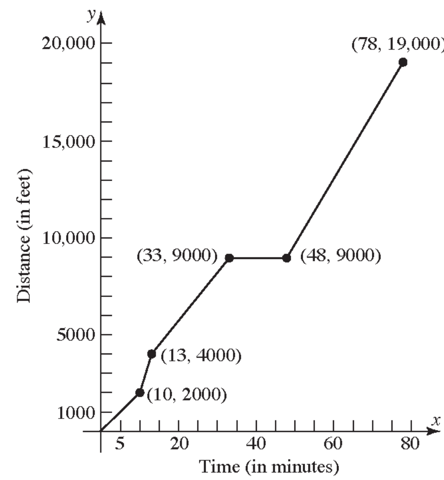
45. (a) III; (b) IV; (c) I; (d) V; (e) II

46. (a) II; (b) V; (c) IV; (d) III; (e) I

47.



48.



49. **a.** 2 hours elapsed; Sobia was between 0 and 3 miles from home.

b. 0.5 hours elapsed; Sobia was 3 miles from home.

c. 0.3 hours elapsed; Sobia was between 0 and 3 miles from home.

d. 0.2 hours elapsed; Sobia was at home.

e. 0.9 hours elapsed; Sobia was between 0 and 2.8 miles from home.

f. 0.3 hours elapsed; Sobia was 2.8 miles from home.

g. 1.1 hours elapsed; Sobia was between 0 and 2.8 miles from home.

h. The farthest distance Sobia is from home is 3 miles.

i. Sobia returned home 2 times.

50. **a.** Michael travels fastest between 7 and 7.4 minutes. That is, $(7, 7.4)$.

b. Michael's speed is zero between 4.2 and 6 minutes. That is, $(4.2, 6)$.

c. Between 0 and 2 minutes, Michael's speed increased from 0 to 30 miles/hour.

d. Between 4.2 and 6 minutes, Michael was stopped (i.e, his speed was 0 miles/hour).

e. Between 7 and 7.4 minutes, Michael was traveling at a steady rate of 50 miles/hour.

f. Michael's speed is constant between 2 and 4 minutes, between 4.2 and 6 minutes, between 7 and 7.4 minutes, and between 7.6 and 8 minutes. That is, on the intervals (2, 4), (4.2, 6), (7, 7.4), and (7.6, 8).

51. Answers (graphs) will vary. Points of the form $(5, y)$ and of the form $(x, 0)$ cannot be on the graph of the function.

52. The only such function is $f(x)=0$ because it is the only function for which $f(x)=-f(x)$. Any other such graph would fail the Vertical-Line Test.

53. Answers may vary.

54.
$$\begin{aligned} f(x-2) &= -(x-2)^2+(x-2)-3 \\ &= -(x^2-4x+4)+x-2-3 \\ &= -x^2+5x-9 \end{aligned}$$

55.
$$\begin{aligned} d &= \sqrt{(1-3)^2+(0-(-6))^2} \\ &= \sqrt{(-2)^2+(-6)^2} \\ &= \sqrt{4+36} = \sqrt{40} = 2\sqrt{10} \end{aligned}$$

56.
$$\begin{aligned} y-4 &= \frac{2}{3}(x-(-6)) \\ y-4 &= \frac{2}{3}x+4 \\ y &= \frac{2}{3}x+8 \end{aligned}$$

57. Since the function contains a cube root then the domain is:

$(-\infty,\infty)$

58. $\left[\frac{1}{2}(-12)\right]^2 = 36$

59.
$$\frac{\sqrt{x}-\sqrt{6}}{x-6}\cdot\frac{\sqrt{x}+\sqrt{6}}{\sqrt{x}+\sqrt{6}} =$$
$$\frac{x-6}{(x-6)\left(\sqrt{x}+\sqrt{6}\right)} = \frac{1}{\sqrt{x}+\sqrt{6}}$$

60. The car traveling north travels a distance or 25t and the car traveling west travels a distance of 35t where t is the time of travel. Using the Pythagorean we have:

$$\begin{aligned} 40^2 &= (35t)^2+(25t)^2 \\ 1600 &= 1225t^2+625t^2 \\ 1600 &= 1850t^2 \\ t^2 &= 0.8649 \\ t &= 0.93 \text{ hours} \end{aligned}$$

Converting to minutes we have

$0.93(60) = 55.8$ minutes

61.
$$\begin{aligned} 3x+4 &\le 7 \quad \text{and} \quad 5-2x<13 \\ 3x &\le 3 \qquad\quad -2x<8 \\ x &\le 1 \qquad\qquad\; x>-4 \end{aligned}$$

The solution set is $(-4,1]$.

62.
$$\begin{aligned} &(5x^2-7x+2)-(8x-10) \\ &= 5x^2-7x+2-8x+10 \\ &= 5x^2-15x+12 \end{aligned}$$

63. $[-3,10]$

Section 2.3

1. $2<x<5$

2. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{8-3}{3-(-2)} = \frac{5}{5} = 1$

3. x-axis: $y \to -y$

$$\begin{aligned} (-y) &= 5x^2-1 \\ -y &= 5x^2-1 \\ y &= -5x^2+1 \quad \text{different} \end{aligned}$$

y-axis: $x \to -x$

$$\begin{aligned} y &= 5(-x)^2-1 \\ y &= 5x^2-1 \quad \text{same} \end{aligned}$$

origin: $x \to -x$ and $y \to -y$

$(-y) = 5(-x)^2 - 1$

$-y = 5x^2 - 1$

$y = -5x^2 + 1$ different

The equation has symmetry with respect to the y-axis only.

4. $y - y_1 = m(x - x_1)$

$y - (-2) = 5(x - 3)$

$y + 2 = 5(x - 3)$

5. $y = x^2 - 9$

x-intercepts:

$0 = x^2 - 9$

$x^2 = 9 \to x = \pm 3$

y-intercept:

$y = (0)^2 - 9 = -9$

The intercepts are $(-3, 0)$, $(3, 0)$, and $(0, -9)$.

6. increasing

7. even; odd

8. True

9. True

10. False; odd functions are symmetric with respect to the origin. Even functions are symmetric with respect to the y-axis.

11. c

12. d

13. Yes

14. No, it is increasing.

15. No

16. Yes

17. f is increasing on the intervals $[-8, -2], [0, 2], [5, 7]$.

18. f is decreasing on the intervals: $[-10, -8], [-2, 0], [2, 5]$.

19. Yes. The local maximum at $x = 2$ is 10.

20. No. There is a local minimum at $x = 5$; the local minimum is 0.

21. f has local maxima at $x = -2$ and $x = 2$. The local maxima are 6 and 10, respectively.

22. f has local minima at $x = -8$, $x = 0$ and $x = 5$. The local minima are –4, 0, and 0, respectively.

23. f has absolute minimum of -4 at $x = -8$.

24. f has absolute maximum of 10 at $x = 2$.

25. **a.** Intercepts: (–2, 0), (2, 0), and (0, 3).

b. Domain: $\{x \mid -4 \le x \le 4\}$ or $[-4, 4]$;

Range: $\{y \mid 0 \le y \le 3\}$ or $[0, 3]$.

c. Increasing: [–2, 0] and [2, 4];
Decreasing: [–4, –2] and [0, 2].

d. Since the graph is symmetric with respect to the y-axis, the function is <u>even</u>.

26. **a.** Intercepts: (–1, 0), (1, 0), and (0, 2).

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;

Range: $\{y \mid 0 \le y \le 3\}$ or $[0, 3]$.

c. Increasing: [–1, 0] and [1, 3];
Decreasing: [–3, –1] and [0, 1].

d. Since the graph is symmetric with respect to the y-axis, the function is <u>even</u>.

27. **a.** Intercepts: (0, 1).

b. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y > 0\}$ or $(0, \infty)$.

c. Increasing: $(-\infty, \infty)$; Decreasing: never.

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is <u>neither</u> even nor odd.

28. **a.** Intercepts: (1, 0).

b. Domain: $\{x \mid x > 0\}$ or $(0, \infty)$;

Range: $\{y \mid y \text{ is any real number}\}$.

c. Increasing: $[0, \infty)$; Decreasing: never.

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is neither even nor odd.

29. a. Intercepts: $(-\pi, 0), (\pi, 0)$, and $(0, 0)$.

b. Domain: $\{x \mid -\pi \le x \le \pi\}$ or $[-\pi, \pi]$;

Range: $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

c. Increasing: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$;

Decreasing: $\left[-\pi, -\frac{\pi}{2}\right]$ and $\left[\frac{\pi}{2}, \pi\right]$.

d. Since the graph is symmetric with respect to the origin, the function is odd.

30. a. Intercepts: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right)$, and $(0, 1)$.

b. Domain: $\{x \mid -\pi \le x \le \pi\}$ or $[-\pi, \pi]$;

Range: $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

c. Increasing: $[-\pi, 0]$; Decreasing: $[0, \pi]$.

d. Since the graph is symmetric with respect to the y-axis, the function is even.

31. a. Intercepts: $\left(\frac{1}{3}, 0\right), \left(\frac{5}{2}, 0\right)$, and $\left(0, \frac{1}{2}\right)$.

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;

Range: $\{y \mid -1 \le y \le 2\}$ or $[-1, 2]$.

c. Increasing: $[2, 3]$; Decreasing: $[-1, 1]$;

Constant: $[-3, -1]$ and $[1, 2]$

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is neither even nor odd.

32. a. Intercepts: $(-2.3, 0), (3, 0)$, and $(0, 1)$.

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;

Range: $\{y \mid -2 \le y \le 2\}$ or $[-2, 2]$.

c. Increasing: $[-3, -2]$ and $[0, 2]$;

Decreasing: $[2, 3]$; Constant: $[-2, 0]$.

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is neither even nor odd.

33. a. f has a local maximum value of 3 at $x = 0$.

b. f has a local minimum value of 0 at both $x = -2$ and $x = 2$.

34. a. f has a local maximum value of 2 at $x = 0$.

b. f has a local minimum value of 0 at both $x = -1$ and $x = 1$.

35. a. f has a local maximum value of 1 at $x = \frac{\pi}{2}$.

b. f has a local minimum value of –1 at $x = -\frac{\pi}{2}$.

36. a. f has a local maximum value of 1 at $x = 0$.

b. f has a local minimum value of –1 both at $x = -\pi$ and $x = \pi$.

37. $f(x) = 4x^3$

$f(-x) = 4(-x)^3 = -4x^3 = -f(x)$

Therefore, f is odd.

38. $f(x) = 2x^4 - x^2$

$f(-x) = 2(-x)^4 - (-x)^2 = 2x^4 - x^2 = f(x)$

Therefore, f is even.

39. $g(x) = 10 - x^2$

$g(-x) = 10 - (-x)^2 = 10 - x^2 = g(x)$

Therefore, g is even.

40. $h(x) = 3x^3 + 5$

$h(-x) = 3(-x)^3 + 5 = -3x^3 + 5$

h is neither even nor odd.

41. $F(x) = \sqrt[3]{4x}$

$F(-x) = \sqrt[3]{-4x} = -\sqrt[3]{4x} = -F(x)$

Therefore, F is odd.

42. $G(x)=\sqrt{x}$
$G(-x)=\sqrt{-x}$
G is neither even nor odd.

43. $f(x)=x+|x|$
$f(-x)=-x+|-x|=-x+|x|$
f is neither even nor odd.

44. $f(x)=\sqrt[3]{2x^2+1}$
$f(-x)=\sqrt[3]{2(-x)^2+1}=\sqrt[3]{2x^2+1}=f(x)$
Therefore, f is even.

45. $g(x)=\dfrac{1}{x^2+8}$
$g(-x)=\dfrac{1}{(-x)^2+8}=\dfrac{1}{x^2+8}=g(x)$
Therefore, g is even.

46. $h(x)=\dfrac{x}{x^2-1}$
$h(-x)=\dfrac{-x}{(-x)^2-1}=\dfrac{-x}{x^2-1}=-h(x)$
Therefore, h is odd.

47. $h(x)=\dfrac{-x^3}{3x^2-9}$
$h(-x)=\dfrac{-(-x)^3}{3(-x)^2-9}=\dfrac{x^3}{3x^2-9}=-h(x)$
Therefore, h is odd.

48. $F(x)=\dfrac{2x}{|x|}$
$F(-x)=\dfrac{2(-x)}{|-x|}=\dfrac{-2x}{|x|}=-F(x)$
Therefore, F is odd.

49. f has an absolute maximum of 4 at $x=1$.
f has an absolute minimum of 1 at $x=5$.
f has an local maximum value of 3 at $x=3$.
f has an local minimum value of 2 at $x=2$.

50. f has an absolute maximum of 4 at $x=4$.
f has an absolute minimum of 0 at $x=5$.
f has an local maximum value of 4 at $x=4$.
f has an local minimum value of 1 at $x=1$.

51. f has an absolute minimum of 1 at $x=1$.
f has an absolute maximum of 4 at $x=3$.
f has an local minimum value of 1 at $x=1$.
f has an local maximum value of 4 at $x=3$.

52. f has an absolute minimum of 1 at $x=0$.
f has no absolute maximum.
f has no local minimum.
f has no local maximum.

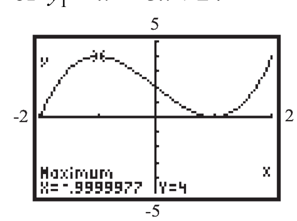
53. f has an absolute minimum of 0 at $x=0$.
f has no absolute maximum.
f has an local minimum value of 0 at $x=0$.
f has an local minimum value of 2 at $x=3$.
f has an local maximum value of 3 at $x=2$.

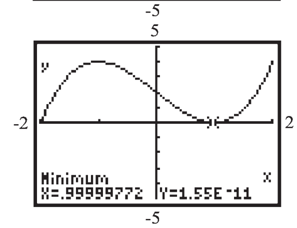
54. f has an absolute maximum of 4 at $x=2$.
f has no absolute minimum.
f has an local maximum value of 4 at $x=2$.
f has an local minimum value of 2 at $x=0$.

55. f has no absolute maximum or minimum.
f has no local maximum or minimum.

56. f has no absolute maximum or minimum.
f has no local maximum or minimum.

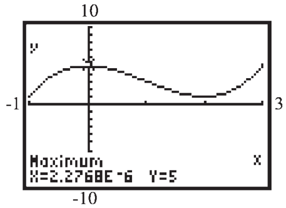
57. $f(x)=x^3-3x+2$ on the interval $(-2,2)$
Use MAXIMUM and MINIMUM on the graph of $y_1=x^3-3x+2$.

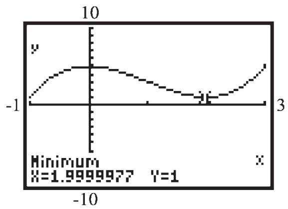




local maximum: $f(-1)=4$
local minimum: $f(1)=0$
f is increasing on: $[-2,-1]$ and $[1,2]$;
f is decreasing on: $[-1,1]$

58. $f(x)=x^3-3x^2+5$ on the interval $(-1,3)$
Use MAXIMUM and MINIMUM on the graph of $y_1=x^3-3x^2+5$.





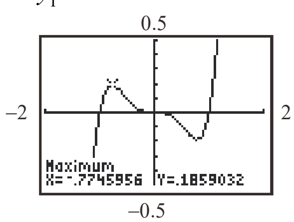
local maximum: $f(0)=5$

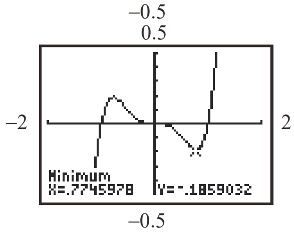
local minimum: $f(2)=1$

f is increasing on: $[-1,0]$ and $[2,3]$;

f is decreasing on: $[0,2]$

59. $f(x)=x^5-x^3$ on the interval $(-2,2)$
Use MAXIMUM and MINIMUM on the graph of $y_1=x^5-x^3$.





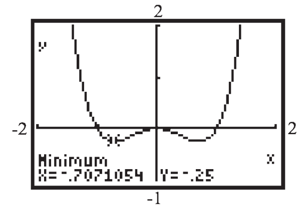
local maximum: $f(-0.77)=0.19$

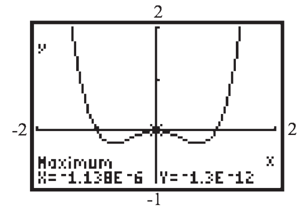
local minimum: $f(0.77)=-0.19$

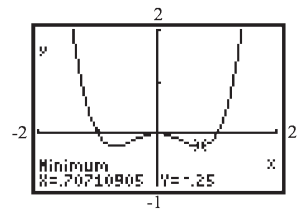
f is increasing on: $[-2,-0.77]$ and $[0.77,2]$;

f is decreasing on: $[-0.77,0.77]$

60. $f(x)=x^4-x^2$ on the interval $(-2,2)$
Use MAXIMUM and MINIMUM on the graph of $y_1=x^4-x^2$.







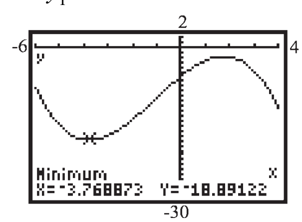
local maximum: $f(0)=0$

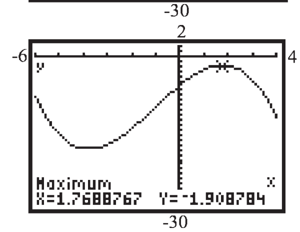
local minimum:
$f(-0.71)=-0.25$; $f(0.71)=-0.25$

f is increasing on: $[-0.71,0]$ and $[0.71,2]$;

f is decreasing on: $[-2,-0.71]$ and $[0,0.71]$

61. $f(x)=-0.2x^3-0.6x^2+4x-6$ on the interval $(-6,4)$
Use MAXIMUM and MINIMUM on the graph of $y_1=-0.2x^3-0.6x^2+4x-6$.





local maximum: $f(1.77)=-1.91$

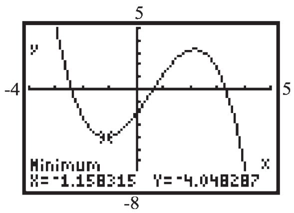
local minimum: $f(-3.77)=-18.89$

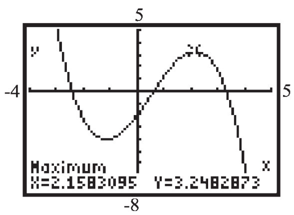
f is increasing on: $[-3.77,1.77]$;

f is decreasing on: $[-6,-3.77]$ and $[1.77,4]$

62. $f(x) = -0.4x^3 + 0.6x^2 + 3x - 2$ on the interval $(-4, 5)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.4x^3 + 0.6x^2 + 3x - 2$.





local maximum: $f(2.16) = 3.25$

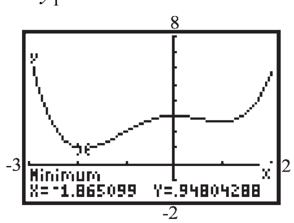
local minimum: $f(-1.16) = -4.05$

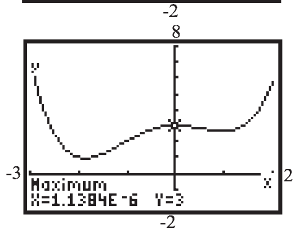
f is increasing on: $[-1.16, 2.16]$;

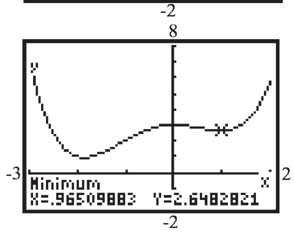
f is decreasing on: $[-4, -1.16]$ and $[2.16, 5]$

63. $f(x) = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3$ on the interval $(-3, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3$.







local maximum: $f(0) = 3$

local minimum:

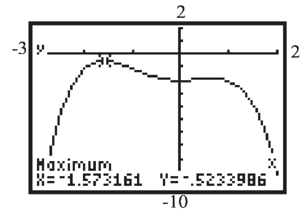
$f(-1.87) = 0.95$, $f(0.97) = 2.65$

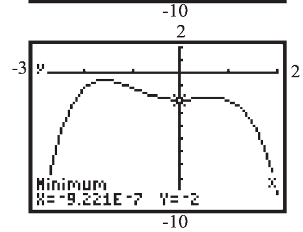
f is increasing on: $[-1.87, 0]$ and $[0.97, 2]$;

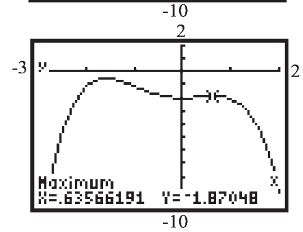
f is decreasing on: $[-3, -1.87]$ and $[0, 0.97]$

64. $f(x) = -0.4x^4 - 0.5x^3 + 0.8x^2 - 2$ on the interval $(-3, 2)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.4x^4 - 0.5x^3 + 0.8x^2 - 2$.







local maxima: $f(-1.57) = -0.52$,

$f(0.64) = -1.87$

local minimum: $(0, -2)$ $f(0) = -2$

f is increasing on: $[-3, -1.57]$ and $[0, 0.64]$;

f is decreasing on: $[-1.57, 0]$ and $[0.64, 2]$

65. $f(x) = -2x^2 + 4$

a. Average rate of change of *f* from $x = 0$ to $x = 2$

$$\frac{f(2) - f(0)}{2 - 0} = \frac{\left(-2(2)^2 + 4\right) - \left(-2(0)^2 + 4\right)}{2}$$

$$= \frac{(-4) - (4)}{2} = \frac{-8}{2} = -4$$

b. Average rate of change of *f* from $x = 1$ to $x = 3$:

$$\frac{f(3)-f(1)}{3-1}=\frac{\left(-2(3)^2+4\right)-\left(-2(1)^2+4\right)}{2}$$
$$=\frac{(-14)-(2)}{2}=\frac{-16}{2}=-8$$

c. Average rate of change of *f* from $x = 1$ to $x = 4$:

$$\frac{f(4)-f(1)}{4-1}=\frac{\left(-2(4)^2+4\right)-\left(-2(1)^2+4\right)}{3}$$
$$=\frac{(-28)-(2)}{3}=\frac{-30}{3}=-10$$

66. $f(x)=-x^3+1$

a. Average rate of change of *f* from $x = 0$ to $x = 2$:

$$\frac{f(2)-f(0)}{2-0}=\frac{\left(-(2)^3+1\right)-\left(-(0)^3+1\right)}{2}$$
$$=\frac{-7-1}{2}=\frac{-8}{2}=-4$$

b. Average rate of change of *f* from $x = 1$ to $x = 3$:

$$\frac{f(3)-f(1)}{3-1}=\frac{\left(-(3)^3+1\right)-\left(-(1)^3+1\right)}{2}$$
$$=\frac{-26-(0)}{2}=\frac{-26}{2}=-13$$

c. Average rate of change of *f* from $x = -1$ to $x = 1$:

$$\frac{f(1)-f(-1)}{1-(-1)}=\frac{\left(-(1)^3+1\right)-\left(-(-1)^3+1\right)}{2}$$
$$=\frac{0-2}{2}=\frac{-2}{2}=-1$$

67. $g(x)=x^3-4x+7$

a. Average rate of change of *g* from $x=-3$ to $x=-2$:

$$\frac{g(-2)-g(-3)}{-2-(-3)}$$
$$=\frac{\left[(-2)^3-4(-2)+7\right]-\left[(-3)^3-4(-3)+7\right]}{1}$$
$$=\frac{(7)-(-8)}{1}=\frac{15}{1}=15$$

b. Average rate of change of *g* from $x=-1$ to $x=1$:

$$\frac{g(1)-g(-1)}{1-(-1)}$$
$$=\frac{\left[(1)^3-4(1)+7\right]-\left[(-1)^3-4(-1)+7\right]}{2}$$
$$=\frac{(4)-(10)}{2}=\frac{-6}{2}=-3$$

c. Average rate of change of *g* from $x=1$ to $x=3$:

$$\frac{g(3)-g(1)}{3-1}$$
$$=\frac{\left[(3)^3-4(3)+7\right]-\left[(1)^3-4(1)+7\right]}{2}$$
$$=\frac{(22)-(4)}{2}=\frac{18}{2}=9$$

68. $h(x)=x^2-2x+3$

a. Average rate of change of *h* from $x=-1$ to $x=1$:

$$\frac{h(1)-h(-1)}{1-(-1)}$$
$$=\frac{\left[(1)^2-2(1)+3\right]-\left[(-1)^2-2(-1)+3\right]}{2}$$
$$=\frac{(2)-(6)}{2}=\frac{-4}{2}=-2$$

b. Average rate of change of *h* from $x=0$ to $x=2$:

$$\frac{h(2)-h(0)}{2-0}$$
$$=\frac{\left[(2)^2-2(2)+3\right]-\left[(0)^2-2(0)+3\right]}{2}$$
$$=\frac{(3)-(3)}{2}=\frac{0}{2}=0$$

c. Average rate of change of h from $x=2$ to $x=5$:

$$\frac{h(5)-h(2)}{5-2}$$
$$=\frac{\left[(5)^2-2(5)+3\right]-\left[(2)^2-2(2)+3\right]}{3}$$
$$=\frac{(18)-(3)}{3}=\frac{15}{3}=5$$

69. $f(x)=5x-2$

a. Average rate of change of f from 1 to 3:

$$\frac{\Delta y}{\Delta x}=\frac{f(3)-f(1)}{3-1}=\frac{13-3}{3-1}=\frac{10}{2}=5$$

Thus, the average rate of change of f from 1 to 3 is 5.

b. From (a), the slope of the secant line joining $(1,f(1))$ and $(3,f(3))$ is 5. We use the point-slope form to find the equation of the secant line:

$$y-y_1=m_{\text{sec}}(x-x_1)$$
$$y-3=5(x-1)$$
$$y-3=5x-5$$
$$y=5x-2$$

70. $f(x)=-4x+1$

a. Average rate of change of f from 2 to 5:

$$\frac{\Delta y}{\Delta x}=\frac{f(5)-f(2)}{5-2}=\frac{-19-(-7)}{5-2}$$
$$=\frac{-12}{3}=-4$$

Therefore, the average rate of change of f from 2 to 5 is -4.

b. From (a), the slope of the secant line joining $(2,f(2))$ and $(5,f(5))$ is -4. We use the point-slope form to find the equation of the secant line:

$$y-y_1=m_{\text{sec}}(x-x_1)$$
$$y-(-7)=-4(x-2)$$
$$y+7=-4x+8$$
$$y=-4x+1$$

71. $g(x)=x^2-2$

a. Average rate of change of g from -2 to 1:

$$\frac{\Delta y}{\Delta x}=\frac{g(1)-g(-2)}{1-(-2)}=\frac{-1-2}{1-(-2)}=\frac{-3}{3}=-1$$

Therefore, the average rate of change of g from -2 to 1 is -1.

b. From (a), the slope of the secant line joining $(-2,g(-2))$ and $(1,g(1))$ is -1. We use the point-slope form to find the equation of the secant line:

$$y-y_1=m_{\text{sec}}(x-x_1)$$
$$y-2=-1(x-(-2))$$
$$y-2=-x-2$$
$$y=-x$$

72. $g(x)=x^2+1$

a. Average rate of change of g from -1 to 2:

$$\frac{\Delta y}{\Delta x}=\frac{g(2)-g(-1)}{2-(-1)}=\frac{5-2}{2-(-1)}=\frac{3}{3}=1$$

Therefore, the average rate of change of g from -1 to 2 is 1.

b. From (a), the slope of the secant line joining $(-1,g(-1))$ and $(2,g(2))$ is 1. We use the point-slope form to find the equation of the secant line:

$$y-y_1=m_{\text{sec}}(x-x_1)$$
$$y-2=1(x-(-1))$$
$$y-2=x+1$$
$$y=x+3$$

73. $h(x)=x^2-2x$

a. Average rate of change of h from 2 to 4:

$$\frac{\Delta y}{\Delta x}=\frac{h(4)-h(2)}{4-2}=\frac{8-0}{4-2}=\frac{8}{2}=4$$

Therefore, the average rate of change of h from 2 to 4 is 4.

b. From (a), the slope of the secant line joining $(2,h(2))$ and $(4,h(4))$ is 4. We use the point-slope form to find the equation of the secant line:

$$y-y_1=m_{\text{sec}}(x-x_1)$$
$$y-0=4(x-2)$$
$$y=4x-8$$

74. $h(x) = -2x^2 + x$

a. Average rate of change from 0 to 3:

$$\frac{\Delta y}{\Delta x} = \frac{h(3)-h(0)}{3-0} = \frac{-15-0}{3-0}$$
$$= \frac{-15}{3} = -5$$

Therefore, the average rate of change of h from 0 to 3 is -5.

b. From (a), the slope of the secant line joining $(0, h(0))$ and $(3, h(3))$ is -5. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 0 = -5(x - 0)$$
$$y = -5x$$

75. a. $g(x) = x^3 - 27x$

$$g(-x) = (-x)^3 - 27(-x)$$
$$= -x^3 + 27x$$
$$= -(x^3 - 27x)$$
$$= -g(x)$$

Since $g(-x) = -g(x)$, the function is odd.

b. Since $g(x)$ is odd then it is symmetric about the origin so there exist a local maximum at $x = -3$.

$$g(-3) = (-3)^3 - 27(-3) = -27 + 81 = 54$$

So there is a local maximum of 54 at $x = -3$.

76. $f(x) = -x^3 + 12x$

a. $f(-x) = -(-x)^3 + 12(-x)$

$$= x^3 - 12x$$
$$= -(-x^3 + 12x)$$
$$= -f(x)$$

Since $f(-x) = -f(x)$, the function is odd.

b. Since $f(x)$ is odd then it is symmetric about the origin so there exist a local maximum at $x = -3$.

$$f(-2) = -(-2)^3 + 12(-2) = 8 - 24 = -16$$

So there is a local minimum value of -16 at $x = -2$.

77. $F(x) = -x^4 + 8x^2 + 9$

a. $F(-x) = -(-x)^4 + 8(-x)^2 + 9$

$$= -x^4 + 8x + 9$$
$$= F(x)$$

Since $F(-x) = F(x)$, the function is even.

b. Since the function is even, its graph has y-axis symmetry. The second local maximum value is 25 and occurs at $x = -2$.

c. Because the graph has y-axis symmetry, the area under the graph between $x = 0$ and $x = 3$ bounded below by the x-axis is the same as the area under the graph between $x = -3$ and $x = 0$ bounded below the x-axis. Thus, the area is 50.4 square units.

78. $G(x) = -x^4 + 32x^2 + 144$

a. $G(-x) = -(-x)^4 + 32(-x)^2 + 144$

$$= -x^4 + 32x^2 + 144$$
$$= G(x)$$

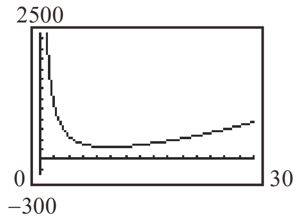
Since $G(-x) = G(x)$, the function is even.

b. Since the function is even, its graph has y-axis symmetry. The second local maximum is in quadrant II and is 400 and occurs at $x = -4$.

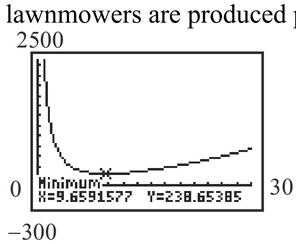
c. Because the graph has y-axis symmetry, the area under the graph between $x = 0$ and $x = 6$ bounded below by the x-axis is the same as the area under the graph between $x = -6$ and $x = 0$ bounded below the x-axis. Thus, the area is 1612.8 square units.

79. $\overline{C}(x)=0.3x^2+21x-251+\dfrac{2500}{x}$

a. $y_1=0.3x^2+21x-251+\dfrac{2500}{x}$



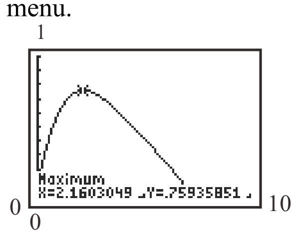
b. Use MINIMUM. Rounding to the nearest whole number, the average cost is minimized when approximately 10 lawnmowers are produced per hour.



c. The minimum average cost is approximately \$239 per mower.

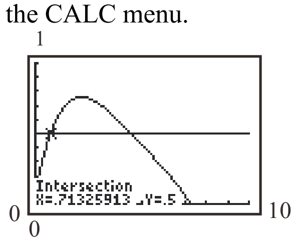
80. a. $C(t)=-.002t^4+.039t^3-.285t^2+.766t+.085$

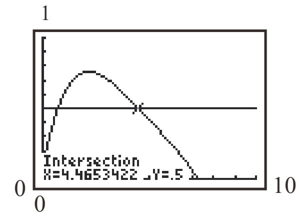
Graph the function on a graphing utility and use the Maximum option from the CALC menu.



The concentration will be highest after about 2.16 hours.

b. Enter the function in Y1 and 0.5 in Y2. Graph the two equations in the same window and use the Intersect option from the CALC menu.





After taking the medication, the woman can feed her child within the first 0.71 hours (about 42 minutes) or after 4.47 hours (about 4hours 28 minutes) have elapsed.

81. a.
$$\text{avg. rate of change}=\frac{P(2.5)-P(0)}{2.5-0}=\frac{0.18-0.09}{2.5-0}=\frac{0.09}{2.5}=0.036\text{ gram per hour}$$

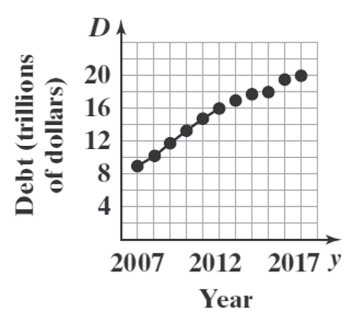
On average, the population is increasing at a rate of 0.036 gram per hour from 0 to 2.5 hours.

b.
$$\text{avg. rate of change}=\frac{P(6)-P(4.5)}{6-4.5}=\frac{0.50-0.35}{6-4.5}=\frac{0.15}{1.5}=0.1\text{ gram per hour}$$

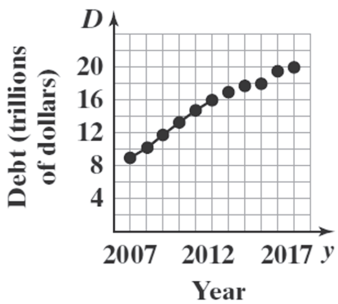
On average, the population is increasing at a rate of 0.1 gram per hour from 4.5 to 6 hours.

c. The average rate of change is increasing as time passes. This indicates that the population is increasing at an increasing rate.

82. a.



b.



c. avg. rate of change $= \dfrac{P(2014)-P(2012)}{2014-2012}$
$= \dfrac{9512-9273}{2}$
$= \dfrac{239}{2}$
$= \$119.50$ dollars/yr

d. avg. rate of change $= \dfrac{P(2016)-P(2014)}{2013-2011}$
$= \dfrac{9938-9512}{2}$
$= \dfrac{426}{2}$
$= \$213.00$ dollars/yr

e. avg. rate of change $= \dfrac{P(2018)-P(2016)}{2016-2014}$
$= \dfrac{10409-9938}{2}$
$= \dfrac{471}{2}$
$= \$235.50$ dollars/yr

f. The average rate of change is increasing.

g. avg. rate of change $= \dfrac{P(2021)-P(2019)}{2021-2019}$
$= \dfrac{9177-10507}{2}$
$= -\dfrac{1330}{2}$
$= -\$665.00$ dollars/yr

The debt decreased.

h. Quarantine from Covid19 most likely had a effect on spending habits.

83. $f(x) = x^2$

a. Average rate of change of *f* from $x = 0$ to $x = 1$:
$$\frac{f(1)-f(0)}{1-0} = \frac{1^2-0^2}{1} = \frac{1}{1} = 1$$

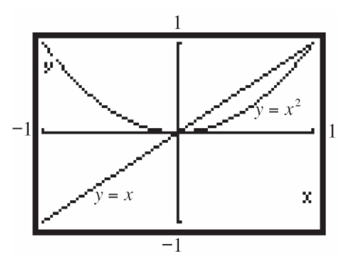
b. Average rate of change of *f* from $x = 0$ to $x = 0.5$:
$$\frac{f(0.5)-f(0)}{0.5-0} = \frac{(0.5)^2-0^2}{0.5} = \frac{0.25}{0.5} = 0.5$$

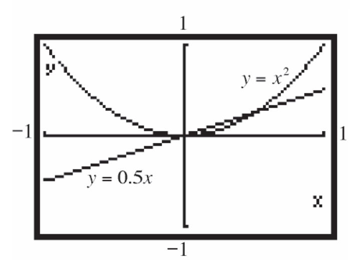
c. Average rate of change of *f* from $x = 0$ to $x = 0.1$:
$$\frac{f(0.1)-f(0)}{0.1-0} = \frac{(0.1)^2-0^2}{0.1} = \frac{0.01}{0.1} = 0.1$$

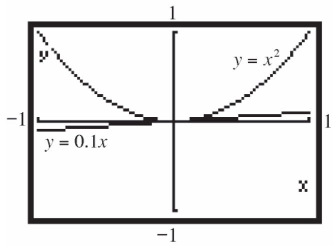
d. Average rate of change of *f* from $x = 0$ to $x = 0.01$:
$$\frac{f(0.01)-f(0)}{0.01-0} = \frac{(0.01)^2-0^2}{0.01}$$
$$= \frac{0.0001}{0.01} = 0.01$$

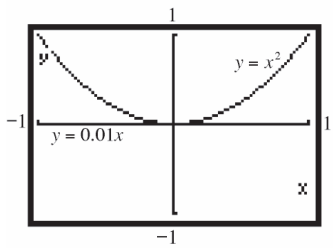
e. Average rate of change of *f* from $x = 0$ to $x = 0.001$:
$$\frac{f(0.001)-f(0)}{0.001-0} = \frac{(0.001)^2-0^2}{0.001}$$
$$= \frac{0.000001}{0.001} = 0.001$$

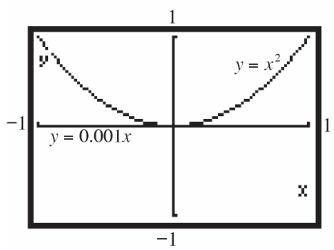
f. Graphing the secant lines:











g. The secant lines are beginning to look more and more like the tangent line to the graph of *f* at the point where $x = 0$.

h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number zero.

84. $f(x) = x^2$

a. Average rate of change of *f* from $x = 1$ to $x = 2$:

$$\frac{f(2) - f(1)}{2 - 1} = \frac{2^2 - 1^2}{1} = \frac{3}{1} = 3$$

b. Average rate of change of *f* from $x = 1$ to $x = 1.5$:

$$\frac{f(1.5) - f(1)}{1.5 - 1} = \frac{(1.5)^2 - 1^2}{0.5} = \frac{1.25}{0.5} = 2.5$$

c. Average rate of change of *f* from $x = 1$ to $x = 1.1$:

$$\frac{f(1.1) - f(1)}{1.1 - 1} = \frac{(1.1)^2 - 1^2}{0.1} = \frac{0.21}{0.1} = 2.1$$

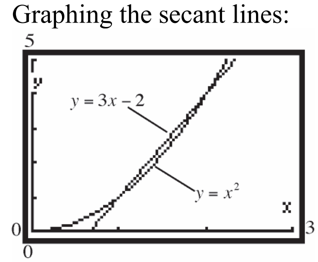
d. Average rate of change of *f* from $x = 1$ to $x = 1.01$:

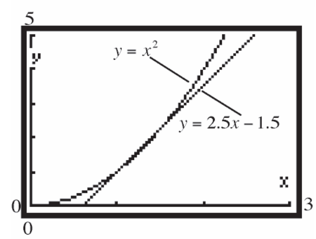
$$\frac{f(1.01) - f(1)}{1.01 - 1} = \frac{(1.01)^2 - 1^2}{0.01} = \frac{0.0201}{0.01} = 2.01$$

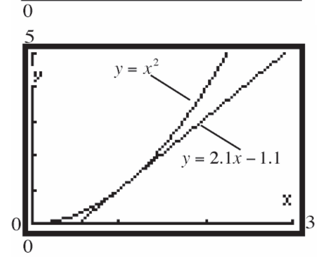
e. Average rate of change of *f* from $x = 1$ to $x = 1.001$:

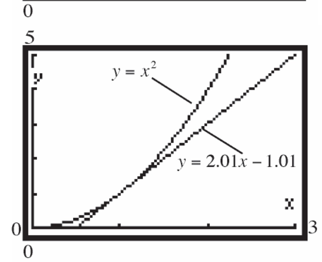
$$\frac{f(1.001) - f(1)}{1.001 - 1} = \frac{(1.001)^2 - 1^2}{0.001} = \frac{0.002001}{0.001} = 2.001$$

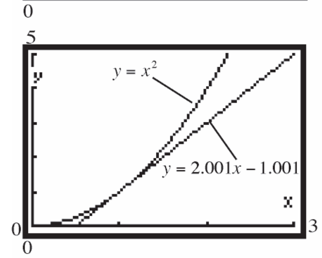
f. Graphing the secant lines:











g. The secant lines are beginning to look more and more like the tangent line to the graph of *f* at the point where $x = 1$.

h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number 2.

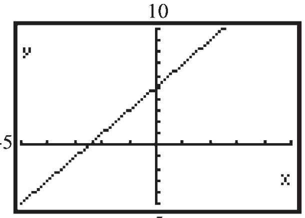
85. $f(x) = 2x + 5$

a. $$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{2(x+h) + 5 - 2x - 5}{h} = \frac{2h}{h} = 2$$

b. When $x = 1$:
$h = 0.5 \Rightarrow m_{\text{sec}} = 2$
$h = 0.1 \Rightarrow m_{\text{sec}} = 2$
$h = 0.01 \Rightarrow m_{\text{sec}} = 2$
as $h \to 0,\ m_{\text{sec}} \to 2$

c. Using the point $(1, f(1)) = (1, 7)$ and slope, $m = 2$, we get the secant line:
$$y - 7 = 2(x - 1)$$
$$y - 7 = 2x - 2$$
$$y = 2x + 5$$

d. Graphing:



The graph and the secant line coincide.

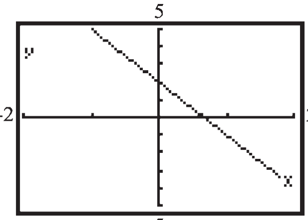
86. $f(x) = -3x + 2$

a. $$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{-3(x+h) + 2 - (-3x + 2)}{h} = \frac{-3h}{h} = -3$$

b. When $x = 1$,
$h = 0.5 \Rightarrow m_{\text{sec}} = -3$
$h = 0.1 \Rightarrow m_{\text{sec}} = -3$
$h = 0.01 \Rightarrow m_{\text{sec}} = -3$
as $h \to 0,\ m_{\text{sec}} \to -3$

c. Using point $(1, f(1)) = (1, -1)$ and slope $= -3$, we get the secant line:
$$y - (-1) = -3(x - 1)$$
$$y + 1 = -3x + 3$$
$$y = -3x + 2$$

d. Graphing:



The graph and the secant line coincide.

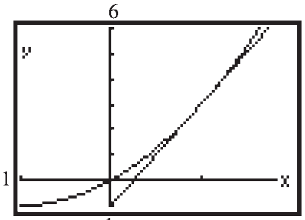
87. $f(x) = x^2 + 2x$

a. $$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h}$$
$$= \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h}$$
$$= \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$$
$$= \frac{2xh + h^2 + 2h}{h}$$
$$= 2x + h + 2$$

b. When $x = 1$,
$h = 0.5 \Rightarrow m_{\text{sec}} = 2 \cdot 1 + 0.5 + 2 = 4.5$
$h = 0.1 \Rightarrow m_{\text{sec}} = 2 \cdot 1 + 0.1 + 2 = 4.1$
$h = 0.01 \Rightarrow m_{\text{sec}} = 2 \cdot 1 + 0.01 + 2 = 4.01$
as $h \to 0,\ m_{\text{sec}} \to 2 \cdot 1 + 0 + 2 = 4$

c. Using point $(1, f(1)) = (1, 3)$ and slope $= 4.01$, we get the secant line:
$$y - 3 = 4.01(x - 1)$$
$$y - 3 = 4.01x - 4.01$$
$$y = 4.01x - 1.01$$

d. Graphing:



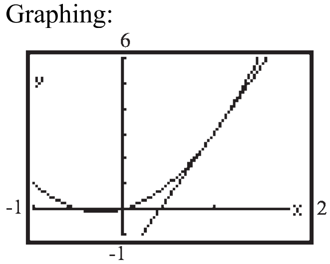
88. $f(x)=2x^2+x$

a. $$m_{\sec}=\frac{f(x+h)-f(x)}{h}$$
$$=\frac{2(x+h)^2+(x+h)-(2x^2+x)}{h}$$
$$=\frac{2(x^2+2xh+h^2)+x+h-2x^2-x}{h}$$
$$=\frac{2x^2+4xh+2h^2+x+h-2x^2-x}{h}$$
$$=\frac{4xh+2h^2+h}{h}$$
$$=4x+2h+1$$

b. When $x=1$,
$h=0.5\Rightarrow m_{\sec}=4\cdot 1+2(0.5)+1=6$
$h=0.1\Rightarrow m_{\sec}=4\cdot 1+2(0.1)+1=5.2$
$h=0.01\Rightarrow m_{\sec}=4\cdot 1+2(0.01)+1=5.02$
as $h\to 0$, $m_{\sec}\to 4\cdot 1+2(0)+1=5$

c. Using point $(1,f(1))=(1,3)$ and slope = 5.02, we get the secant line:
$y-3=5.02(x-1)$
$y-3=5.02x-5.02$
$y=5.02x-2.02$

d. Graphing:



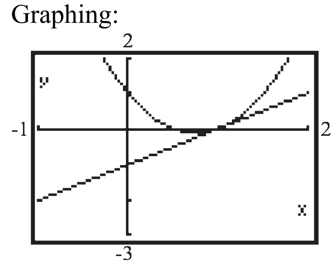
89. $f(x)=2x^2-3x+1$

a. $$m_{\sec}=\frac{f(x+h)-f(x)}{h}$$
$$=\frac{2(x+h)^2-3(x+h)+1-\left(2x^2-3x+1\right)}{h}$$
$$=\frac{2(x^2+2xh+h^2)-3x-3h+1-2x^2+3x-1}{h}$$
$$=\frac{2x^2+4xh+2h^2-3x-3h+1-2x^2+3x-1}{h}$$
$$=\frac{4xh+2h^2-3h}{h}$$
$$=4x+2h-3$$

b. When $x=1$,
$h=0.5\Rightarrow m_{\sec}=4\cdot 1+2(0.5)-3=2$
$h=0.1\Rightarrow m_{\sec}=4\cdot 1+2(0.1)-3=1.2$
$h=0.01\Rightarrow m_{\sec}=4\cdot 1+2(0.01)-3=1.02$
as $h\to 0$, $m_{\sec}\to 4\cdot 1+2(0)-3=1$

c. Using point $(1,f(1))=(1,0)$ and slope = 1.02, we get the secant line:
$y-0=1.02(x-1)$
$y=1.02x-1.02$

d. Graphing:



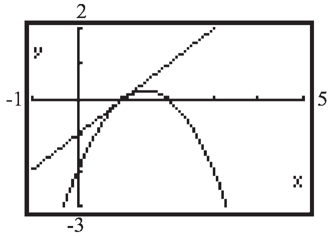
90. $f(x)=-x^2+3x-2$

a. $$m_{\sec}=\frac{f(x+h)-f(x)}{h}$$
$$=\frac{-(x+h)^2+3(x+h)-2-\left(-x^2+3x-2\right)}{h}$$
$$=\frac{-(x^2+2xh+h^2)+3x+3h-2+x^2-3x+2}{h}$$
$$=\frac{-x^2-2xh-h^2+3x+3h-2+x^2-3x+2}{h}$$
$$=\frac{-2xh-h^2+3h}{h}$$
$$=-2x-h+3$$

b. When $x=1$,
$h=0.5\Rightarrow m_{\sec}=-2\cdot 1-0.5+3=0.5$
$h=0.1\Rightarrow m_{\sec}=-2\cdot 1-0.1+3=0.9$
$h=0.01\Rightarrow m_{\sec}=-2\cdot 1-0.01+3=0.99$
as $h\to 0$, $m_{\sec}\to -2\cdot 1-0+3=1$

c. Using point $(1,f(1))=(1,0)$ and slope = 0.99, we get the secant line:
$y-0=0.99(x-1)$
$y=0.99x-0.99$

d. Graphing:



91. $f(x)=\dfrac{1}{x}$

a. $$m_{\text{sec}}=\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(\frac{1}{x+h}-\frac{1}{x}\right)}{h}=\frac{\left(\frac{x-(x+h)}{(x+h)x}\right)}{h}$$
$$=\left(\frac{x-x-h}{(x+h)x}\right)\left(\frac{1}{h}\right)=\left(\frac{-h}{(x+h)x}\right)\left(\frac{1}{h}\right)$$
$$=-\frac{1}{(x+h)x}$$

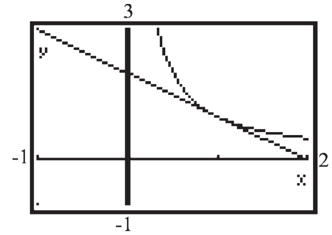
b. When $x=1$,

$$h=0.5\Rightarrow m_{\text{sec}}=-\frac{1}{(1+0.5)(1)}=-\frac{1}{1.5}=-\frac{2}{3}\approx-0.667$$
$$h=0.1\Rightarrow m_{\text{sec}}=-\frac{1}{(1+0.1)(1)}=-\frac{1}{1.1}=-\frac{10}{11}\approx-0.909$$
$$h=0.01\Rightarrow m_{\text{sec}}=-\frac{1}{(1+0.01)(1)}=-\frac{1}{1.01}=-\frac{100}{101}\approx-0.990$$
$$\text{as } h\to0,\quad m_{\text{sec}}\to-\frac{1}{(1+0)(1)}=-\frac{1}{1}=-1$$

c. Using point $(1,f(1))=(1,1)$ and slope $=-\dfrac{100}{101}$, we get the secant line:

$$y-1=-\frac{100}{101}(x-1)$$
$$y-1=-\frac{100}{101}x+\frac{100}{101}$$
$$y=-\frac{100}{101}x+\frac{201}{101}$$

d. Graphing:



92. $f(x)=\dfrac{1}{x^2}$

a. $$m_{\text{sec}}=\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(\frac{1}{(x+h)^2}-\frac{1}{x^2}\right)}{h}$$
$$=\frac{\left(\frac{x^2-(x+h)^2}{(x+h)^2x^2}\right)}{h}$$
$$=\left(\frac{x^2-\left(x^2+2xh+h^2\right)}{(x+h)^2x^2}\right)\left(\frac{1}{h}\right)$$
$$=\left(\frac{-2xh-h^2}{(x+h)^2x^2}\right)\left(\frac{1}{h}\right)$$
$$=\frac{-2x-h}{(x+h)^2x^2}=\frac{-2x-h}{\left(x^2+2xh+h^2\right)x^2}$$

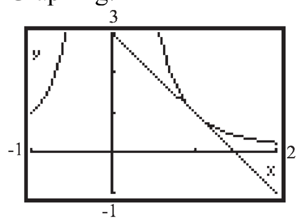
b. When $x=1$,

$$h=0.5\Rightarrow m_{\text{sec}}=\frac{-2\cdot1-0.5}{(1+0.5)^2\,1^2}=-\frac{10}{9}\approx-1.1111$$
$$h=0.1\Rightarrow m_{\text{sec}}=\frac{-2\cdot1-0.1}{(1+0.1)^2\,1^2}=-\frac{210}{121}\approx-1.7355$$
$$h=0.01\Rightarrow m_{\text{sec}}=\frac{-2\cdot1-0.01}{(1+0.01)^2\,1^2}=-\frac{20{,}100}{10{,}201}\approx-1.9704$$
$$\text{as } h\to0,\quad m_{\text{sec}}\to\frac{-2\cdot1-0}{(1+0)^2\,1^2}=-2$$

c. Using point $(1, f(1)) = (1,1)$ and slope $= -1.9704$, we get the secant line:

$$y - 1 = -1.9704(x-1)$$
$$y - 1 = -1.9704x + 1.9704$$
$$y = -1.9704x + 2.9704$$

d. Graphing:



93. $f(2) = 12$ and $f(-1) = 8,$ so

$$\frac{f(2) - f(-1)}{2-(-1)} = \frac{12}{3} = 4$$
$$f'(x) = 4 \rightarrow$$
$$3x^2 + 4x - 1 = 4$$
$$3x^2 + 4x - 5 = 0$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-5)}}{2(3)} = \frac{-4 \pm \sqrt{76}}{6}$$
$$= \frac{-4 \pm 2\sqrt{19}}{6} = \frac{-2 \pm \sqrt{19}}{3}$$
$$x = \frac{-2-\sqrt{19}}{3} \approx -2.1 \text{ is outside the interval}$$
$$x = \frac{-2+\sqrt{19}}{3} \approx .079 \text{ is in the interval.}$$

The only such number is $\frac{-2+\sqrt{19}}{3}$.

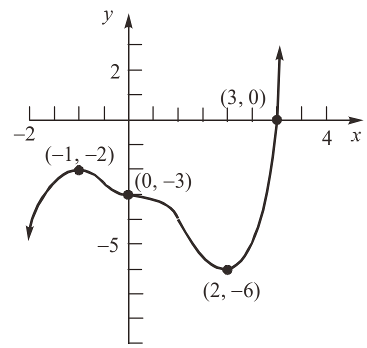
94. $g(-x) = -2f\left(-\frac{-x}{3}\right) = -2f\left(\frac{x}{3}\right)$. Since f is odd,

$$f\left(-\frac{x}{3}\right) = -f\left(\frac{x}{3}\right)$$
$$g(-x) = -2f\left(\frac{x}{3}\right) = 2f\left(-\frac{x}{3}\right) = -g(x)$$

so g is odd.

So g is odd.

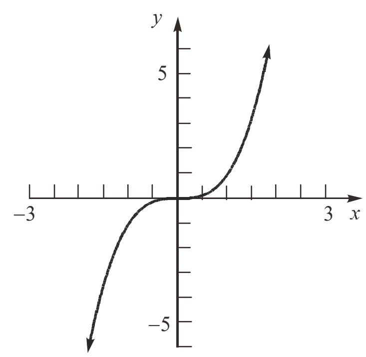
95. Answers will vary. One possibility follows:



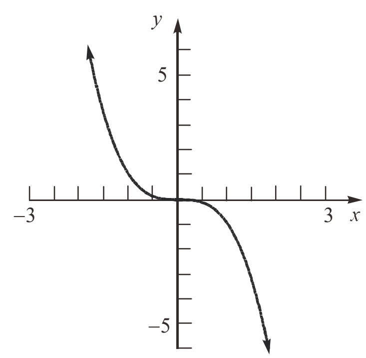
96. Answers will vary. See solution to Problem 89 for one possibility.

97. A function that is increasing on an interval can have at most one x-intercept on the interval. The graph of f could not "turn" and cross it again or it would start to decrease.

98. An increasing function is a function whose graph goes up as you read from left to right.

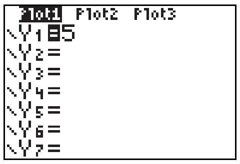


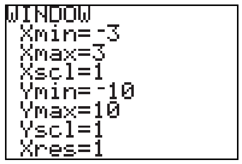
A decreasing function is a function whose graph goes down as you read from left to right.

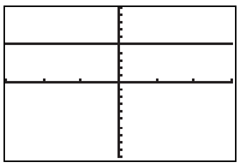


99. To be an even function we need $f(-x) = f(x)$ and to be an odd function we need $f(-x) = -f(x)$. In order for a function be both even and odd, we would need $f(x) = -f(x)$. This is only possible if $f(x) = 0$.

100. The graph of $y = 5$ is a horizontal line.







The local maximum is $y = 5$ and it occurs at each *x*-value in the interval.

101. Not necessarily. It just means $f(5) > f(2)$. The function could have both increasing and decreasing intervals.

102. $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{b - b}{x_2 - x_1} = 0$

$$\frac{f(2) - f(-2)}{2 - (-2)} = \frac{0 - 0}{4} = 0$$

103. $\sqrt{540} = \sqrt{36(15)} = 6\sqrt{15}$

104.
$$\begin{aligned}(4a - b)^2 &= (4a - b)(4a - b)\\ &= 16a^2 - 4ab - 4ab + b^2\\ &= 16a^2 - 8ab + b^2\end{aligned}$$

105. $C(x) = 0.80x + 40$

106.
$$S = \frac{k}{T}$$
$$33 = \frac{k}{10}$$
$$k = 330$$
$$S = \frac{330}{T}$$
$$S = \frac{330}{10}$$
$$S = 8.25 \text{ days}$$

107.
$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 3)^2 + (y - (-2))^2 = \left(\frac{\sqrt{6}}{2}\right)^2$$
$$(x - 3)^2 + (y + 2)^2 = \frac{3}{2}$$

108.
$$\begin{aligned}&\left(x^2 - 2\right)^3 2\left(3x^5 + 1\right)15x^4 + \left(3x^5 + 1\right)^2 3\left(x^2 - 2\right)^3 2x\\ &= 6x\left(x^2 - 2\right)^2\left(3x^5 + 1\right)\left[\left(x^2 - 2\right)5x^3 + \left(3x^5 + 1\right)\right]\\ &= 6x\left(x^2 - 2\right)^2\left(3x^5 + 1\right)\left[5x^5 - 10x^3 + 3x^5 + 1\right]\\ &= 6x\left(x^2 - 2\right)^2\left(3x^5 + 1\right)\left[8x^5 - 10x^3 + 1\right]\end{aligned}$$

109.
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) =$$
$$\left(\frac{-2 + \frac{3}{5}}{2}, \frac{1 + (-4)}{2}\right) =$$
$$\left(\frac{-\frac{7}{5}}{2}, \frac{-3}{2}\right) = \left(-\frac{7}{10}, -\frac{3}{2}\right)$$

110.
$$|3x + 7| - 3 = 5$$
$$|3x + 7| = 8$$
$$3x + 7 = -8 \quad \text{or} \quad 3x + 7 = 8$$
$$3x = -15 \qquad 3x = 1$$
$$x = -5 \qquad x = \frac{1}{3}$$

The solution set is $\left\{-5, \frac{1}{3}\right\}$.

111.
$$x^6 + 7x^3 = 8$$
$$x^6 + 7x^3 - 8 = 0$$
$$(x^3 + 8)(x^3 - 1) = 0$$
$$x^3 + 8 = 0 \quad \text{or} \quad x^3 - 1 = 0$$
$$x^3 = -8 \qquad x^3 = 1$$
$$x = -2 \qquad x = 1$$

The solution set is $\{-2, 1\}$.

112. $3y^2 \cdot D + 3x^2 - 3xy^2 - 3x^2 y \cdot D = 0$

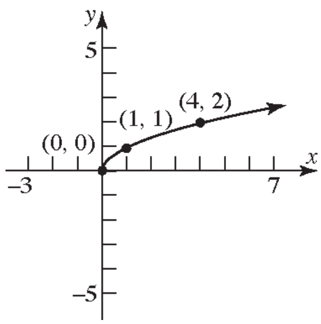
$$3y^2 \cdot D - 3x^2 y \cdot D = -3x^2 + 3xy^2$$

$$D\left(3y^2 - 3x^2 y\right) = -3x^2 + 3xy^2$$

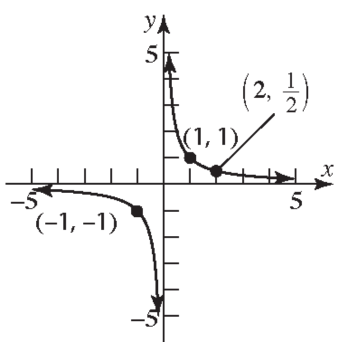
$$D = \frac{-3x^2 + 3xy^2}{3y^2 - 3x^2 y} = \frac{xy^2 - x^2}{y^2 - x^2 y} = \frac{x\left(y^2 - x\right)}{y\left(y - x^2\right)}$$

Section 2.4

1. $y = \sqrt{x}$



2. $y = \dfrac{1}{x}$



3. $y = x^3 - 8$

<u>*y*-intercept:</u>

Let $x = 0$, then $y = (0)^3 - 8 = -8$.

<u>*x*-intercept:</u>

Let $y = 0$, then $0 = x^3 - 8$

$$x^3 = 8$$

$$x = 2$$

The intercepts are $(0,-8)$ and $(2,0)$.

4. $(-\infty, 0)$

5. piecewise-defined

6. True

7. False; the cube root function is odd and increasing on the interval $(-\infty, \infty)$.

8. False; the domain and range of the reciprocal function are both the set of real numbers except for 0.

9. b

10. a

11. C

12. A

13. E

14. G

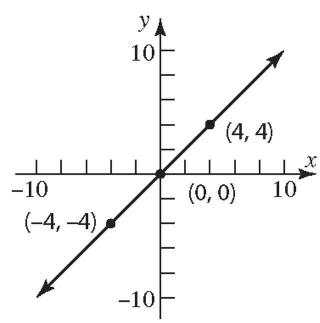
15. B

16. D

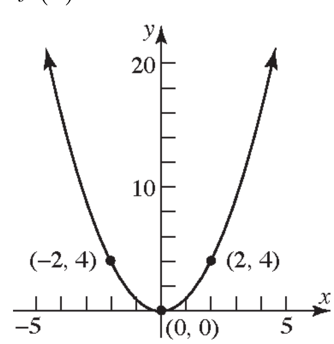
17. F

18. H

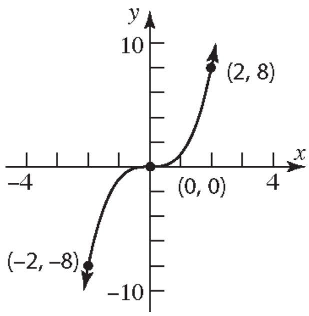
19. $f(x) = x$



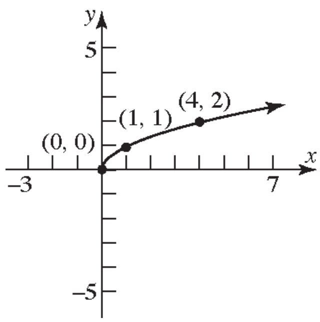
20. $f(x) = x^2$



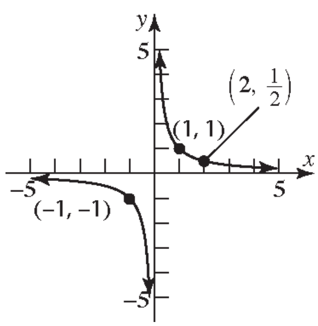
21. $f(x)=x^3$



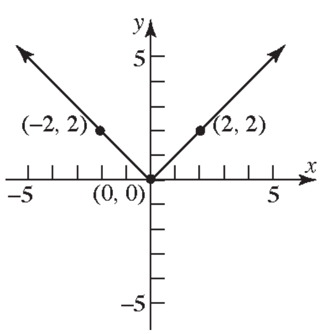
22. $f(x)=\sqrt{x}$



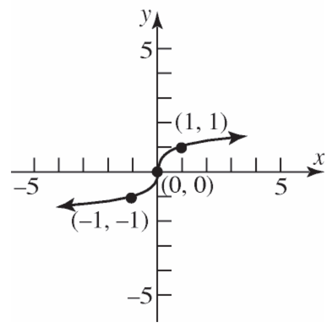
23. $f(x)=\dfrac{1}{x}$



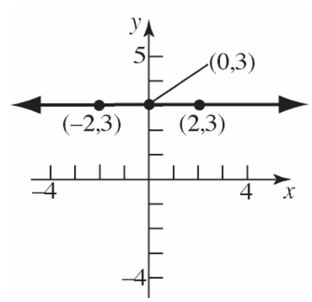
24. $f(x)=|x|$



25. $f(x)=\sqrt[3]{x}$



26. $f(x)=3$



27. **a.** $f(-3)=-(-3)^2=-9$

b. $f(0)=4$

c. $f(2)=3(3)-2=7$

28. **a.** $f(-2)=-3(-2)=6$

b. $f(-1)=0$

c. $f(0)=2(0)^2+1=1$

29. **a.** $f(-2)=2(-2)+4=0$

b. $f(0)=2(0)+4=4$

c. $f(1)=2(1)+4=6$

d. $f(3)=(3)^3-1=26$

30. **a.** $f(-1)=(-1)^3=-1$

b. $f(0)=(0)^3=0$

c. $f(1)=3(1)+2=5$

d. $f(3)=3(3)+2=11$

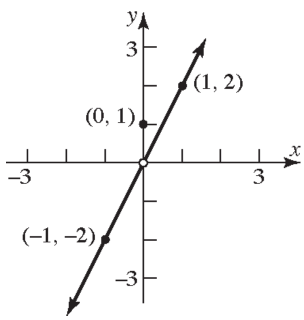
31. $f(x)=\begin{cases}2x & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$

a. Domain: $\{x\,|\,x \text{ is any real number}\}$

b. x-intercept: none
y-intercept:
$f(0)=1$

The only intercept is $(0,1)$.

c. Graph:



d. Range: $\{y\,|\,y\neq 0\}$; $(-\infty,0)\cup(0,\infty)$

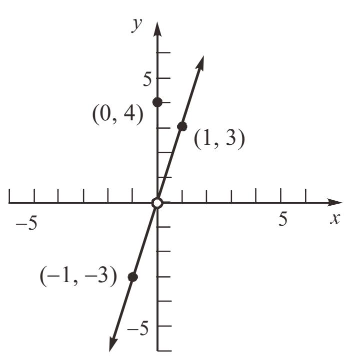
32. $f(x)=\begin{cases}3x & \text{if } x\neq 0\\ 4 & \text{if } x=0\end{cases}$

a. Domain: $\{x\,|\,x \text{ is any real number}\}$

b. x-intercept: none
y-intercept: $f(0)=4$

The only intercept is $(0,4)$.

c. Graph:



d. Range: $\{y\,|\,y\neq 0\}$; $(-\infty,0)\cup(0,\infty)$

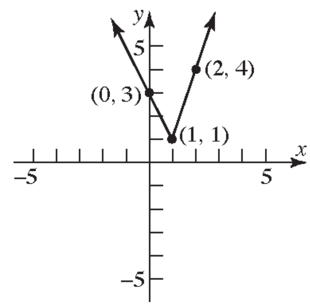
33. $f(x)=\begin{cases}-2x+3 & \text{if } x<1\\ 3x-2 & \text{if } x\geq 1\end{cases}$

a. Domain: $\{x\,|\,x \text{ is any real number}\}$

b. x-intercept: none
y-intercept: $f(0)=-2(0)+3=3$

The only intercept is $(0,3)$.

c. Graph:



d. Range: $\{y\,|\,y\geq 1\}$; $[1,\infty)$

34. $f(x)=\begin{cases}x+3 & \text{if } x<-2\\ -2x-3 & \text{if } x\geq -2\end{cases}$

a. Domain: $\{x\,|\,x \text{ is any real number}\}$

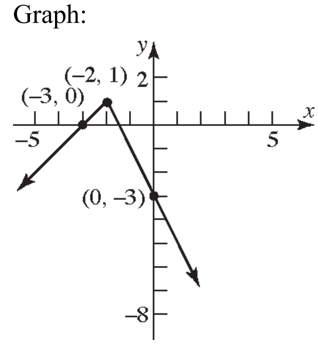
b.
$$\begin{aligned} x+3&=0 \qquad & -2x-3&=0\\ x&=-3 & -2x&=3\\ & & x&=-\frac{3}{2}\end{aligned}$$

x-intercepts: $-3,-\frac{3}{2}$

y-intercept: $f(0)=-2(0)-3=-3$

The intercepts are $(-3,0)$, $\left(-\frac{3}{2},0\right)$, and $(0,-3)$.

c. Graph:



d. Range: $\{y \mid y \le 1\}$; $(-\infty, 1]$

35. $f(x) = \begin{cases} x+3 & \text{if } -2 \le x < 1 \\ 5 & \text{if } x = 1 \\ -x+2 & \text{if } x > 1 \end{cases}$

a. Domain: $\{x \mid x \ge -2\}$; $[-2, \infty)$

b. $x+3=0$ $\qquad$ $-x+2=0$

$x=-3$ $\qquad$ $-x=-2$

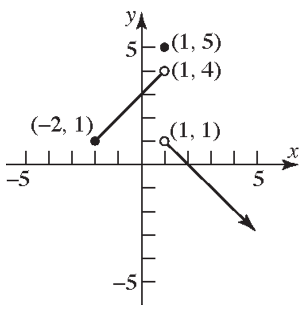
(not in domain) $\qquad$ $x=2$

x-intercept: 2

y-intercept: $f(0) = 0+3 = 3$

The intercepts are $(2,0)$ and $(0,3)$.

c. Graph:



d. Range: $\{y \mid y < 4,\ y = 5\}$; $(-\infty, 4) \cup \{5\}$

36. $f(x) = \begin{cases} 2x+5 & \text{if } -3 \le x < 0 \\ -3 & \text{if } x = 0 \\ -5x & \text{if } x > 0 \end{cases}$

a. Domain: $\{x \mid x \ge -3\}$; $[-3, \infty)$

b. $2x+5=0$ $\qquad$ $-5x=0$

$2x=-5$ $\qquad$ $x=0$

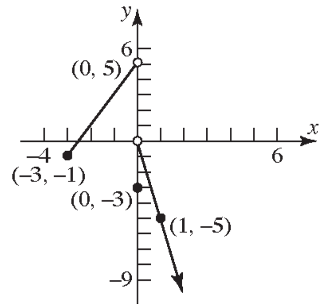
$x=-\dfrac{5}{2}$ $\qquad$ (not in domain of piece)

x-intercept: $-\dfrac{5}{2}$

y-intercept: $f(0) = -3$

The intercepts are $\left(-\dfrac{5}{2}, 0\right)$ and $(0,-3)$.

c. Graph:



d. Range: $\{y \mid y < 5\}$; $(-\infty, 5)$

37. $f(x) = \begin{cases} 1+x & \text{if } x < 0 \\ x^2 & \text{if } x \ge 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. $1+x=0$ $\qquad$ $x^2=0$

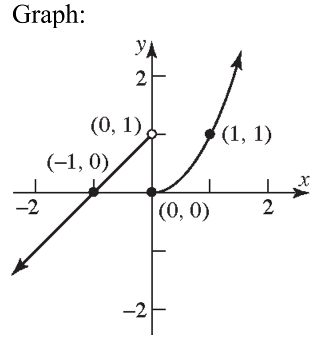
$x=-1$ $\qquad$ $x=0$

x-intercepts: $-1, 0$

y-intercept: $f(0) = 0^2 = 0$

The intercepts are $(-1,0)$ and $(0,0)$.

c. Graph:



d. Range: $\{y \mid y \text{ is any real number}\}$

38. $f(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 0 \\ \sqrt[3]{x} & \text{if } x \ge 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. $\dfrac{1}{x} = 0$ $\qquad$ $\sqrt[3]{x} = 0$

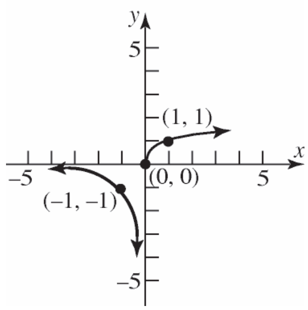
(no solution) $\qquad$ $x = 0$

x-intercept: 0

y-intercept: $f(0) = \sqrt[3]{0} = 0$

The only intercept is $(0,0)$.

c. Graph:



d. Range: $\{y \mid y \text{ is any real number}\}$

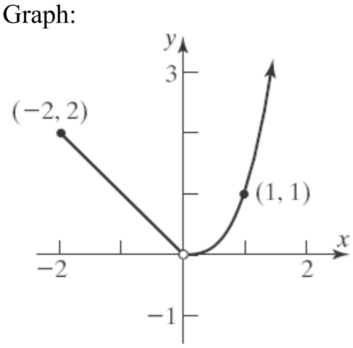
39. $f(x)=\begin{cases}|x| & \text{if } -2 \le x < 0 \\ x^3 & \text{if } x > 0\end{cases}$

a. Domain: $\{x \mid -2 \le x < 0 \text{ and } x > 0\}$ or $\{x \mid x \ge -2, x \ne 0\}$; $[-2,0)\cup(0,\infty)$.

b. x-intercept: none
There are no x-intercepts since there are no values for x such that $f(x)=0$.

y-intercept:
There is no y-intercept since $x=0$ is not in the domain.

c. Graph:



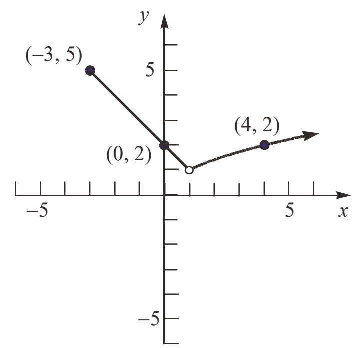
d. Range: $\{y \mid y > 0\}$; $(0,\infty)$

40. $f(x)=\begin{cases}2-x & \text{if } -3 \le x < 1 \\ \sqrt{x} & \text{if } x > 1\end{cases}$

a. Domain: $\{x \mid -3 \le x < 1 \text{ and } x > 1\}$ or $\{x \mid x \ge -3, x \ne 1\}$; $[-3,1)\cup(1,\infty)$.

b. $2-x=0$ $\quad$ $\sqrt{x}=0$
$x=2$ $\quad$ $x=0$
(not in domain of piece)
no x-intercepts
y-intercept: $f(0)=2-0=2$
The intercept is $(0,2)$.

c. Graph:



d. Range: $\{y \mid y > 1\}$; $(1,\infty)$

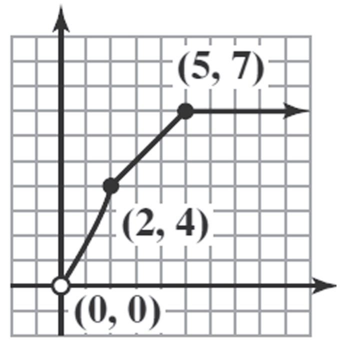
41. $f(x)=\begin{cases}x^2 & \text{if } 0 < x \le 2 \\ x+2 & \text{if } 2 < x < 5 \\ 7 & \text{if } \ge 5\end{cases}$

a. Domain: $\{x \mid x > 0\}$; $(0,\infty)$.

b. $x^2=0$
$x=0$
(not in domain of piece)

$x+2=0$ $\quad$ $7=0$
$x=-2$ $\quad$ (not possible)
(not in domain of piece)
No intercepts.

c. Graph:



d. Range: $\{y \mid 0 < y \le 7\}$; $(0, 7]$

42. $f(x) = \begin{cases} 3x+5 & \text{if } -3 \le x < 0 \\ 5 & \text{if } 0 \le x \le 2 \\ x^2+1 & \text{if } x > 2 \end{cases}$

a. Domain: $\{x \mid x \ge -3\}$; $[3, \infty)$.

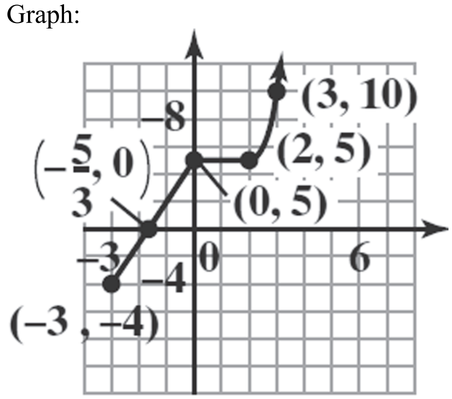
b. $3x+5=0$ $\quad x = -\frac{5}{3}$ $\qquad 5 = 0$ (not possible) $\qquad x^2+1=0$ $\quad x^2=-1$ (not possible)

x-intercept: $-\frac{5}{3}$

y-intercept: $f(0) = 5$

The intercepts are $(0,5)$ and $\left(-\frac{5}{3}, 0\right)$.

c. Graph:



d. Range: $[-4, \infty)$

43. Answers may vary. One possibility follows:

$f(x) = \begin{cases} -x & \text{if } -1 \le x \le 0 \\ \frac{1}{2}x & \text{if } 0 < x \le 2 \end{cases}$

44. Answers may vary. One possibility follows:

$f(x) = \begin{cases} x & \text{if } -1 \le x \le 0 \\ 1 & \text{if } 0 < x \le 2 \end{cases}$

45. Answers may vary. One possibility follows:

$f(x) = \begin{cases} -x & \text{if } x \le 0 \\ -x+2 & \text{if } 0 < x \le 2 \end{cases}$

46. Answers may vary. One possibility follows:

$f(x) = \begin{cases} 2x+2 & \text{if } -1 \le x \le 0 \\ x & \text{if } x > 0 \end{cases}$

47. a. $f(1.7) = \text{int}(2(1.7)) = \text{int}(3.4) = 3$

b. $f(2.8) = \text{int}(2(2.8)) = \text{int}(5.6) = 5$

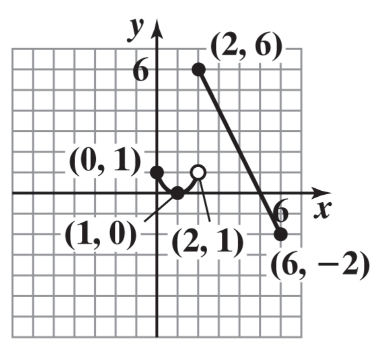
c. $f(-3.6) = \text{int}(2(-3.6)) = \text{int}(-7.2) = -8$

48. a. $f(1.2) = \text{int}\left(\frac{1.2}{2}\right) = \text{int}(0.6) = 0$

b. $f(1.6) = \text{int}\left(\frac{1.6}{2}\right) = \text{int}(0.8) = 0$

c. $f(-1.8) = \text{int}\left(\frac{-1.8}{2}\right) = \text{int}(-0.9) = -1$

49. a.

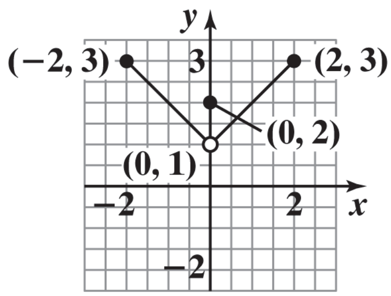


b. The domain is $[0, 6]$.

c. Absolute max: $f(2) = 6$

Absolute min: $f(6) = -2$

50. a.



b. The domain is $\left[-2,2\right]$.

c. Absolute max: $f(-2)=f(2)=3$

Absolute min: none

51. $C=\begin{cases}0.13x+17 & \text{if } 0\le x\le 3\\ 0.4x-23.5 & \text{if } x>150\end{cases}$

a. $C\left(76\right)=0.13\left(76\right)+17=\$26.88$

b. $C\left(133\right)=0.13\left(133\right)+17=\$34.29$

c. $C\left(189\right)=0.4\left(189\right)-23.5=\$52.10$

52. $F\left(x\right)=\begin{cases}3-3\operatorname{int}\left(1-x\right) & 0<x\le 2\\ 6-4\operatorname{int}\left(2-x\right) & 2<x\le 4\\ 36 & 4<x\le 8\\ 77 & 8<x\le 24\end{cases}$

a. $F\left(0.5\right)=3-3\operatorname{int}(1-0.5)=3$

Parking for 0.5 hours costs \$3.

b. $F\left(2\right)=3-3\operatorname{int}\left(1-2\right)=6$

Parking for 2 hours costs \$6.

c. $F\left(3.25\right)=6-4\operatorname{int}\left(2-3.25\right)=14$

Parking for 3.5 hours costs \$14.

d. $F\left(8\right)=36$

Parking for 8 hours costs \$36.

53. a. Charge for 2 dozen: $C=2.00(24)$

$=\$48.00$

b. Charge for 120 cupcakes:

$C=100+1.5(x-50)$

$=100+1.5(120-50)$

$=\$205$

c. For $0\le x\le 50$:

$C=2x$

For $x\le 200$:

$C=100+1.5\left(x-50\right)$

For $x>200$:

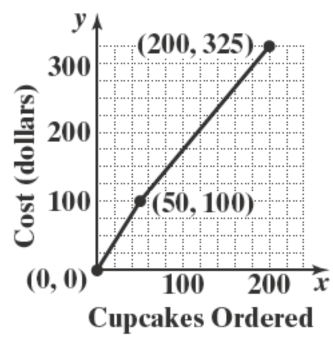
$C=100+1.5\left(150\right)+1.15(x-200)$

$=325+1.15(x-200)$

The cost function:

$$C=\begin{cases}2x & \text{if } 0\le x\le 50\\ 100+1.5(x-50) & \text{if } 50<x\le 200\\ 325+1.15(x-200) & \text{if } x>200\end{cases}$$

d. Graph:



54. a. Taxes for \$48,000:

$T=0.08(48000)$

$=\$3840$

b. Taxes for \$132,000:

$T=0.08(75000)+0.16(57000)$

$=\$15,120$

c. Taxes for \$231,000:

$T=0.08(75000)+0.16(125000)+0.22(31000)$

$=\$32,820$

d. For $0\le x\le 75,000$:

$T=0.08x$

For $75,000<x\le 200,000$:

$T=0.08(75,000)+0.16(x-75,000)$

$=6000+0.16(x-75,000)$

For $200,000<x\le 350,000$:

$T=0.08(75,000)+0.16(12,5000)$

$+0.22(x-200,000)$

$=26000+0.22(x-200000)$

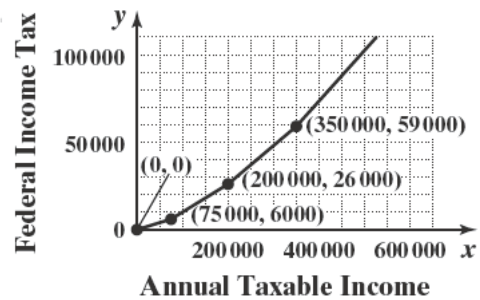
For $x>350000$:

$T=59000+0.3(x-350000)$

The tax function:

$$T(x)=\begin{cases}0.08x & \text{if} \quad 0 \le x \le 75{,}000\\ 6000+0.16(x-75{,}000) & \text{if} \quad 75{,}000 < x \le 200{,}000\\ 26{,}000+0.22(x-200{,}000) & \text{if} \quad 200{,}000 < x \le 350{,}000\\ 59{,}000+0.3(x-350{,}000) & \text{if} \quad x > 350{,}000\end{cases}$$

e.



55. For schedule X:

$$f(x)=\begin{cases}0.10x & \text{if} \quad 0 < x \le 11{,}000\\ 1100+0.12(x-11{,}000) & \text{if} \quad 11{,}000 < x \le 44{,}725\\ 5{,}175+0.22(x-44{,}725) & \text{if} \quad 44{,}725 < x \le 95{,}375\\ 16{,}290+0.24(x-95{,}375) & \text{if} \quad 95{,}375 < x \le 182{,}100\\ 37{,}104+0.32(x-182{,}100) & \text{if} \quad 182{,}100 < x \le 231{,}250\\ 52{,}832+0.35(x-231{,}250) & \text{if} \quad 231{,}250 < x \le 578{,}126\\ 174{,}238+0.37(x-578{,}126) & \text{if} \quad x > 578{,}126\end{cases}$$

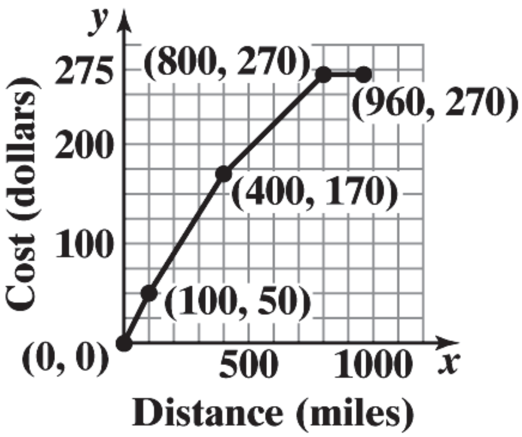
56. For Schedule $\text{Y}-1$:

$$f(x)=\begin{cases}0.10x & \text{if} \quad 0 < x \le 22{,}000\\ 2200+0.12(x-22{,}000) & \text{if} \quad 22{,}000 < x \le 89{,}450\\ 10{,}294+0.22(x-89{,}450) & \text{if} \quad 89{,}450 < x \le 190{,}750\\ 32{,}580+0.24(x-190{,}750) & \text{if} \quad 190{,}750 < x \le 364{,}200\\ 74{,}208+0.32(x-364{,}200) & \text{if} \quad 364{,}200 < x \le 462{,}500\\ 105{,}664+0.35(x-462{,}500) & \text{if} \quad 462{,}500 < x \le 693{,}750\\ 186{,}601+0.37(x-693{,}750) & \text{if} \quad x > 693{,}750\end{cases}$$

57. a. Let x represent the number of miles and C be the cost of transportation.

$$C(x)=\begin{cases}0.50x & \text{if } 0 \le x \le 100\\ 0.50(100)+0.40(x-100) & \text{if } 100 < x \le 400\\ 0.50(100)+0.40(300)+0.25(x-400) & \text{if } 400 < x \le 800\\ 0.50(100)+0.40(300)+0.25(400)+0(x-800) & \text{if } 800 < x \le 960\end{cases}$$

$$C(x)=\begin{cases}0.50x & \text{if } 0\le x\le 100\\ 10+0.40x & \text{if } 100<x\le 400\\ 70+0.25x & \text{if } 400<x\le 800\\ 270 & \text{if } 800<x\le 960\end{cases}$$

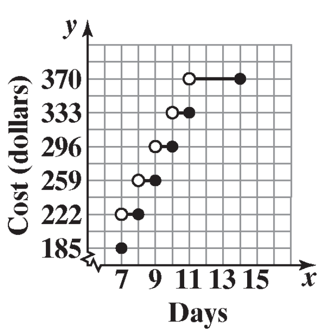


b. For hauls between 100 and 400 miles the cost is: $C(x)=10+0.40x$.

c. For hauls between 400 and 800 miles the cost is: $C(x)=70+0.25x$.

58. Let x = number of days car is used. The cost of renting is given by

$$C(x)=\begin{cases}185 & \text{if } x=7\\ 222 & \text{if } 7<x\le 8\\ 259 & \text{if } 8<x\le 9\\ 296 & \text{if } 9<x\le 10\\ 333 & \text{if } 10<x\le 11\\ 370 & \text{if } 11<x\le 14\end{cases}$$



59. **a.** Let s = the credit score of an individual who wishes to borrow \$300,000 with an 80% LTV ratio. The adverse market delivery charge is given by

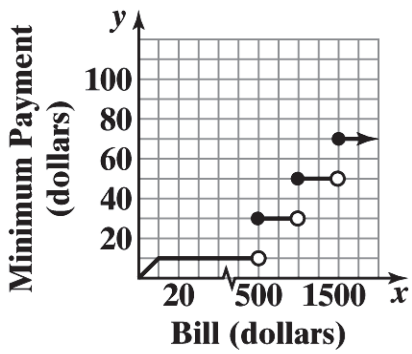
$$C(s)=\begin{cases}9000 & \text{if } s\le 659\\ 8250 & \text{if } 660\le s\le 679\\ 5250 & \text{if } 680\le s\le 699\\ 3750 & \text{if } 700\le s\le 719\\ 2250 & \text{if } 720\le s\le 739\\ 1500 & \text{if } s\ge 740\end{cases}$$

b. 725 is between 720 and 739 so the charge would be \$2250.

c. 670 is between 660 and 679 so the charge would be \$8250.

60. Let x = the amount of the bill in dollars. The minimum payment due is given by

$$f(x)=\begin{cases}x & \text{if } 0\le x<10\\ 10 & \text{if } 10\le x<500\\ 30 & \text{if } 500\le x<1000\\ 50 & \text{if } 1000\le x<1500\\ 70 & \text{if } x\ge 1500\end{cases}$$



61. **a.** $W = 10°C$

b. $W = 33 - \dfrac{(10.45 + 10\sqrt{5} - 5)(33 - 10)}{22.04} \approx 4°C$

c. $W = 33 - \dfrac{(10.45 + 10\sqrt{15} - 15)(33 - 10)}{22.04} \approx -3°C$

d. $W = 33 - 1.5958(33 - 10) = -4°C$

e. When $0 \le v < 1.79$, the wind speed is so small that there is no effect on the temperature.

f. When the wind speed exceeds 20, the wind chill depends only on the air temperature.

62. **a.** $W = -10°C$

b. $W = 33 - \dfrac{\left(10.45 + 10\sqrt{5} - 5\right)\left(33 - (-10)\right)}{22.04}$
$\approx -21°C$

c. $W = 33 - \dfrac{\left(10.45 + 10\sqrt{15} - 15\right)\left(33 - (-10)\right)}{22.04}$
$\approx -34°C$

d. $W = 33 - 1.5958\left(33 - (-10)\right) = -36°C$

63. Let x = the sales made and $S(x)$ = the total salary.

For $0 \le x \le 250{,}000$: $S(x) = \$45{,}000 + 0.04x$

For $250{,}000 < x \le 500{,}000$:

$S(x) = 55{,}000 + 0.06(x - 250{,}000)$

For $x > 500{,}000$:

$S(x) = 70{,}000 + 0.09(x - 500{,}000)$

64. Use intervals $(0,8),[8,16),[16,32),[32,38)$ (exclude 0 and 38 since those would be the walls). Depth for the intervals $[8,16)$ and $[32,38)$ are constant (8 ft and 3 ft respectively). The other two are linear functions. On $(0,8)$ the endpoint coordinates can be thought or as $(0,3)$ and $(8,8)$.

$m = \dfrac{8-3}{8-0} = \dfrac{5}{8} \quad y = \dfrac{5}{8}x + 3$

On $[16,32)$ the endpoint coordinates can be thought of as $(16,8)$ and $(32,3)$.

$m = \dfrac{3-8}{32-16} = -\dfrac{5}{16}$

$8 = -\dfrac{5}{16}(16) + b$

$13 = b$

$y = -\dfrac{5}{16}x + 13$

Therefore,

$$d(x) = \begin{cases} \dfrac{5}{8}x + 3 & \text{if } 0 < x < 8 \\ 8 & \text{if } 8 \le x < 16 \\ -\dfrac{5}{16}x + 13 & \text{if } 16 \le x < 32 \\ 3 & \text{if } 32 \le x < 38 \end{cases}$$

65. The function *f* changes definition at 2 and the function *g* changes definition at 0. Combining these together, the sum function will change definitions at 0 and 2.

On the interval $(-\infty, 0]$:

$(f+g)(x) = f(x) + g(x) = (2x+3) + (-4x+1)$.
$= -2x + 4$

On the interval $(0,2)$:

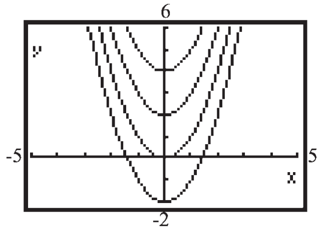
$(f+g)(x) = f(x) + g(x) = (2x+3) + (x-7)$.
$= 3x - 4$

On the interval $[2,\infty)$:

$(f+g)(x) = f(x) + g(x) = (x^2+5x) + (x-7)$
$= x^2 + 6x - 7$

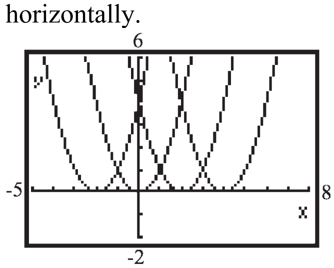
So, $(f+g)(x) = \begin{cases} -2x+4 & \text{if } x \le 0 \\ 3x-4 & \text{if } 0 < x \le 2 \\ x^2+6x-7 & \text{if } x \ge 2 \end{cases}$

66. Each graph is that of $y = x^2$, but shifted vertically.



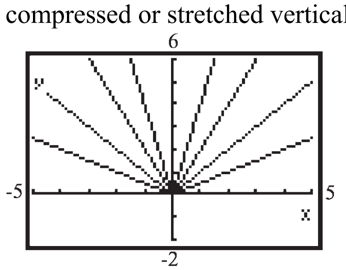
If $y = x^2 + k,\ k > 0$, the shift is up k units; if $y = x^2 - k,\ k > 0$, the shift is down k units. The graph of $y = x^2 - 4$ is the same as the graph of $y = x^2$, but shifted down 4 units. The graph of $y = x^2 + 5$ is the graph of $y = x^2$, but shifted up 5 units.

67. Each graph is that of $y = x^2$, but shifted horizontally.



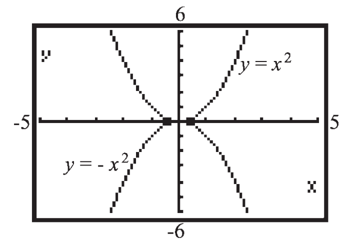
If $y = (x-k)^2,\ k > 0$, the shift is to the right k units; if $y = (x+k)^2,\ k > 0$, the shift is to the left k units. The graph of $y = (x+4)^2$ is the same as the graph of $y = x^2$, but shifted to the left 4 units. The graph of $y = (x-5)^2$ is the graph of $y = x^2$, but shifted to the right 5 units.

68. Each graph is that of $y = |x|$, but either compressed or stretched vertically.

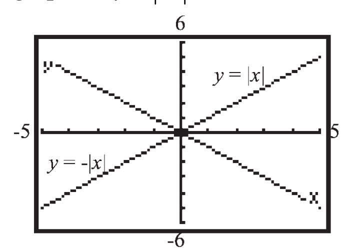


If $y = k|x|$ and $k > 1$, the graph is stretched vertically; if $y = k|x|$ and $0 < k < 1$, the graph is compressed vertically. The graph of $y = \frac{1}{4}|x|$ is the same as the graph of $y = |x|$, but compressed vertically. The graph of $y = 5|x|$ is the same as the graph of $y = |x|$, but stretched vertically.

69. The graph of $y = -x^2$ is the reflection of the graph of $y = x^2$ about the x-axis.

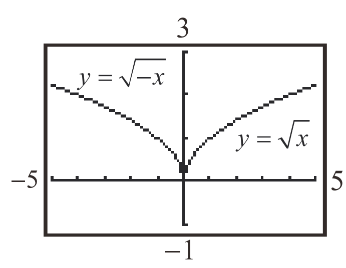


The graph of $y = -|x|$ is the reflection of the graph of $y = |x|$ about the x-axis.

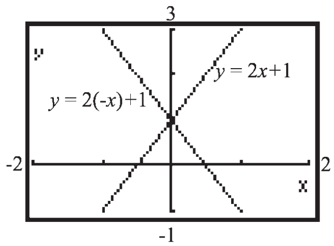


Multiplying a function by –1 causes the graph to be a reflection about the x-axis of the original function's graph.

70. The graph of $y = \sqrt{-x}$ is the reflection about the y-axis of the graph of $y = \sqrt{x}$.

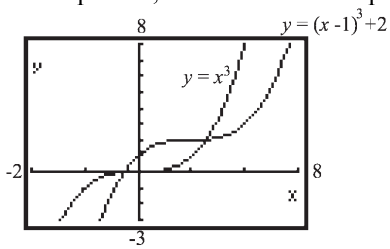


The same type of reflection occurs when graphing $y = 2x + 1$ and $y = 2(-x) + 1$.

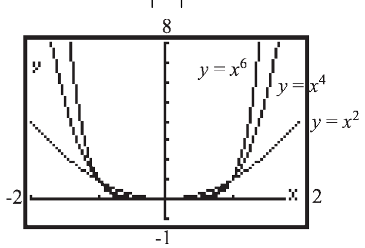


The graph of $y = f(-x)$ is the reflection about the y-axis of the graph of $y = f(x)$.

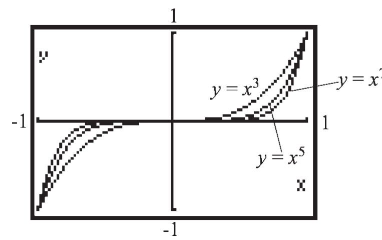
71. The graph of $y = (x-1)^3 + 2$ is a shifting of the graph of $y = x^3$ one unit to the right and two units up. Yes, the result could be predicted.



72. The graphs of $y = x^n$, n a positive even integer, **are all U-shaped and open upward. All go** through the points $(-1, 1)$, $(0, 0)$, and $(1, 1)$. As n increases, the graph of the function is narrower for $|x| > 1$ and flatter for $|x| < 1$.



73. The graphs of $y = x^n$, n a positive odd integer, all have the same general shape. All go through the points $(-1,-1)$, $(0, 0)$, and $(1, 1)$. As n increases, the graph of the function increases at a greater rate for $|x| > 1$ and is flatter around 0 for $|x| < 1$.



74. $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

Yes, it is a function.

Domain = $\{x \mid x \text{ is any real number}\}$ or $(-\infty, \infty)$

Range = $\{0, 1\}$ or $\{y \mid y = 0 \text{ or } y = 1\}$

y-intercept: $x = 0 \Rightarrow x$ is rational $\Rightarrow y = 1$

So the y-intercept is $y = 1$.

x-intercept: $y = 0 \Rightarrow x$ is irrational

So the graph has infinitely many x-intercepts, namely, there is an x-intercept at each irrational value of x.

$f(-x) = 1 = f(x)$ when x is rational;

$f(-x) = 0 = f(x)$ when x is irrational.

Thus, f is even.

The graph of f consists of 2 infinite clusters of distinct points, extending horizontally in both directions. One cluster is located 1 unit above the x-axis, and the other is located along the x-axis.

75. Answers will vary.

76. $(x^{-3}y^5)^{-2} = x^6 y^{-10} = \dfrac{x^6}{y^{10}}$

77.
$$\begin{aligned} x^2+y^2&=6y+16\\ x^2+y^2-6y&=16\\ x^2+(y^2-6y+9)&=16+9\\ x^2+(y-3)^2&=5^2 \end{aligned}$$

Center (h,k): (0, 3); Radius = 5

78.
$$\begin{aligned} 4x-5(2x-1)&=4-7(x+1)\\ 4x-10x+5&=4-7x-7\\ -6x+5&=-7x-3\\ x&=-8 \end{aligned}$$

The solution set is: $\{-8\}$

79. Let x represent the amount of money invested in a mutual fund. Then $60{,}000-x$ represents the amount of money invested in CD's. Since the total interest is to be \$3700, we have:

$$\begin{aligned} 0.08x+0.03(60{,}000-x)&=3700\\ (100)\left(0.08x+0.03(60{,}000-x)\right)&=(3700)(100)\\ 8x+3(60{,}000-x)&=370{,}000\\ 8x+180{,}000-3x&=370{,}000\\ 5x+180{,}000&=370{,}000\\ 5x&=190{,}000\\ x&=38{,}000 \end{aligned}$$

\$38,000 should be invested in a mutual fund at 8% and \$22,000 should be invested in CD's at 3%.

80.
$$\begin{array}{r|rrrr} -2 & 1 & 3 & 0 & -6\\ & & -2 & -2 & 4\\ \hline & 1 & 1 & 6 & -2 \end{array}$$

Quotient: x^2+x-2
Remainder: -2

81. $\dfrac{3}{2}+2i$

82. $-2x^7$

83.
$$\begin{aligned} \sqrt{\left(5t^2\right)^2+\left(25t^{7/2}\right)^2}&=\sqrt{25t^4+625t^7}\\ &=\sqrt{25t^4\left(1+25t^3\right)}\\ &=5t^2\sqrt{1+25t^3} \end{aligned}$$

84. The radicand cannot be negative so:
$x+7\ge 0$
$x\ge -7$
The domain is $\{x\mid x\ge -7\}$.

85.
$$\begin{aligned} &3x^3y-2x^2y^2+18x-12y=\\ &\left(3x^3y-2x^2y^2\right)+\left(18x-12y\right)=\\ &x^2y\left(3x-2y\right)+6\left(3x-4y\right)=\\ &\left(3x-2y\right)\left(x^2y+6\right) \end{aligned}$$

Section 2.5

1. horizontal; right
2. y
3. False
4. True; the graph of $y=-f(x)$ is the reflection about the x-axis of the graph of $y=f(x)$.
5. d
6. b
7. B
8. E
9. H
10. D
11. I
12. A
13. L
14. C
15. F
16. J
17. G
18. K
19. $y=(x-4)^3$
20. $y=(x+4)^3$
21. $y=x^3+4$
22. $y=x^3-4$

23. $y = (-x)^3 = -x^3$

24. $y = -x^3$

25. $y = 5x^3$

26. $y = \left(\frac{1}{4}x\right)^3 = \frac{1}{64}x^3$

27. $y = (2x)^3 = 8x^3$

28. $y = \frac{1}{4}x^3$

29. (1) $y = \sqrt{x} + 2$
(2) $y = -\left(\sqrt{x} + 2\right)$
(3) $y = -\left(\sqrt{-x} + 2\right) = -\sqrt{-x} - 2$

30. (1) $y = -\sqrt{x}$
(2) $y = -\sqrt{x-3}$
(3) $y = -\sqrt{x-3} - 2$

31. (1) $y = 3\sqrt{x}$
(2) $y = 3\sqrt{x} + 4$
(3) $y = 3\sqrt{x+5} + 4$

32. (1) $y = \sqrt{x} + 2$
(2) $y = \sqrt{-x} + 2$
(3) $y = \sqrt{-(x+3)} + 2 = \sqrt{-x-3} + 2$

33. (c); To go from $y = f(x)$ to $y = -f(x)$ we reflect about the x-axis. This means we change the sign of the y-coordinate for each point on the graph of $y = f(x)$. Thus, the point $(3, 6)$ would become $(3, -6)$.

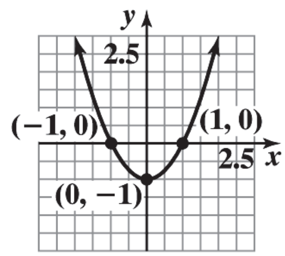
34. (d); To go from $y = f(x)$ to $y = f(-x)$, we reflect each point on the graph of $y = f(x)$ about the y-axis. This means we change the sign of the x-coordinate for each point on the graph of $y = f(x)$. Thus, the point $(3, 6)$ would become $(-3, 6)$.

35. (c); To go from $y = f(x)$ to $y = 2f(x)$, we stretch vertically by a factor of 2. Multiply the y-coordinate of each point on the graph of $y = f(x)$ by 2. Thus, the point $(1, 3)$ would become $(1, 6)$.

36. (c); To go from $y = f(x)$ to $y = f(2x)$, we compress horizontally by a factor of 2. Divide the x-coordinate of each point on the graph of $y = f(x)$ by 2. Thus, the point $(4, 2)$ would become $(2, 2)$.

37. $f(x) = x^2 - 1$

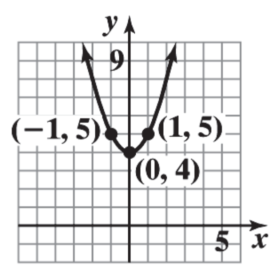
Using the graph of $y = x^2$, vertically shift downward 1 unit.



The domain is $(-\infty, \infty)$ and the range is $[-1, \infty)$.

38. $f(x) = x^2 + 4$

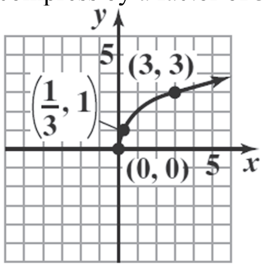
Using the graph of $y = x^2$, vertically shift upward 4 units.



The domain is $(-\infty, \infty)$ and the range is $[4, \infty)$.

39. $g(x)=\sqrt{3x}$

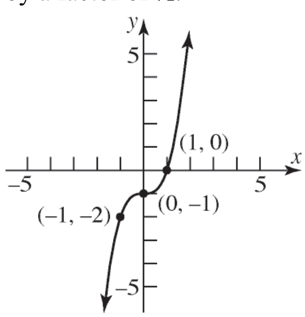
Using the graph of $y=\sqrt{x}$, horizontally compress by a factor of 3.



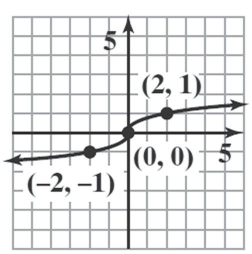
The domain is $[0,\infty)$ and the range is $[0,\infty)$.

40. $g(x)=\sqrt[3]{\frac{1}{2}x}$

Using the graph of $y=\sqrt[3]{x}$, horizontally stretch by a factor of ½.

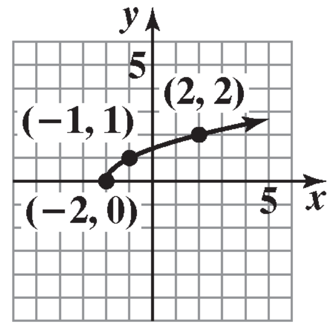


The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.



41. $h(x)=\sqrt{x+2}$

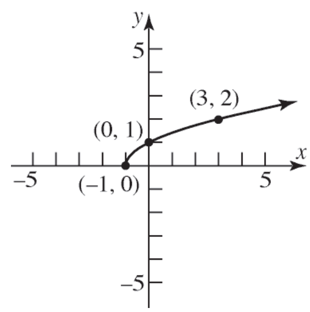
Using the graph of $y=\sqrt{x}$, horizontally shift to the left 2 units.



The domain is $[-2,\infty)$ and the range is $[0,\infty)$.

42. $h(x)=\sqrt{x+1}$

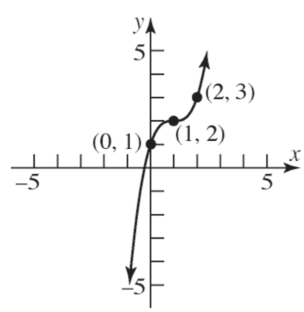
Using the graph of $y=\sqrt{x}$, horizontally shift to the left 1 unit.



The domain is $[-1,\infty)$ and the range is $[0,\infty)$.

43. $f(x)=(x-1)^3+2$

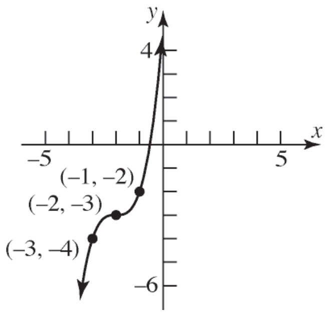
Using the graph of $y=x^3$, horizontally shift to the right 1 unit $\left[y=(x-1)^3\right]$, then vertically shift up 2 units $\left[y=(x-1)^3+2\right]$.



The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

44. $f(x)=(x+2)^3-3$

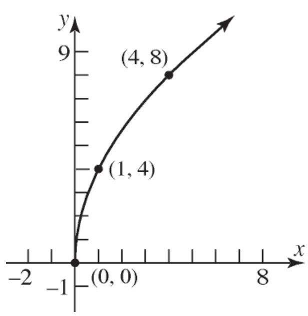
Using the graph of $y=x^3$, horizontally shift to the left 2 units $\left[y=(x+2)^3\right]$, then vertically shift down 3 units $\left[y=(x+2)^3-3\right]$.



The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

45. $g(x)=4\sqrt{x}$

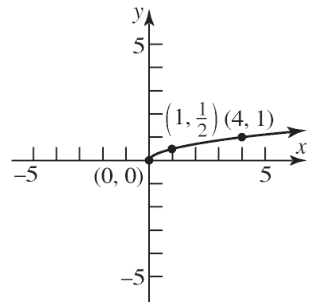
Using the graph of $y=\sqrt{x}$, vertically stretch by a factor of 4.



The domain is $[0,\infty)$ and the range is $[0,\infty)$.

46. $g(x)=\dfrac{1}{2}\sqrt{x}$

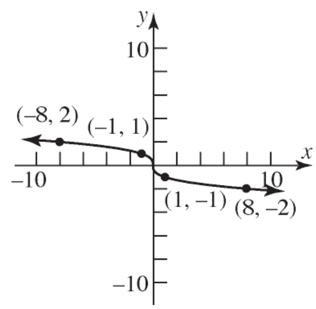
Using the graph of $y=\sqrt{x}$, vertically compress by a factor of $\dfrac{1}{2}$.



The domain is $[0,\infty)$ and the range is $[0,\infty)$.

47. $f(x)=-\sqrt[3]{x}$

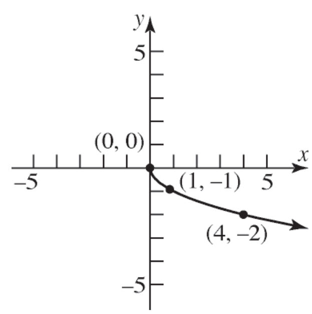
Using the graph of $y=\sqrt[3]{x}$, reflect the graph about the x-axis.



The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

48. $f(x)=-\sqrt{x}$

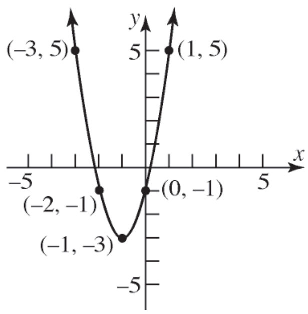
Using the graph of $y=\sqrt{x}$, reflect the graph about the x-axis.



The domain is $[0,\infty)$ and the range is $(-\infty,0]$.

49. $f(x) = 2(x+1)^2 - 3$

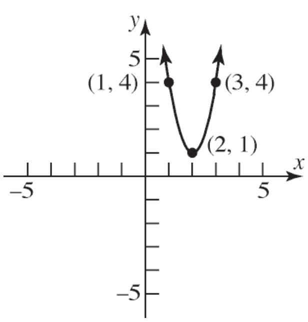
Using the graph of $y = x^2$, horizontally shift to the left 1 unit $\left[y = (x+1)^2\right]$, vertically stretch by a factor of 2 $\left[y = 2(x+1)^2\right]$, and then vertically shift downward 3 units $\left[y = 2(x+1)^2 - 3\right]$.



The domain is $(-\infty, \infty)$ and the range is $[-3, \infty)$.

50. $f(x) = 3(x-2)^2 + 1$

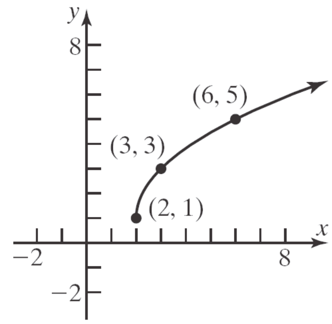
Using the graph of $y = x^2$, horizontally shift to the right 2 units $\left[y = (x-2)^2\right]$, vertically stretch by a factor of 3 $\left[y = 3(x-2)^2\right]$, and then vertically shift upward 1 unit $\left[y = 3(x-2)^2 + 1\right]$.



The domain is $(-\infty, \infty)$ and the range is $[1, \infty)$.

51. $g(x) = 2\sqrt{x-2} + 1$

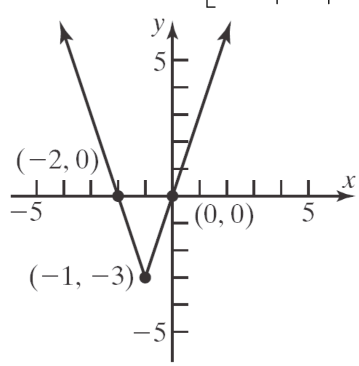
Using the graph of $y = \sqrt{x}$, horizontally shift to the right 2 units $\left[y = \sqrt{x-2}\right]$, vertically stretch by a factor of 2 $\left[y = 2\sqrt{x-2}\right]$, and vertically shift upward 1 unit $\left[y = 2\sqrt{x-2} + 1\right]$.



The domain is $[2, \infty)$ and the range is $[1, \infty)$.

52. $g(x) = 3|x+1| - 3$

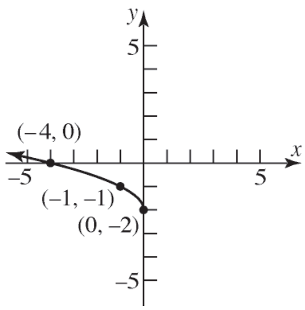
Using the graph of $y = |x|$, horizontally shift to the left 1 unit $\left[y = |x+1|\right]$, vertically stretch by a factor of 3 $\left[y = 3|x+1|\right]$, and vertically shift downward 3 units $\left[y = 3|x+1| - 3\right]$.



The domain is $(-\infty, \infty)$ and the range is $[-3, \infty)$.

53. $h(x) = \sqrt{-x} - 2$

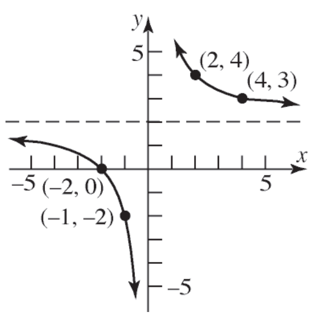
Using the graph of $y = \sqrt{x}$, reflect the graph about the y-axis $\left[y = \sqrt{-x} \right]$ and vertically shift downward 2 units $\left[y = \sqrt{-x} - 2 \right]$.



The domain is $\left(-\infty, 0\right]$ and the range is $\left[-2, \infty\right)$.

54. $h(x) = \dfrac{4}{x} + 2 = 4\left(\dfrac{1}{x}\right) + 2$

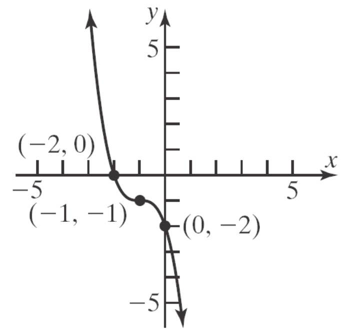
Stretch the graph of $y = \dfrac{1}{x}$ vertically by a factor of 4 $\left[y = 4 \cdot \dfrac{1}{x} = \dfrac{4}{x} \right]$ and vertically shift upward 2 units $\left[y = \dfrac{4}{x} + 2 \right]$.



The domain is $\left(-\infty, 0\right) \cup \left(0, \infty\right)$ and the range is $\left(-\infty, 2\right) \cup \left(2, \infty\right)$.

55. $f(x) = -(x+1)^3 - 1$

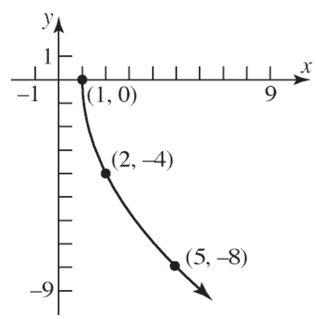
Using the graph of $y = x^3$, horizontally shift to the left 1 unit $\left[y = \left(x+1\right)^3 \right]$, reflect the graph about the x-axis $\left[y = -\left(x+1\right)^3 \right]$, and vertically shift downward 1 unit $\left[y = -\left(x+1\right)^3 - 1 \right]$.



The domain is $\left(-\infty, \infty\right)$ and the range is $\left(-\infty, \infty\right)$.

56. $f(x) = -4\sqrt{x-1}$

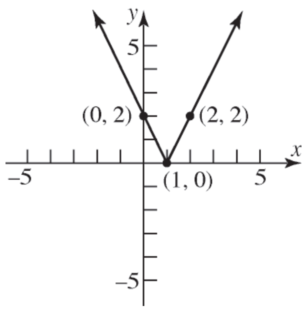
Using the graph of $y = \sqrt{x}$, horizontally shift to the right 1 unit $\left[y = \sqrt{x-1} \right]$, reflect the graph about the x-axis $\left[y = -\sqrt{x-1} \right]$, and stretch vertically by a factor of 4 $\left[y = -4\sqrt{x-1} \right]$.



The domain is $\left[1, \infty\right)$ and the range is $\left(-\infty, 0\right]$.

57. $g(x)=2|1-x|=2|-(-1+x)|=2|x-1|$

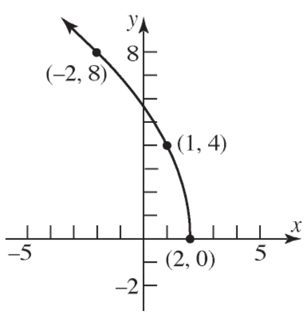
Using the graph of $y=|x|$, horizontally shift to the right 1 unit $\left[y=|x-1|\right]$, and vertically stretch by a factor or 2 $\left[y=2|x-1|\right]$.



The domain is $(-\infty,\infty)$ and the range is $[0,\infty)$.

58. $g(x)=4\sqrt{2-x}=4\sqrt{-(x-2)}$

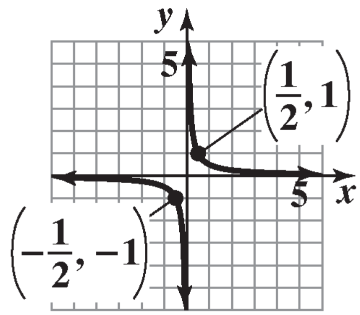
Using the graph of $y=\sqrt{x}$, reflect the graph about the y-axis $\left[y=\sqrt{-x}\right]$, horizontally shift to the right 2 units $\left[y=\sqrt{-(x-2)}\right]$, and vertically stretch by a factor of 4 $\left[y=4\sqrt{-(x-2)}\right]$.



The domain is $(-\infty,2]$ and the range is $[0,\infty)$.

59. $h(x)=\dfrac{1}{2x}$

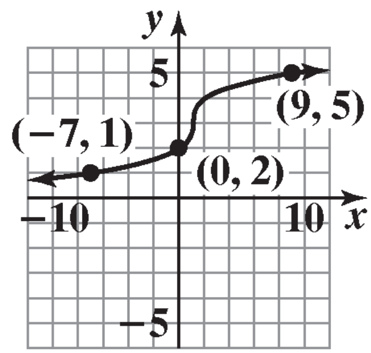
Using the graph of $y=\dfrac{1}{x}$, vertically compress by a factor of $\dfrac{1}{2}$.



The domain is $(-\infty,0)\cup(0,\infty)$ and the range is $(-\infty,0)\cup(0,\infty)$.

60. $f(x)=\sqrt[3]{x-1}+3$

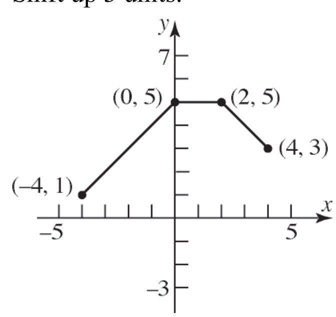
Using the graph of $f(x)=\sqrt[3]{x}$, horizontally shift to the right 1 unit $\left[y=\sqrt[3]{x-1}\right]$, then vertically shift up 3 units $\left[y=\sqrt[3]{x-1}+3\right]$.



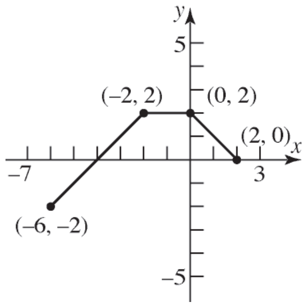
The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

61. **a.** $F(x)=f(x)+3$

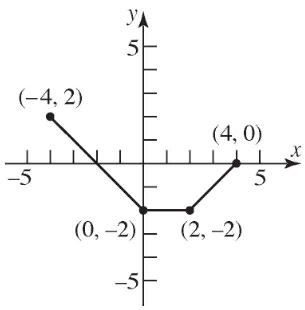
Shift up 3 units.



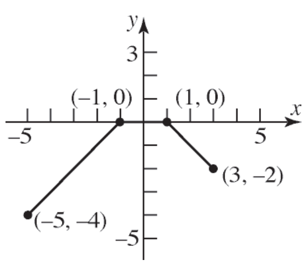
b. $G(x) = f(x+2)$
Shift left 2 units.



c. $P(x) = -f(x)$
Reflect about the x-axis.

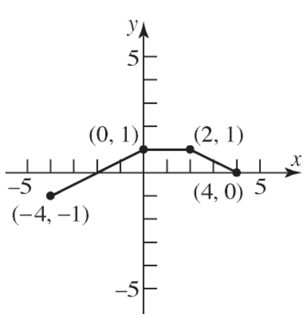


d. $H(x) = f(x+1) - 2$
Shift left 1 unit and shift down 2 units.

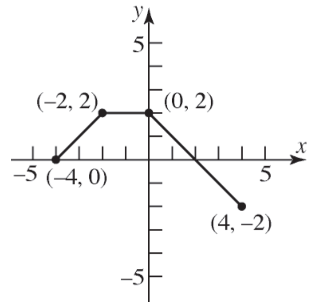


e. $Q(x) = \dfrac{1}{2} f(x)$

Compress vertically by a factor of $\dfrac{1}{2}$.

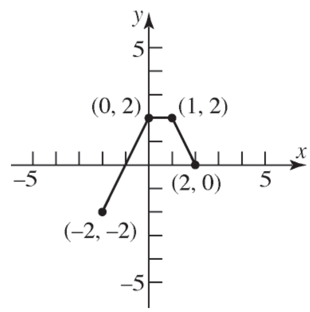


f. $g(x) = f(-x)$
Reflect about the y-axis.

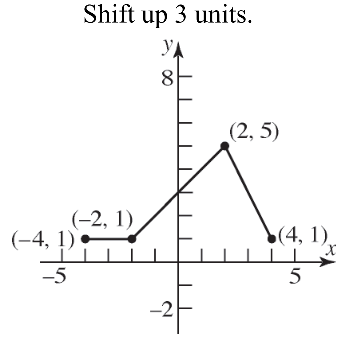


g. $h(x) = f(2x)$

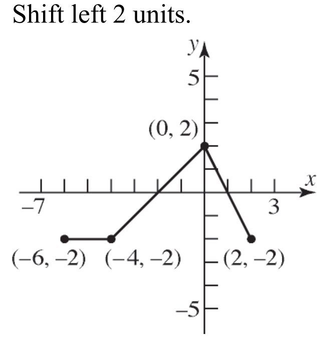
Compress horizontally by a factor of $\dfrac{1}{2}$.



62. **a.** $F(x) = f(x) + 3$
Shift up 3 units.

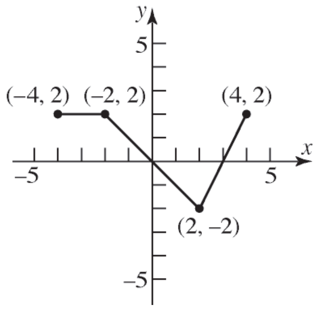


b. $G(x) = f(x+2)$
Shift left 2 units.



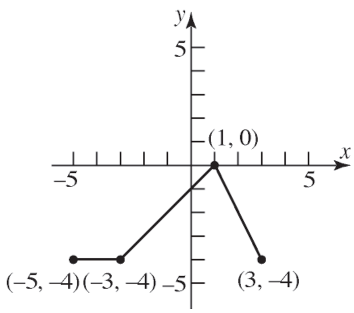
c. $P(x) = -f(x)$

Reflect about the x-axis.



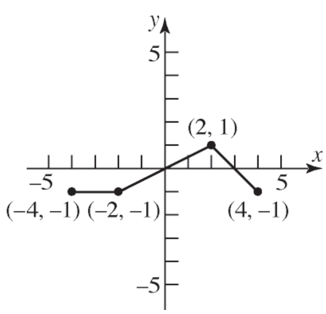
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



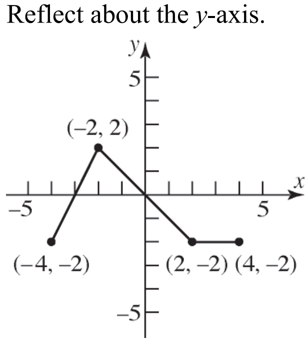
e. $Q(x) = \frac{1}{2}f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



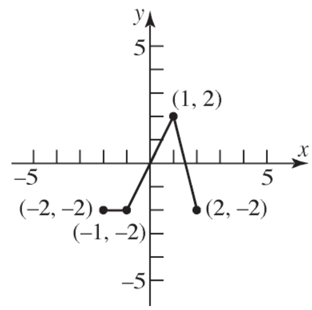
f. $g(x) = f(-x)$

Reflect about the y-axis.



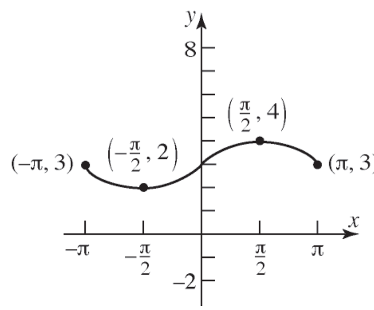
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



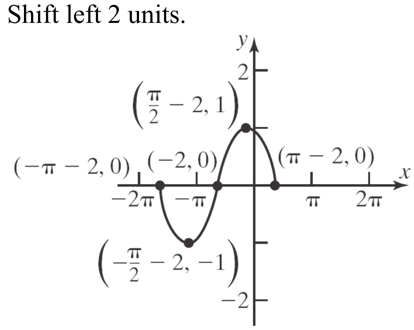
63. a. $F(x) = f(x) + 3$

Shift up 3 units.



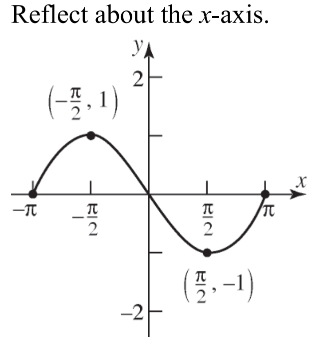
b. $G(x) = f(x+2)$

Shift left 2 units.



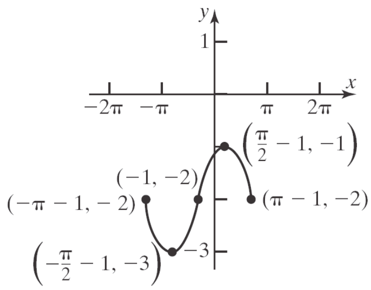
c. $P(x) = -f(x)$

Reflect about the x-axis.



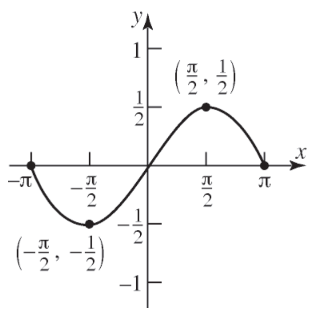
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



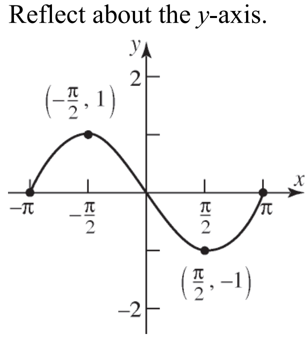
e. $Q(x) = \dfrac{1}{2} f(x)$

Compress vertically by a factor of $\dfrac{1}{2}$.



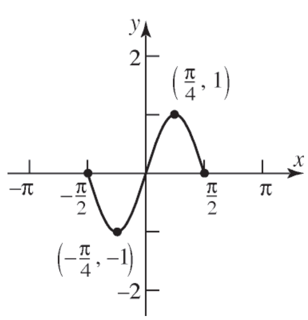
f. $g(x) = f(-x)$

Reflect about the y-axis.



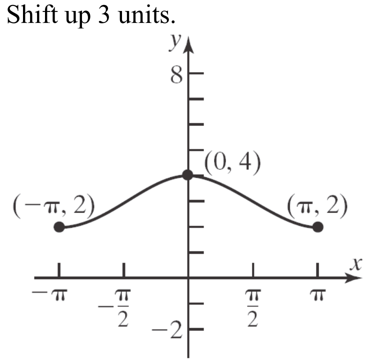
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\dfrac{1}{2}$.



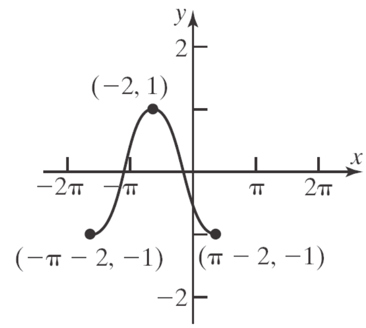
64. a. $F(x) = f(x) + 3$

Shift up 3 units.



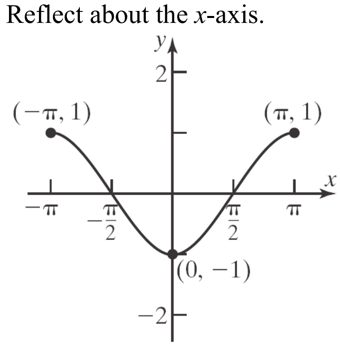
b. $G(x) = f(x+2)$

Shift left 2 units.



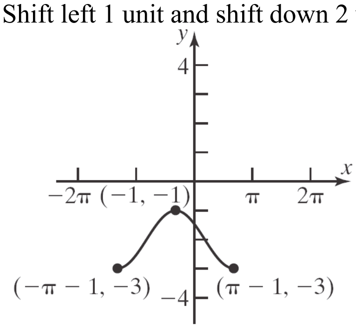
c. $P(x) = -f(x)$

Reflect about the x-axis.



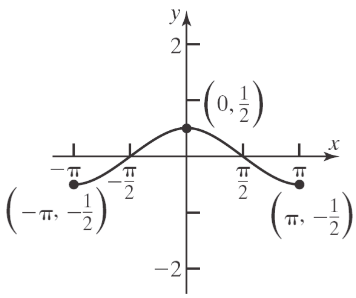
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



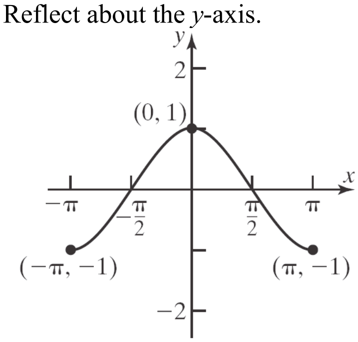
e. $Q(x)=\frac{1}{2}f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



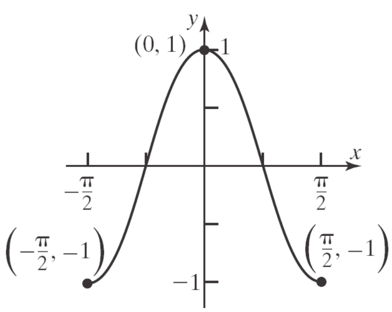
f. $g(x)=f(-x)$

Reflect about the y-axis.



g. $h(x)=f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.

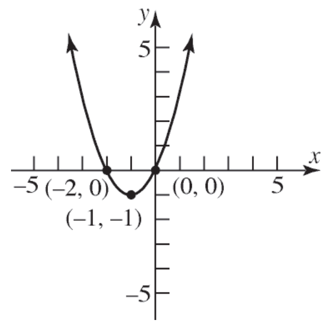


65. $f(x)=x^2+2x$

$f(x)=(x^2+2x+1)-1$

$f(x)=(x+1)^2-1$

Using $f(x)=x^2$, shift left 1 unit and shift down 1 unit.

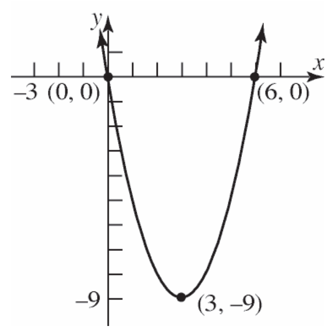


66. $f(x)=x^2-6x$

$f(x)=(x^2-6x+9)-9$

$f(x)=(x-3)^2-9$

Using $f(x)=x^2$, shift right 3 units and shift down 9 units.

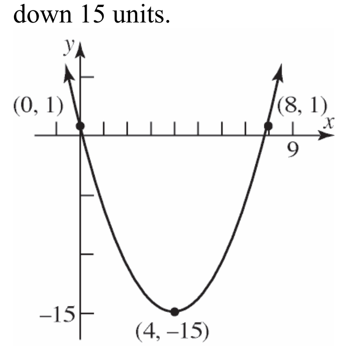


67. $f(x)=x^2-8x+1$

$f(x)=\left(x^2-8x+16\right)+1-16$

$f(x)=\left(x-4\right)^2-15$

Using $f(x)=x^2$, shift right 4 units and shift down 15 units.

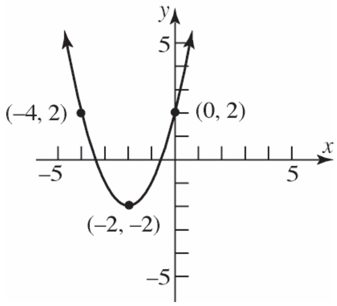


68. $f(x)=x^2+4x+2$

$f(x)=\left(x^2+4x+4\right)+2-4$

$f(x)=\left(x+2\right)^2-2$

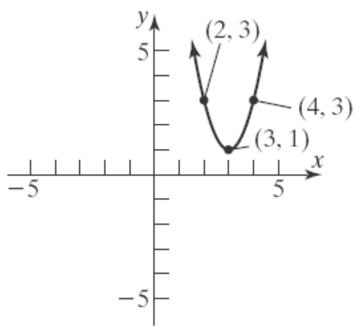
Using $f(x)=x^2$, shift left 2 units and shift down 2 units.



69. $f(x)=2x^2-12x+19$

$$\begin{aligned}&=2\left(x^2-6x\right)+19\\&=2\left(x^2-6x+9\right)+19-18\\&=2(x-3)^2+1\end{aligned}$$

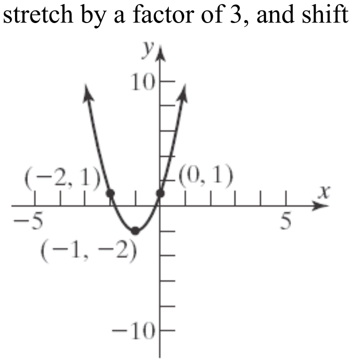
Using $f(x)=x^2$, shift right 3 units, vertically stretch by a factor of 2, and then shift up 1 unit.



70. $f(x)=3x^2+6x+1$

$$\begin{aligned}&=3\left(x^2+2x\right)+1\\&=3\left(x^2+2x+1\right)+1-3\\&=3(x+1)^2-2\end{aligned}$$

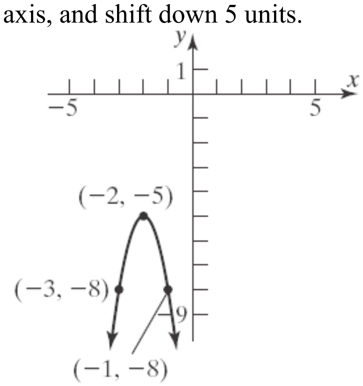
Using $f(x)=x^2$, shift left 1 unit, vertically stretch by a factor of 3, and shift down 2 units.



71. $f(x)=-3x^2-12x-17$

$$\begin{aligned}&=-3\left(x^2+4x\right)-17\\&=-3\left(x^2+4x+4\right)-17+12\\&=-3(x+2)^2-5\end{aligned}$$

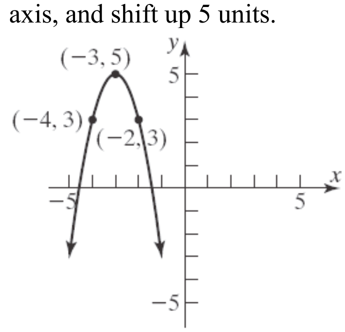
Using $f(x)=x^2$, shift left 2 units, stretch vertically by a factor of 3, reflect about the *x*-axis, and shift down 5 units.



72. $f(x)=-2x^2-12x-13$

$$\begin{aligned}&=-2\left(x^2+6x\right)-13\\&=-2\left(x^2+6x+9\right)-13+18\\&=-2(x+3)^2+5\end{aligned}$$

Using $f(x)=x^2$, shift left 3 units, stretch vertically by a factor of 2, reflect about the *x*-axis, and shift up 5 units.



73. **a.** The graph of $y=f(x+2)$ is the same as the graph of $y=f(x)$, but shifted 2 units to the left. Therefore, the *x*-intercepts are -7 and 1.

b. The graph of $y=f(x-2)$ is the same as the graph of $y=f(x)$, but shifted 2 units to the right. Therefore, the x-intercepts are -3 and 5.

c. The graph of $y = 4f(x)$ is the same as the graph of $y = f(x)$, but stretched vertically by a factor of 4. Therefore, the x-intercepts are still -5 and 3 since the y-coordinate of each is 0.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, the x-intercepts are 5 and -3.

74. a. The graph of $y = f(x+4)$ is the same as the graph of $y = f(x)$, but shifted 4 units to the left. Therefore, the x-intercepts are -12 and -3.

b. The graph of $y = f(x-3)$ is the same as the graph of $y = f(x)$, but shifted 3 units to the right. Therefore, the x-intercepts are -5 and 4.

c. The graph of $y = 2f(x)$ is the same as the graph of $y = f(x)$, but stretched vertically by a factor of 2. Therefore, the x-intercepts are still -8 and 1 since the y-coordinate of each is 0.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, the x-intercepts are 8 and -1.

75. a. The graph of $y = f(x+2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the graph of $f(x+2)$ is increasing on the interval $[-3,3]$.

b. The graph of $y = f(x-5)$ is the same as the graph of $y = f(x)$, but shifted 5 units to the right. Therefore, the graph of $f(x-5)$ is increasing on the interval $[4,10]$.

c. The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$, but reflected about the x-axis. Therefore, we can say that the graph of $y = -f(x)$ must be *decreasing* on the interval $[-1,5]$.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, we can say that the graph of $y = f(-x)$ must be *decreasing* on the interval $[-5,1]$.

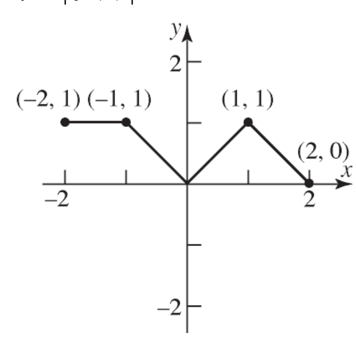
76. a. The graph of $y = f(x+2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the graph of $f(x+2)$ is decreasing on the interval $[-4,5]$.

b. The graph of $y = f(x-5)$ is the same as the graph of $y = f(x)$, but shifted 5 units to the right. Therefore, the graph of $f(x-5)$ is decreasing on the interval $[3,12]$.

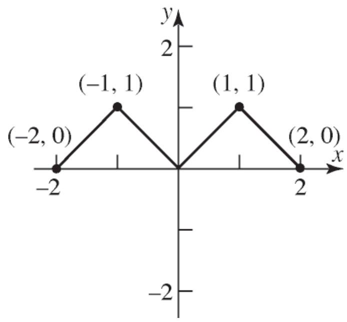
c. The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$, but reflected about the x-axis. Therefore, we can say that the graph of $y = -f(x)$ must be *increasing* on the interval $[-2,7]$.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, we can say that the graph of $y = f(-x)$ must be *increasing* on the interval $[-7,2]$.

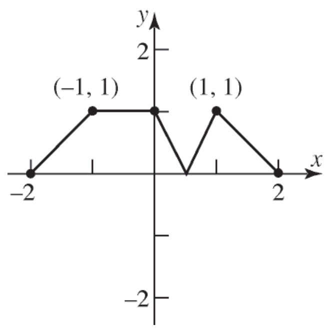
77. a. $y = |f(x)|$



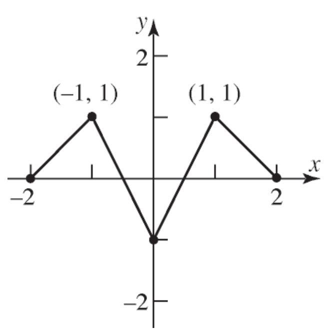
b. $y = f(|x|)$



78. a. To graph $y = |f(x)|$, the part of the graph for *f* that lies in quadrants III or IV is reflected about the *x*-axis.



b. To graph $y = f(|x|)$, the part of the graph for *f* that lies in quadrants II or III is replaced by the reflection of the part in quadrants I and IV reflected about the *y*-axis.



79. a. The graph of $y = f(x+3) - 5$ is the graph of $y = f(x)$ but shifted left 3 units and down 5 units. Thus, the point $(1,3)$ becomes the point $(-2,-2)$.

b. The graph of $y = -2f(x-2)+1$ is the graph of $y = f(x)$ but shifted right 2 units, stretched vertically by a factor of 2, reflected about the *x*-axis, and shifted up 1 unit. Thus, the point $(1,3)$ becomes the point $(3,-5)$.

c. The graph of $y = f(2x+3)$ is the graph of $y = f(x)$ but shifted left 3 units and horizontally compressed by a factor of 2. Thus, the point $(1,3)$ becomes the point $(-1,3)$.

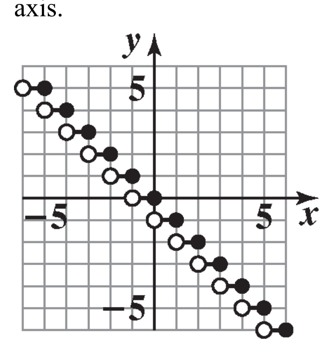
80. a. The graph of $y = g(x+1)-3$ is the graph of $y = g(x)$ but shifted left 1 unit and down 3 units. Thus, the point $(-3,5)$ becomes the point $(-4,2)$.

b. The graph of $y = -3g(x-4)+3$ is the graph of $y = g(x)$ but shifted right 4 units, stretched vertically by a factor of 3, reflected about the *x*-axis, and shifted up 3 units. Thus, the point $(-3,5)$ becomes the point $(1,-12)$.

c. The graph of $y = g(3x+9)$ is the graph of $y = f(x)$ but shifted left 9 units and horizontally compressed by a factor of 3. Thus, the point $(-3,5)$ becomes the point $(-4,5)$.

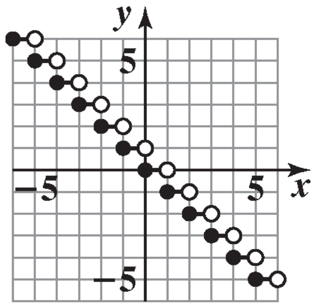
81. a. $f(x) = \text{int}(-x)$

Reflect the graph of $y = \text{int}(x)$ about the *y*-axis.



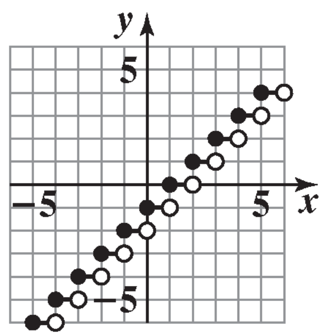
b. $g(x) = -\text{int}(x)$

Reflect the graph of $y = \text{int}(x)$ about the x-axis.



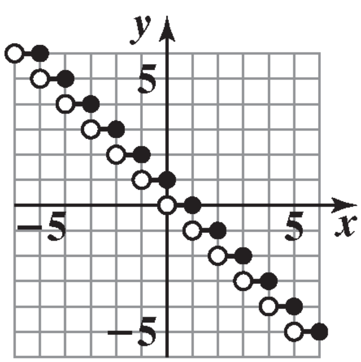
82. a. $f(x) = \text{int}(x-1)$

Shift the graph of $y = \text{int}(x)$ right 1 unit.



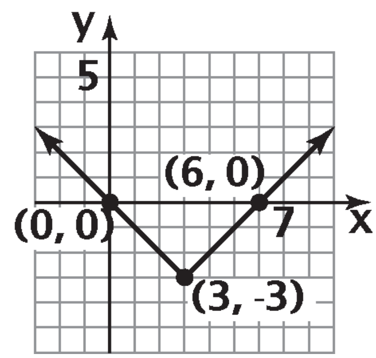
b. $g(x) = \text{int}(1-x) = \text{int}(-(x-1))$

Using the graph of $y = \text{int}(x)$, reflect the graph about the y-axis $\left[y = \text{int}(-x)\right]$, horizontally shift to the right 1 unit$\left[y = \text{int}(-(x-1))\right]$.



83. a. $f(x) = |x-3| - 3$

Using the graph of $y = |x|$, horizontally shift to the right 3 units $\left[y = |x-3|\right]$ and vertically shift downward 3 units $\left[y = |x-3| - 3\right]$.



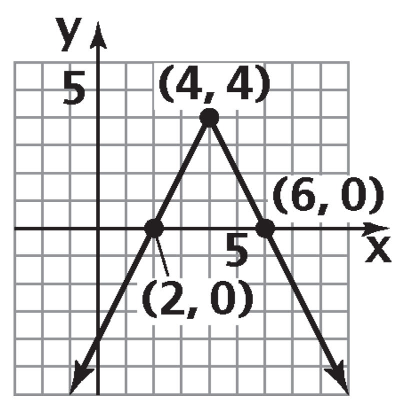
b. $A = \dfrac{1}{2}bh$

$= \dfrac{1}{2}(6)(3) = 9$

The area is 9 square units.

84. a. $f(x) = -2|x-4| + 4$

Using the graph of $y = |x|$, horizontally shift to the right 4 units $\left[y = |x-4|\right]$, vertically stretch by a factor of 2 and flip on the x-axis $\left[y = -2|x-4|\right]$, and vertically shift upward 4 units $\left[y = -2|x-4| + 4\right]$.



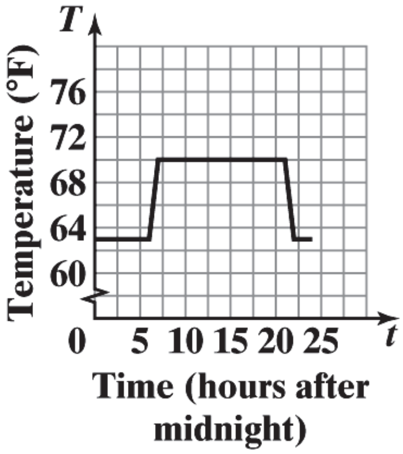
b. $A = \dfrac{1}{2}bh$

$= \dfrac{1}{2}(4)(4) = 8$

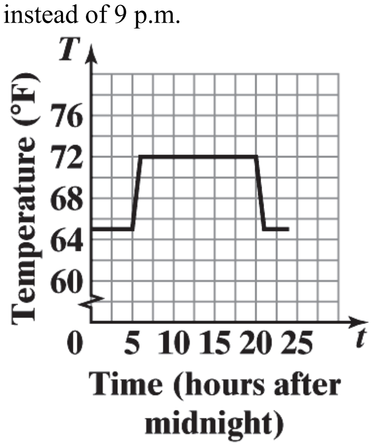
The area is 8 square units.

85. a. From the graph, the thermostat is set at $72°F$ during the daytime hours. The thermostat appears to be set at $65°F$ overnight.

b. To graph $y = T(t) - 2$, the graph of $T(t)$ is shifted down 2 units. This change will lower the temperature in the house by 2 degrees.



c. To graph $y = T(t+1)$, the graph of $T(t)$ should be shifted left one unit. This change will cause the program to switch between the daytime temperature and overnight temperature one hour sooner. The home will begin warming up at 5 a.m. instead of 6 a.m. and will begin cooling down at 8 p.m. instead of 9 p.m.



86. a. $f(0) = -0.12(0)^2 + 1.29(0) + 9.96 = 9.96$

The estimated hours of daylight in January is 9.96.

$$f(6) = -0.12(6)^2 + 1.29(6) + 9.96$$
$$= 13.38$$

The estimated hours of daylight in July is 13.38.

$$f(11) = -0.12(11)^2 + 1.29(11) + 9.96$$
$$= 9.63$$

The estimated hours of daylight in December is 13.38.

b. x represents the number of months after July.

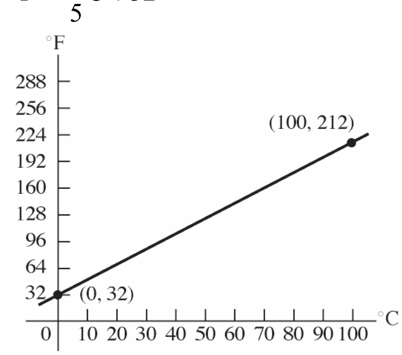
c. January corresponds to:

$F(6) = f(6-6) = f(0)$

$f(0) = 9.96$

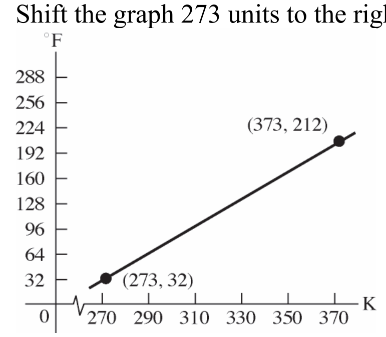
d. Orlando is in the Northern Hemisphere and Alice Springs is in the Southern Hemisphere; therefore, they have "opposite" seasons in each month. For example, it is winder in Orlando in January while it is summer in Alice Spring.

87. $F = \frac{9}{5}C + 32$

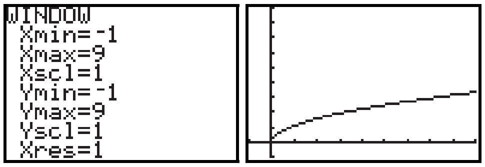


$F = \frac{9}{5}(K - 273) + 32$

Shift the graph 273 units to the right.

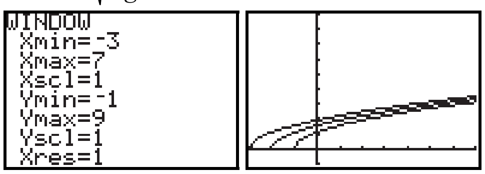


88. **a.** $T = 2\pi\sqrt{\dfrac{l}{g}}$



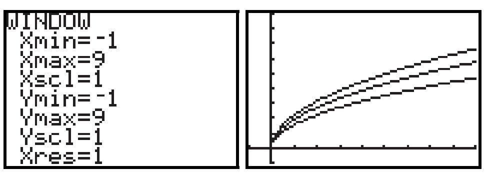
b. $T_1 = 2\pi\sqrt{\dfrac{l+1}{g}}\,; T_2 = 2\pi\sqrt{\dfrac{l+2}{g}}\,;$

$T_3 = 2\pi\sqrt{\dfrac{l+3}{g}}$



c. As the length of the pendulum increases, the period increases.

d. $T_1 = 2\pi\sqrt{\dfrac{2l}{g}}\,;\ T_2 = 2\pi\sqrt{\dfrac{3l}{g}}\,;\ T_3 = 2\pi\sqrt{\dfrac{4l}{g}}$



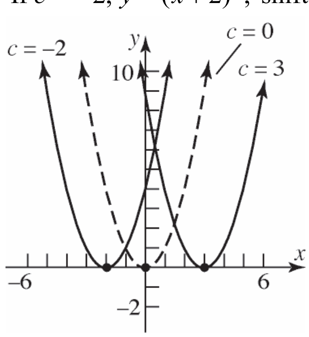
e. If the length of the pendulum is multiplied by k, the period is multiplied by $\sqrt{k}$.

89. $y = (x-c)^2$

If $c = 0$, $y = x^2$.

If $c = 3$, $y = (x-3)^2$; shift right 3 units.

If $c = -2$, $y = (x+2)^2$; shift left 2 units.

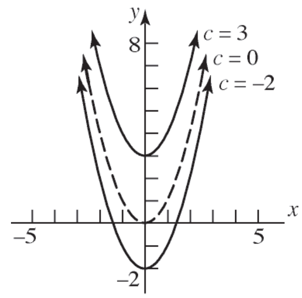


90. $y = x^2 + c$

If $c = 0$, $y = x^2$.

If $c = 3$, $y = x^2 + 3$; shift up 3 units.

If $c = -2$, $y = x^2 - 2$; shift down 2 units.



91. $f(x-5)$ is a shift right 5 units; increasing on $[2,8]$ and $[16,24]$; decreasing of $[8,16]$.

$f(2x-5)$ compresses horizontally by a factor of ½; increasing on $[1,4]$ and $[8,12]$; decreasing on $[4,8]$. $-f(2x-5)$ reflects about the x-axis; increasing on $[4,8]$; decreasing on $[1,4]$ and $[8,12]$. $-3f(2x-5)$ stretches vertically by a factor or 3 but does not affect increasing/decreasing. Therefore $-3f(2x-5)$ is increasing on $[4,8]$.

92. Write the general normal density as

$f(x) = \dfrac{1}{\sigma}\cdot\dfrac{1}{\sqrt{2\pi}}\exp\left[-\dfrac{\left(\dfrac{x-\mu}{\sigma}\right)^2}{2}\right]$. Starting

with the standard normal density,

$f(x) = \dfrac{1}{\sqrt{2\pi}}\exp\left[-\dfrac{x^2}{2}\right]$, stretch/compress

horizontally by a factor of σ to get

$f(x) = \dfrac{1}{\sqrt{2\pi}}\exp\left[-\dfrac{\left(\dfrac{x}{\sigma}\right)^2}{2}\right]$

$\left[\text{multiply all the } x\text{-coordinates by } \dfrac{1}{\sigma}\right]$, then shift

the graph horizontally $|\mu|$ units (left if $\mu < 0$ and right if $\mu > 0$) to get

$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{\left(\frac{x-\mu}{\sigma}\right)^2}{2}\right]$. Then

stretch/compress vertically by a factor of $\frac{1}{\sigma}$ to get

$$f(x) = \frac{1}{\sigma} \cdot \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{\left(\frac{x-\mu}{\sigma}\right)^2}{2}\right]$$

$\left[\text{multiply all the } y\text{-coordinates by } \frac{1}{\sigma}\right]$.

1. Stretch/compress horizontally by a factor or σ (stretch if $\sigma > 1$)
2. Shift horizontally $|\mu|$ units (left if $\mu < 0$ and right if $\mu > 0$).
3.Stretch/compress vertically by a factor of $\frac{1}{\sigma}$ (compress if $\sigma > 1$)

93. The graph of $y = 4f(x)$ is a vertical stretch of the graph of *f* by a factor of 4, while the graph of $y = f(4x)$ is a horizontal compression of the graph of *f* by a factor of $\frac{1}{4}$.

94. The graph of $y = f(x) - 2$ will shift the graph of $y = f(x)$ down by 2 units. The graph of $y = f(x-2)$ will shift the graph of $y = f(x)$ to the right by 2 units.

95. The graph of $y = \sqrt{-x}$ is the graph of $y = \sqrt{x}$ but reflected about the *y*-axis. Therefore, our region is simply rotated about the *y*-axis and does not change shape. Instead of the region being bounded on the right by $x = 4$, it is bounded on the left by $x = -4$. Thus, the area of the second region would also be $\frac{16}{3}$ square units.

96. The range of $f(x) = x^2$ is $[0, \infty)$. The graph of $g(x) = f(x) + k$ is the graph of f shifted up k units if k > 0 and shifted down $|k|$ units if k < 0, so the range of g is $[k, \infty)$.

97. The domain of $g(x) = \sqrt{x}$ is $[0, \infty)$. The graph of $g(x-k)$ is the graph of g shifted k units to the right, so the domaine of g is $[k, \infty)$.

98.
$$3x - 5y = 30$$
$$-5y = -3x + 30$$
$$y = \frac{3}{5}x - 6$$

The slope is $\frac{3}{5}$ and the y-intercept is -6.

99. The total time run is $\frac{13.1}{7} + \frac{13.1}{2} = 8.4214$. The total distance is 26.2 mile. Thus the average speed is $\frac{26.2}{8.4214} = 3.11 \text{ mph}$.

100.
$$W = kT$$
$$7 = k4$$
$$k = \frac{7}{4}$$
$$W = \frac{7}{4}T = \frac{7}{4}(9) = 15.75 \text{ gal}$$

101. $y^2 = x + 4$

x-intercepts:
$(0)^2 = x + 4$
$0 = x + 4$
$x = -4$

y-intercepts:
$y^2 = 0 + 4$
$y^2 = 4$
$y = \pm 2$

The intercepts are $(-4, 0)$, $(0, -2)$ and $(0, 2)$.

<u>Test x-axis symmetry</u>: Let $y = -y$
$(-y)^2 = x + 4$
$y^2 = x + 4$ same

<u>Test y-axis symmetry</u>: Let $x = -x$
$y^2 = -x + 4$ different

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$.
$(-y)^2 = -x + 4$
$y^2 = -x + 4$ different

Therefore, the graph will have x-axis symmetry.

102. The denominator must not be zero.
$x^2 - 5x - 14 = 0$
$(x - 7)(x + 2) = 0$
$x = 7, x = -2$
So the domain is: $\{x \mid x \neq 7, x \neq -2\}$

103. $-16t^2 + 96t + 200 = 88$
$-16t^2 + 96t + 112 = 0$
$-16(t^2 - 6t - 7) = 0$
$-16(t - 7)(t + 1) = 0$
$t = 7, t = -1$
Since t represents time the only answer that is reasonable is 7 seconds.

104. $\sqrt[3]{16x^5y^6z} = \sqrt[3]{8 \cdot 2x^3x^2y^6z} = 2xy^2\sqrt[3]{2x^2z}$

105.
$$\frac{f(x+h) - f(x)}{h} =$$
$$\frac{3(x+h)^2 + 2(x+h) - 1 - (3x^2 + 2x - 1)}{h} =$$
$$\frac{3(x^2 + 2xh + h^2) + 2x + 2h - 1 - 3x^2 - 2x + 1}{h} =$$
$$\frac{3x^2 + 6xh + 3h^2 + 2x + 2h - 1 - 3x^2 - 2x + 1}{h}$$
$$\frac{6xh + h^2 + 2h}{h} = \frac{h(6x + h + 2)}{h} = 6x + h + 2$$

106. $z^3 + 216 = (z)^3 + (6)^3$
$= (z + 6)(z^2 - 6z + 36)$

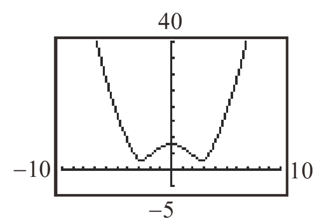
Section 2.6

1. a. The distance d from P to the origin is $d = \sqrt{x^2 + y^2}$. Since P is a point on the graph of $y = x^2 - 8$, we have:
$$d(x) = \sqrt{x^2 + (x^2 - 8)^2} = \sqrt{x^4 - 15x^2 + 64}$$

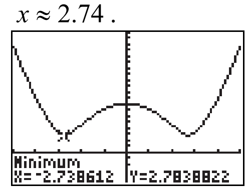
b. $d(0) = \sqrt{0^4 - 15(0)^2 + 64} = \sqrt{64} = 8$

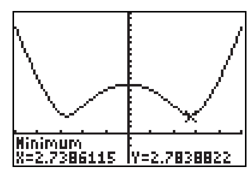
c. $d(1) = \sqrt{(1)^4 - 15(1)^2 + 64}$
$= \sqrt{1 - 15 + 64} = \sqrt{50} = 5\sqrt{2} \approx 7.07$

d.



e. d is smallest when $x \approx -2.74$ or when $x \approx 2.74$.



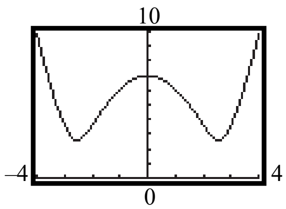


2. a. The distance d from P to $(0, -1)$ is $d = \sqrt{x^2 + (y+1)^2}$. Since P is a point on the graph of $y = x^2 - 8$, we have:
$$d(x) = \sqrt{x^2 + (x^2 - 8 + 1)^2}$$
$$= \sqrt{x^2 + \left(x^2 - 7\right)^2} = \sqrt{x^4 - 13x^2 + 49}$$

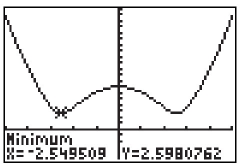
b. $d(0) = \sqrt{0^4 - 13(0)^2 + 49} = \sqrt{49} = 7$

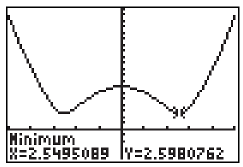
c. $d(-1) = \sqrt{(-1)^4 - 13(-1)^2 + 49} = \sqrt{37} \approx 6.08$

d.



e. d is smallest when $x \approx -2.55$ or when $x \approx 2.55$.



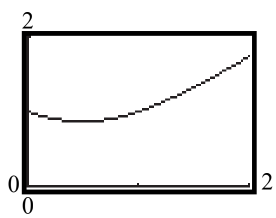


3. a. The distance d from P to the point (1, 0) is $d = \sqrt{(x-1)^2 + y^2}$. Since P is a point on the graph of $y = \sqrt{x}$, we have:

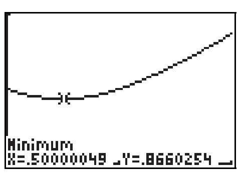
$$d(x) = \sqrt{(x-1)^2 + \left(\sqrt{x}\right)^2} = \sqrt{x^2 - x + 1}$$

where $x \geq 0$.

b.



c. d is smallest when $x = \frac{1}{2}$.

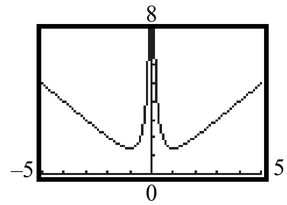


d. $d(x) = \sqrt{\frac{1}{2}^2 - \frac{1}{2} + 1} = \frac{\sqrt{3}}{2}$

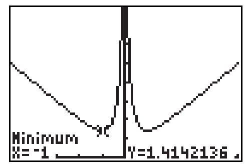
4. a. The distance d from P to the origin is $d = \sqrt{x^2 + y^2}$. Since P is a point on the graph of $y = \frac{1}{x}$, we have:

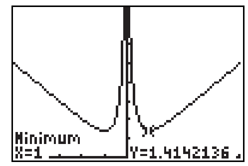
$$d(x) = \sqrt{x^2 + \left(\frac{1}{x}\right)^2} = \sqrt{x^2 + \frac{1}{x^2}}$$

b.



c. d is smallest when $x = -1$ or $x = 1$.





d.

$$d(1) = \frac{\sqrt{1^2 + 1}}{|1|} = \sqrt{2}; d(-1) = \frac{\sqrt{(-1)^2 + 1}}{|-1|} = \sqrt{2}$$

5. By definition, a triangle has area $A = \frac{1}{2} b\,h$, $b =$ base, $h =$ height. From the figure, we know that $b = x$ and $h = y$. Expressing the area of the triangle as a function of x, we have:

$A(x) = \frac{1}{2} xy = \frac{1}{2} x\left(x^3\right) = \frac{1}{2} x^4$.

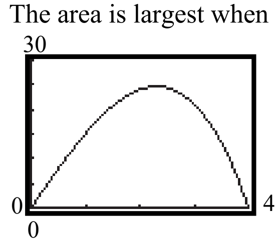
6. By definition, a triangle has area $A = \frac{1}{2} b\,h$, b=base, $h =$ height. Because one vertex of the triangle is at the origin and the other is on the x-axis, we know that $b = x$ and $h = y$. Expressing the area of the triangle as a function of x, we have:

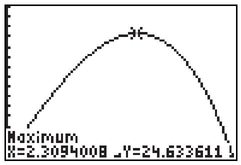
$A(x) = \frac{1}{2} xy = \frac{1}{2} x\left(9 - x^2\right) = \frac{9}{2} x - \frac{1}{2} x^3$.

7. a. $A(x) = xy = x\left(16 - x^2\right)$

b. Domain: $\{x \mid 0 < x < 4\}$

c. The area is largest when $x \approx 2.31$.



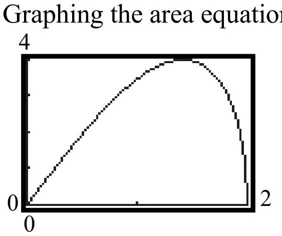


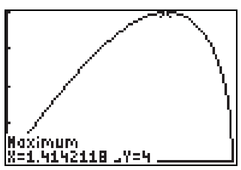
d. The largest area is
$A(2.31) = 2.31\left(16-2.31^2\right) \approx 24.63$ square units.

8. a. $A(x) = 2xy = 2x\sqrt{4-x^2}$

b. $p(x) = 2(2x)+2(y) = 4x+2\sqrt{4-x^2}$

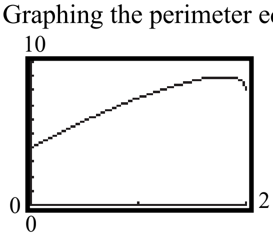
c. Graphing the area equation:

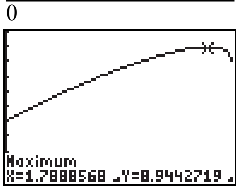




The area is largest when $x \approx 1.41$.

d. Graphing the perimeter equation:





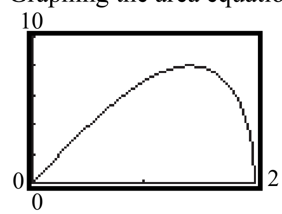
The perimeter is largest when $x \approx 1.79$.

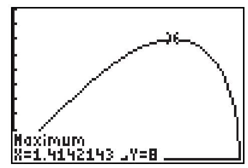
e. The largest area is
$A(1.41) = 2(1.41)\sqrt{4-1.41^2} \approx 4$ square units.
The largest perimeter is
$p(1.79) = 4(1.79)+2\sqrt{4-1.79^2} \approx 8.94$ units.

9. a. In Quadrant I, $x^2+y^2=4 \to y=\sqrt{4-x^2}$
$A(x) = (2x)(2y) = 4x\sqrt{4-x^2}$

b. $p(x) = 2(2x)+2(2y) = 4x+4\sqrt{4-x^2}$

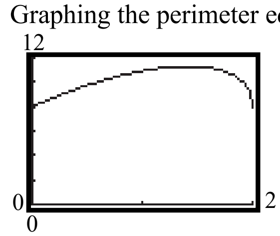
c. Graphing the area equation:

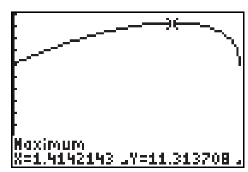




The area is largest when $x \approx 1.41$.

d. Graphing the perimeter equation:





The perimeter is largest when $x \approx 1.41$.

10. a. $A(r) = (2r)(2r) = 4r^2$

b. $p(r) = 4(2r) = 8r$

11. a. $C =$ circumference, $A =$ total area,
$r =$ radius, $x =$ side of square

$C = 2\pi r = 10-4x \quad \Rightarrow \quad r = \frac{5-2x}{\pi}$

Total Area $=$ area$_{\text{square}}$ $+$ area$_{\text{circle}}$ $= x^2 + \pi r^2$

$$A(x) = x^2 + \pi\left(\frac{5-2x}{\pi}\right)^2 = x^2 + \frac{25-20x+4x^2}{\pi}$$

b. Since the lengths must be positive, we have:

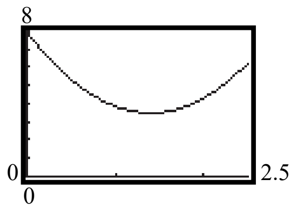
$10-4x>0$ and $x>0$

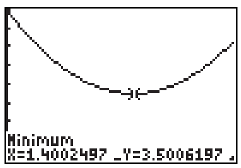
$-4x>-10$ and $x>0$

$x<2.5$ and $x>0$

Domain: $\{x \mid 0<x<2.5\}$

c. The total area is smallest when $x \approx 1.40$ meters.





12. a. C = circumference, A = total area, r = radius, x = side of equilateral triangle

$$C = 2\pi r = 10-3x \Rightarrow r = \frac{10-3x}{2\pi}$$

The height of the equilateral triangle is $\frac{\sqrt{3}}{2}x$.

$$\text{Total Area} = \text{area}_{\text{triangle}} + \text{area}_{\text{circle}}$$
$$= \frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right) + \pi r^2$$
$$A(x) = \frac{\sqrt{3}}{4}x^2 + \pi\left(\frac{10-3x}{2\pi}\right)^2$$
$$= \frac{\sqrt{3}}{4}x^2 + \frac{100-60x+9x^2}{4\pi}$$

b. Since the lengths must be positive, we have:

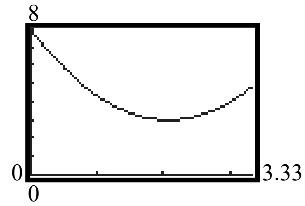
$10-3x>0$ and $x>0$

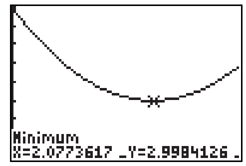
$-3x>-10$ and $x>0$

$x<\frac{10}{3}$ and $x>0$

Domain: $\left\{x \,\middle|\, 0<x<\frac{10}{3}\right\}$

c. The area is smallest when $x \approx 2.08$ meters.





13. a. Since the wire of length x is bent into a circle, the circumference is x. Therefore, $C(x)=x$.

b. Since $C = x = 2\pi r$, $r = \frac{x}{2\pi}$.

$$A(x) = \pi r^2 = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}.$$

14. a. Since the wire of length x is bent into a square, the perimeter is x. Therefore, $p(x)=x$.

b. Since $P = x = 4s$, $s = \frac{1}{4}x$, we have

$$A(x) = s^2 = \left(\frac{1}{4}x\right)^2 = \frac{1}{16}x^2.$$

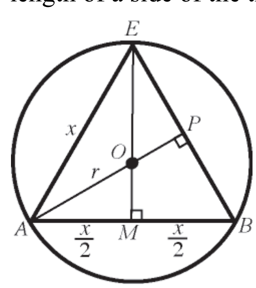
15. a. A = area, r = radius; diameter = $2r$

$A(r) = (2r)(r) = 2r^2$

b. p = perimeter

$p(r) = 2(2r) + 2r = 6r$

16. C = circumference, r = radius; x = length of a side of the triangle



Since ΔABC is equilateral, $EM = \frac{\sqrt{3}x}{2}$.

Therefore, $OM = \dfrac{\sqrt{3}x}{2} - OE = \dfrac{\sqrt{3}x}{2} - r$

In ΔOAM, $r^2 = \left(\dfrac{x}{2}\right)^2 + \left(\dfrac{\sqrt{3}x}{2} - r\right)^2$

$$r^2 = \frac{x^2}{4} + \frac{3}{4}x^2 - \sqrt{3}\,rx + r^2$$

$$\sqrt{3}\,rx = x^2$$

$$r = \frac{x}{\sqrt{3}}$$

Therefore, the circumference of the circle is

$$C(x) = 2\pi r = 2\pi\left(\frac{x}{\sqrt{3}}\right) = \frac{2\pi\sqrt{3}}{3}x$$

17. Area of the equilateral triangle

$$A = \frac{1}{2}x \cdot \frac{\sqrt{3}}{2}x = \frac{\sqrt{3}}{4}x^2$$

From problem 16, we have $r^2 = \dfrac{x^2}{3}$.

Area inside the circle, but outside the triangle:

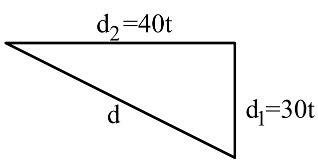
$$A(x) = \pi r^2 - \frac{\sqrt{3}}{4}x^2$$

$$= \pi\frac{x^2}{3} - \frac{\sqrt{3}}{4}x^2 = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)x^2$$

18. $d^2 = d_1{}^2 + d_2{}^2$

$$d^2 = (30t)^2 + (40t)^2$$

$$d(t) = \sqrt{900t^2 + 1600t^2} = \sqrt{2500t^2} = 50t$$



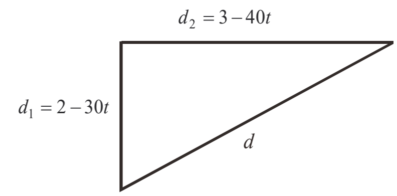
19. a. $d^2 = d_1{}^2 + d_2{}^2$

$$d^2 = (2 - 30t)^2 + (3 - 40t)^2$$

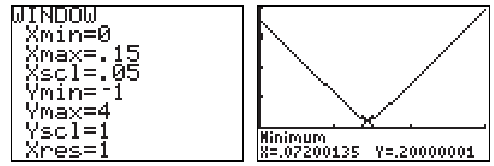
$$d(t) = \sqrt{(2 - 30t)^2 + (3 - 40t)^2}$$

$$= \sqrt{4 - 120t + 900t^2 + 9 - 240t + 1600t^2}$$

$$= \sqrt{2500t^2 - 360t + 13}$$



b. The distance is smallest at $t \approx 0.07$ hours.



20. r = radius of cylinder, h = height of cylinder, V = volume of cylinder

$$r^2 + \left(\frac{h}{2}\right)^2 = R^2 \Rightarrow r^2 + \frac{h^2}{4} = R^2 \Rightarrow r^2 = R^2 - \frac{h^2}{4}$$

$$V = \pi r^2 h$$

$$V(h) = \pi\left(R^2 - \frac{h^2}{4}\right)h = \pi h\left(R^2 - \frac{h^2}{4}\right)$$

21. r = radius of cylinder, h = height of cylinder, V = volume of cylinder

By similar triangles: $\dfrac{H}{R} = \dfrac{H - h}{r}$

$$Hr = R(H - h)$$

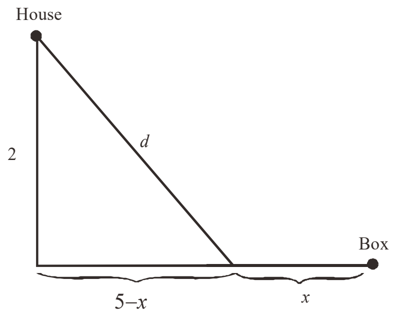
$$Hr = RH - Rh$$

$$Rh = RH - Hr$$

$$h = \frac{RH - Hr}{R} = \frac{H(R - r)}{R}$$

$$V(r) = \pi r^2 h = \pi r^2\left(\frac{H(R - r)}{R}\right) = \frac{\pi H(R - r)r^2}{R}$$

22. a. The total cost of installing the cable along the road is $500x$. If cable is installed x miles along the road, there are $5 - x$ miles between the road to the house and where the cable ends along the road.



$d = \sqrt{(5-x)^2 + 2^2}$

$= \sqrt{25 - 10x + x^2 + 4} = \sqrt{x^2 - 10x + 29}$

The total cost of installing the cable is:

$C(x) = 500x + 700\sqrt{x^2 - 10x + 29}$

Domain: $\{x \mid 0 \le x \le 5\}$

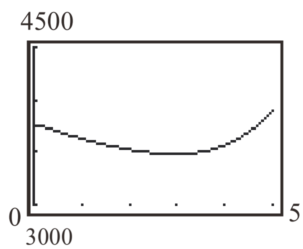
b. $C(1) = 500(1) + 700\sqrt{1^2 - 10(1) + 29}$

$= 500 + 700\sqrt{20} = \$3630.50$

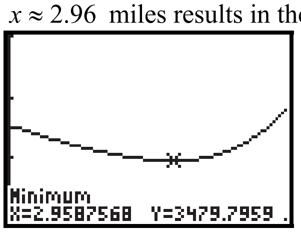
c. $C(3) = 500(3) + 700\sqrt{3^2 - 10(3) + 29}$

$= 1500 + 700\sqrt{8} = \$3479.90$

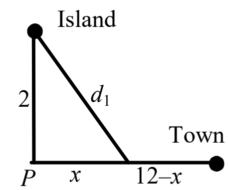
d.



e. Using MINIMUM, the graph indicates that $x \approx 2.96$ miles results in the least cost.



23. a. The time on the boat is given by $\frac{d_1}{3}$. The time on land is given by $\frac{12-x}{5}$.



$d_1 = \sqrt{x^2 + 2^2} = \sqrt{x^2 + 4}$

The total time for the trip is:

$T(x) = \frac{12-x}{5} + \frac{d_1}{3} = \frac{12-x}{5} + \frac{\sqrt{x^2+4}}{3}$

b. Domain: $\{x \mid 0 \le x \le 12\}$

c. $T(4) = \frac{12-4}{5} + \frac{\sqrt{4^2+4}}{3}$

$= \frac{8}{5} + \frac{\sqrt{20}}{3} \approx 3.09$ hours

d. $T(8) = \frac{12-8}{5} + \frac{\sqrt{8^2+4}}{3}$

$= \frac{4}{5} + \frac{\sqrt{68}}{3} \approx 3.55$ hours

24. a. Let $A =$ amount of material, $x =$ length of the base, $h =$ height, and $V =$ volume.

$V = x^2 h = 10 \Rightarrow h = \frac{10}{x^2}$

Total Area $A = (\text{Area}_{\text{base}}) + (4)(\text{Area}_{\text{side}})$

$= x^2 + 4xh$

$= x^2 + 4x\left(\frac{10}{x^2}\right)$

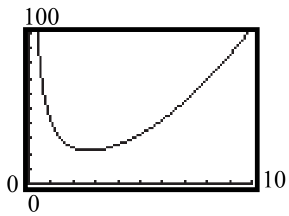
$= x^2 + \frac{40}{x}$

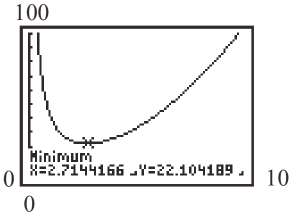
$A(x) = x^2 + \frac{40}{x}$

b. $A(1) = 1^2 + \frac{40}{1} = 1 + 40 = 41 \text{ ft}^2$

c. $A(2) = 2^2 + \frac{40}{2} = 4 + 20 = 24 \text{ ft}^2$

d. $y_1 = x^2 + \dfrac{40}{x}$





The amount of material is least when $x = 2.71$ ft.

e. The largest area is

$A(2.71) = 2.71^2 + \dfrac{40}{2.71} = 22.1 \text{ ft}^2$

25. **a.** length $= 24 - 2x$; width $= 24 - 2x$; height $= x$

$V(x) = x(24-2x)(24-2x) = x(24-2x)^2$

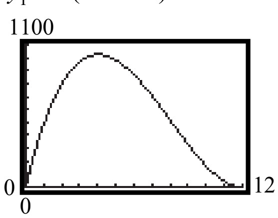
b. $V(3) = 3(24-2(3))^2 = 3(18)^2$

$= 3(324) = 972 \text{ in}^3$.

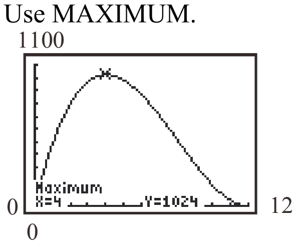
c. $V(10) = 10(24-2(10))^2 = 10(4)^2$

$= 10(16) = 160 \text{ in}^3$.

d. $y_1 = x(24-2x)^2$



Use MAXIMUM.

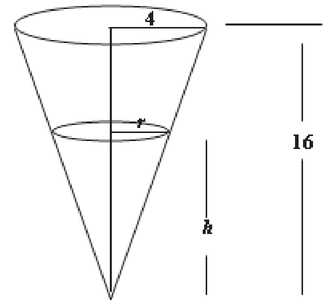


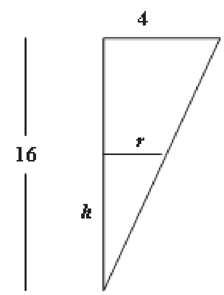
The volume is largest when $x = 4$ inches.

e. The largest volume is

$V(4) = 4(24-2(4))^2 = 1024 \text{ in}^3$

26. Consider the diagrams shown below.





There is a pair of similar triangles in the diagram. This allows us to write

$\dfrac{r}{h} = \dfrac{4}{16} \Rightarrow \dfrac{r}{h} = \dfrac{1}{4} \Rightarrow r = \dfrac{1}{4}h$

Substituting into the volume formula for the conical portion of water gives

$V(h) = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi\left(\dfrac{1}{4}h\right)^2 h = \dfrac{\pi}{48}h^3$.

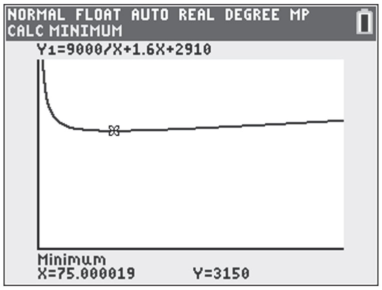
27. **a.** The total cost is the sum of the shipment cost, storage cost, and product cost. Since each shipment will contain x units, there are $600/x$ shipments per year, each costing \$15.

So the shipment cost is $15\left(\dfrac{600}{x}\right) = \left(\dfrac{9000}{x}\right)$.

The storage cost for the year is given as $1.60x$. The product costs is $600(4.85) = 2910$. So, the total cost is

$C(x) = \dfrac{9000}{x} + 1.60x + 2910$.

b.



The retailer should order 75 drives per order for a minimum yearly cost of \$3150.

28. $|2x-3|-5=-2$

$|2x-3|=3$

$2x-3=-3 \text{ or } 2x-3=3$

$2x=0 \quad \text{or} \quad 2x=6$

$x=0 \quad \text{or} \quad x=3$

The solution set is $\{0,3\}$.

29. In order for the 16-foot long Ford Fusion to pass the 50-foot truck, the Ford Fusion must travel the length of the truck and the length of itself in the time frame of 5 seconds. Thus the Fusion must travel an additional 66 feet in 5 seconds.

Convert this to miles-per-hour.

$5 \text{ sec} = \frac{5}{60} \text{ min} = \frac{5}{3600} \text{ hr} = \frac{1}{720} \text{ hr.}$

$66 \text{ ft} = \frac{66}{5280} \text{ mi}$

$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{\frac{66}{5280}}{\frac{1}{720}} = 9 \text{ mph}$

Since the truck is traveling 55 mph, the Fusion must travel 55 + 9 = 64 mph.

30. $m=\frac{y_2-y_1}{x_2-x_1}=\frac{6-(-2)}{1-3}=\frac{8}{-2}=-4$

31. $\frac{10}{4}=\frac{14}{x}$

$10x=4(14)$

$10x=56$

$x=5.6$

32. $x=u-1$

$y=\frac{u-1}{u-1+1}=\frac{u-1}{u}$

33. $\frac{x+5}{3x^{2/3}}+x^{1/3}=\frac{x+5}{3x^{2/3}}+\frac{3x}{3x^{2/3}}$

$=\frac{x+5+3x}{3x^{2/3}}$

$=\frac{4x+5}{3x^{2/3}}$

34. $-\sqrt{3x-2}\ge 4$

$\sqrt{3x-2}\le -4$

No solution since a square root cannot be negative.

35. Since the graph is symmetric is symmetric about the origin then (3, -2) is symmetric to (-3, 2).

36. $v=\frac{2.6t}{d^2}\sqrt{\frac{E}{P}}$

$\frac{vd^2}{2.6t}=\sqrt{\frac{E}{P}}$

$\left(\frac{vd^2}{2.6t}\right)^2=\frac{E}{P}$

$\frac{v^2d^4}{6.76t^2}=\frac{E}{P}$

$\frac{Pv^2d^4}{6.76t^2}=E$

$P=\frac{6.76t^2E}{v^2d^4}$

37. $3x^2 - 7x = 4x - 2$

$3x^2 - 11x + 2 = 0$

$b^2 - 4ac = (-11)^2 - 4(3)(2)$

$= 121 - 24 = 97$

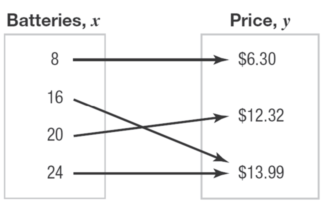
Chapter 2 Review Exercises

1. a. Domain {8, 16, 20, 24}

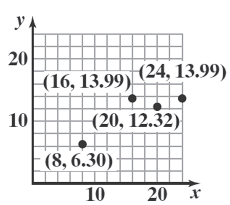
Range {\$6.30, \$12.32, \$13.99}

b. {(8,\$6.30), (16,\$13.99), (20,\$12.32), (24,\$13.99)}

c.



d.



2. This relation represents a function. Domain = {–1, 2, 4}; Range = {0, 3}.

3. Domain {2,4}; Range {-1,1,2}
Not a function

4. not a function; domain [-1, 3]; range [-2, 2]

5. function; domain: all real numbers; range $[-3, \infty)$

6. $f(x) = \dfrac{3x}{x^2 - 1}$

a. $f(2) = \dfrac{3(2)}{(2)^2 - 1} = \dfrac{6}{4-1} = \dfrac{6}{3} = 2$

b. $f(-2) = \dfrac{3(-2)}{(-2)^2 - 1} = \dfrac{-6}{4-1} = \dfrac{-6}{3} = -2$

c. $f(-x) = \dfrac{3(-x)}{(-x)^2 - 1} = \dfrac{-3x}{x^2 - 1}$

d. $-f(x) = -\left(\dfrac{3x}{x^2 - 1}\right) = \dfrac{-3x}{x^2 - 1}$

e. $f(x-2) = \dfrac{3(x-2)}{(x-2)^2 - 1}$

$= \dfrac{3x - 6}{x^2 - 4x + 4 - 1} = \dfrac{3(x-2)}{x^2 - 4x + 3}$

f. $f(2x) = \dfrac{3(2x)}{(2x)^2 - 1} = \dfrac{6x}{4x^2 - 1}$

7. $f(x) = \sqrt{x^2 - 4}$

a. $f(2) = \sqrt{2^2 - 4} = \sqrt{4 - 4} = \sqrt{0} = 0$

b. $f(-2) = \sqrt{(-2)^2 - 4} = \sqrt{4 - 4} = \sqrt{0} = 0$

c. $f(-x) = \sqrt{(-x)^2 - 4} = \sqrt{x^2 - 4}$

d. $-f(x) = -\sqrt{x^2 - 4}$

e. $f(x-2) = \sqrt{(x-2)^2 - 4}$

$= \sqrt{x^2 - 4x + 4 - 4}$

$= \sqrt{x^2 - 4x}$

f. $f(2x) = \sqrt{(2x)^2 - 4} = \sqrt{4x^2 - 4}$

$= \sqrt{4\left(x^2 - 1\right)} = 2\sqrt{x^2 - 1}$

8. $f(x) = \dfrac{x^2 - 4}{x^2}$

a. $f(2) = \dfrac{2^2 - 4}{2^2} = \dfrac{4 - 4}{4} = \dfrac{0}{4} = 0$

b. $f(-2) = \dfrac{(-2)^2 - 4}{(-2)^2} = \dfrac{4 - 4}{4} = \dfrac{0}{4} = 0$

c. $f(-x) = \dfrac{(-x)^2 - 4}{(-x)^2} = \dfrac{x^2 - 4}{x^2}$

d. $-f(x) = -\left(\dfrac{x^2 - 4}{x^2}\right) = \dfrac{4 - x^2}{x^2} = -\dfrac{x^2 - 4}{x^2}$

e. $f(x-2) = \dfrac{(x-2)^2 - 4}{(x-2)^2} = \dfrac{x^2 - 4x + 4 - 4}{(x-2)^2}$

$= \dfrac{x^2 - 4x}{(x-2)^2} = \dfrac{x(x-4)}{(x-2)^2}$

f. $f(2x) = \dfrac{(2x)^2 - 4}{(2x)^2} = \dfrac{4x^2 - 4}{4x^2}$

$= \dfrac{4\left(x^2 - 1\right)}{4x^2} = \dfrac{x^2 - 1}{x^2}$

9. $f(x) = \dfrac{x}{x^2 - 9}$

The denominator cannot be zero:

$x^2 - 9 \neq 0$

$(x+3)(x-3) \neq 0$

$x \neq -3$ or 3

Domain: $\{x \mid x \neq -3, x \neq 3\}$

10. $f(x) = \sqrt{2 - x}$

The radicand must be non-negative:

$2 - x \geq 0$

$x \leq 2$

Domain: $\{x \mid x \leq 2\}$ or $(-\infty, 2]$

11. $g(x) = \dfrac{|x|}{x}$

The denominator cannot be zero:

$x \neq 0$

Domain: $\{x \mid x \neq 0\}$

12. $f(x) = \dfrac{x}{x^2 + 2x - 3}$

The denominator cannot be zero:

$x^2 + 2x - 3 \neq 0$

$(x+3)(x-1) \neq 0$

$x \neq -3$ or 1

Domain: $\{x \mid x \neq -3,\ x \neq 1\}$

13. $f(x) = \dfrac{\sqrt{x+1}}{x^2 - 4}$

The denominator cannot be zero:

$x^2 - 4 \neq 0$

$(x+2)(x-2) \neq 0$

$x \neq -2$ or 2

Also, the radicand must be non-negative:

$x + 1 \geq 0$

$x \geq -1$

Domain: $[-1, 2) \cup (2, \infty)$

14. $f(x) = \dfrac{x}{\sqrt{x+8}}$

The radicand must be non-negative and not zero:

$x + 8 > 0$

$x > -8$

Domain: $\{x \mid x > -8\}$

15. $f(x) = 2 - x \quad g(x) = 3x + 1$

$(f+g)(x) = f(x) + g(x)$

$= 2 - x + 3x + 1 = 2x + 3$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f-g)(x) = f(x) - g(x)$

$= 2 - x - (3x + 1)$

$= 2 - x - 3x - 1$

$= -4x + 1$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f \cdot g)(x) = f(x) \cdot g(x)$

$= (2 - x)(3x + 1)$

$= 6x + 2 - 3x^2 - x$

$= -3x^2 + 5x + 2$

Domain: $\{x \mid x \text{ is any real number}\}$

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{2 - x}{3x + 1}$

$3x + 1 \neq 0$

$3x \neq -1 \Rightarrow x \neq -\dfrac{1}{3}$

Domain: $\left\{x \,\middle|\, x \neq -\dfrac{1}{3}\right\}$

16. $f(x) = 3x^2 + x + 1 \qquad g(x) = 3x$

$(f+g)(x) = f(x) + g(x)$

$= 3x^2 + x + 1 + 3x$

$= 3x^2 + 4x + 1$

Domain: $\{x \mid x \text{ is any real number}\}$

$(f-g)(x) = f(x) - g(x)$

$= 3x^2 + x + 1 - 3x$

$= 3x^2 - 2x + 1$

Domain: $\{x \mid x \text{ is any real number}\}$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$
$$= \left(3x^2 + x + 1\right)(3x)$$
$$= 9x^3 + 3x^2 + 3x$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x^2 + x + 1}{3x}$$

$3x \neq 0 \Rightarrow x \neq 0$

Domain: $\{x \mid x \neq 0\}$

17. $f(x) = \dfrac{x+1}{x-1} \qquad g(x) = \dfrac{1}{x}$

$$(f+g)(x) = f(x) + g(x)$$
$$= \frac{x+1}{x-1} + \frac{1}{x} = \frac{x(x+1) + 1(x-1)}{x(x-1)}$$
$$= \frac{x^2 + x + x - 1}{x(x-1)} = \frac{x^2 + 2x - 1}{x(x-1)}$$

Domain: $\{x \mid x \neq 0, x \neq 1\}$

$$(f-g)(x) = f(x) - g(x)$$
$$= \frac{x+1}{x-1} - \frac{1}{x} = \frac{x(x+1) - 1(x-1)}{x(x-1)}$$
$$= \frac{x^2 + x - x + 1}{x(x-1)} = \frac{x^2 + 1}{x(x-1)}$$

Domain: $\{x \mid x \neq 0, x \neq 1\}$

$$(f \cdot g)(x) = f(x) \cdot g(x) = \left(\frac{x+1}{x-1}\right)\left(\frac{1}{x}\right) = \frac{x+1}{x(x-1)}$$

Domain: $\{x \mid x \neq 0, x \neq 1\}$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{x+1}{x-1}}{\frac{1}{x}} = \left(\frac{x+1}{x-1}\right)\left(\frac{x}{1}\right) = \frac{x(x+1)}{x-1}$$

Domain: $\{x \mid x \neq 0, x \neq 1\}$

18. $f(x) = -2x^2 + x + 1$

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{-2(x+h)^2 + (x+h) + 1 - \left(-2x^2 + x + 1\right)}{h}$$
$$= \frac{-2\left(x^2 + 2xh + h^2\right) + x + h + 1 + 2x^2 - x - 1}{h}$$
$$= \frac{-2x^2 - 4xh - 2h^2 + x + h + 1 + 2x^2 - x - 1}{h}$$
$$= \frac{-4xh - 2h^2 + h}{h} = \frac{h(-4x - 2h + 1)}{h}$$
$$= -4x - 2h + 1$$

19. **a.** Domain: $\{x \mid -4 \leq x \leq 3\}$; $[-4, 3]$

Range: $\{y \mid -3 \leq y \leq 3\}$; $[-3, 3]$

b. Intercept: $(0, 0)$

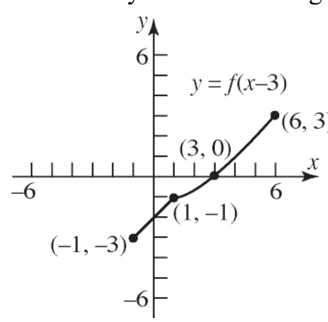
c. $f(-2) = -1$

d. $f(x) = -3$ when $x = -4$

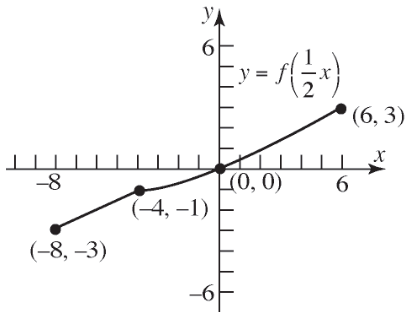
e. $f(x) > 0$ when $0 < x \leq 3$

$\{x \mid 0 < x \leq 3\}$

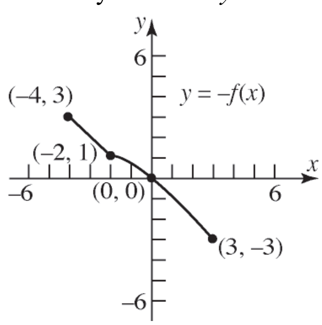
f. To graph $y = f(x-3)$, shift the graph of f horizontally 3 units to the right.



g. To graph $y = f\left(\frac{1}{2}x\right)$, stretch the graph of f horizontally by a factor of 2.



h. To graph $y = -f(x)$, reflect the graph of f vertically about the y-axis.



20. a. Domain: $(-\infty, 4]$

Range: $(-\infty, 3]$

b. Increasing: $(-\infty, -2]$ and $[2, 4]$;

Decreasing: $[-2, 2]$

c. Local minimum is -1 at $x = 2$;
Local maximum is 1 at $x = -2$

d. No absolute minimum;
Absolute maximum is 3 at $x = 4$

e. The graph has no symmetry.

f. The function is neither.

g. x-intercepts: $(-3,0), (0,0), (3,0)$;

y-intercept: (0,0)

21. $f(x) = x^3 - 4x$

$f(-x) = (-x)^3 - 4(-x) = -x^3 + 4x$

$\quad = -\left(x^3 - 4x\right) = -f(x)$

f is odd.

22. $g(x) = \dfrac{4 + x^2}{1 + x^4}$

$g(-x) = \dfrac{4 + (-x)^2}{1 + (-x)^4} = \dfrac{4 + x^2}{1 + x^4} = g(x)$

g is even.

23. $G(x) = 1 - x + x^3$

$G(-x) = 1 - (-x) + (-x)^3$

$\quad = 1 + x - x^3 \neq -G(x)$ or $G(x)$

G is neither even nor odd.

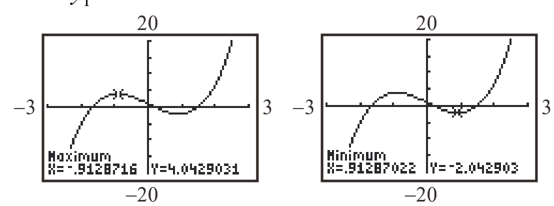
24. $f(x) = \dfrac{x}{1 + x^2}$

$f(-x) = \dfrac{-x}{1 + (-x)^2} = \dfrac{-x}{1 + x^2} = -f(x)$

f is odd.

25. $f(x) = 2x^3 - 5x + 1$ on the interval $(-3, 3)$

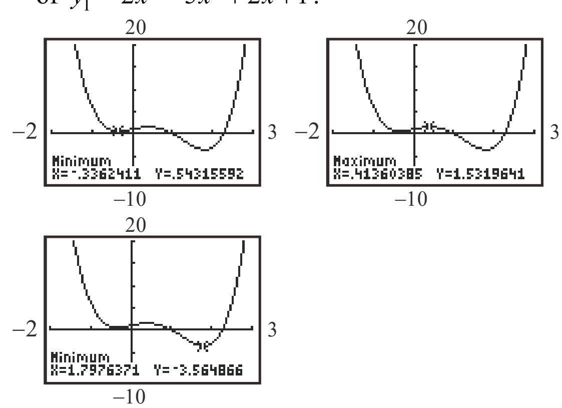
Use MAXIMUM and MINIMUM on the graph of $y_1 = 2x^3 - 5x + 1$.



local maximum value: 4.04 when $x \approx -0.91$
local minimum value: -2.04 when $x = 0.91$
f is increasing on: $[-3, -0.91]$ and $[0.91, 3]$;
f is decreasing on: $[-0.91, 0.91]$.

26. $f(x) = 2x^4 - 5x^3 + 2x + 1$ on the interval $(-2, 3)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = 2x^4 - 5x^3 + 2x + 1$.



local maximum: 1.53 when $x = 0.41$