TT TEXTBOOK TESTBANKS

Solutions Manual for College Algebra 12th Sullivan

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INSTRUCTOR'S SOLUTIONS MANUAL

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Chapter R

Review

Section R.1

1. rational

2. $4+5\cdot 6-3=4+30-3=31$

3. Distributive

4. c

5. a

6. b

7. True

8. False; The Zero-Product Property states that if a product equals 0, then at least one of the factors must equal 0.

9. False; 6 is the Greatest Common Factor of 12 and 18. The Least Common Multiple is the smallest value that both numbers will divide evenly. The LCM for 12 and 18 is 36.

10. True

11. $A\cup B=\{1,3,4,5,9\}\cup\{2,4,6,7,8\}$
$=\{1,2,3,4,5,6,7,8,9\}$

12. $A\cup C=\{1,3,4,5,9\}\cup\{1,3,4,6\}$
$=\{1,3,4,5,6,9\}$

13. $A\cap B=\{1,3,4,5,9\}\cap\{2,4,6,7,8\}=\{4\}$

14. $A\cap C=\{1,3,4,5,9\}\cap\{1,3,4,6\}=\{1,3,4\}$

15. $(A\cup B)\cap C$
$=(\{1,3,4,5,9\}\cup\{2,4,6,7,8\})\cap\{1,3,4,6\}$
$=\{1,2,3,4,5,6,7,8,9\}\cap\{1,3,4,6\}$
$=\{1,3,4,6\}$

16. $(A\cap B)\cup C$
$=(\{1,3,4,5,9\}\cap\{2,4,6,7,8\})\cup\{1,3,4,6\}$
$=\{4\}\cup\{1,3,4,6\}$
$=\{1,3,4,6\}$

17. $\overline{A}=\{0,2,6,7,8\}$

18. $\overline{C}=\{0,2,5,7,8,9\}$

19. $\overline{A\cap B}=\overline{\{1,3,4,5,9\}\cap\{2,4,6,7,8\}}$
$=\overline{\{4\}}=\{0,1,2,3,5,6,7,8,9\}$

20. $\overline{B\cup C}=\overline{\{2,4,6,7,8\}\cup\{1,3,4,6\}}$
$=\overline{\{1,2,3,4,6,7,8\}}=\{0,5,9\}$

21. $\overline{A}\cup\overline{B}=\{0,2,6,7,8\}\cup\{0,1,3,5,9\}$
$=\{0,1,2,3,5,6,7,8,9\}$

22. $\overline{B}\cap\overline{C}=\{0,1,3,5,9\}\cap\{0,2,5,7,8,9\}$
$=\{0,5,9\}$

23. **a.** $\{2,5\}$

b. $\{-6,2,5\}$

c. $\left\{-6,\frac{1}{2},-1.333...,2,5\right\}$

d. $\{\pi\}$

e. $\left\{-6,\frac{1}{2},-1.333...,\pi,2,5\right\}$

24. **a.** $\{1\}$

b. $\{0,1\}$

c. $\left\{-\frac{5}{3},2.060606...=2.\overline{06},1.25,0,1\right\}$

d. $\{\sqrt{5}\}$

e. $\left\{-\frac{5}{3}, 2.060606... = 2.\overline{06}, 1.25, 0, 1, \sqrt{5}\right\}$

25. **a.** $\{1\}$

b. $\{0,1\}$

c. $\left\{0,1,\frac{1}{2},\frac{1}{3},\frac{1}{4}\right\}$

d. None

e. $\left\{0,1,\frac{1}{2},\frac{1}{3},\frac{1}{4}\right\}$

26. **a.** None

b. $\{-1\}$

c. $\{-1.3,-1.2,-1.1,-1\}$

d. None

e. $\{-1.3,-1.2,-1.1,-1\}$

27. **a.** None

b. None

c. None

d. $\left\{\sqrt{2}, \pi, \sqrt{2}+1, \pi+\frac{1}{2}\right\}$

e. $\left\{\sqrt{2}, \pi, \sqrt{2}+1, \pi+\frac{1}{2}\right\}$

28. **a.** None

b. None

c. $\left\{\frac{1}{2}+10.3\right\}$

d. $\left\{-\sqrt{2}, \pi+\sqrt{2}\right\}$

e. $\left\{-\sqrt{2}, \pi+\sqrt{2}, \frac{1}{2}+10.3\right\}$

29. **a.** 18.953 **b.** 18.952

30. **a.** 25.861 **b.** 25.861

31. **a.** 28.653 **b.** 28.653

32. **a.** 99.052 **b.** 99.052

33. **a.** 0.063 **b.** 0.062

34. **a.** 0.054 **b.** 0.053

35. **a.** 9.999 **b.** 9.998

36. **a.** 1.001 **b.** 1.000

37. **a.** 0.429 **b.** 0.428

38. **a.** 0.556 **b.** 0.555

39. **a.** 34.733 **b.** 34.733

40. **a.** 16.200 **b.** 16.200

41. $3+2=5$

42. $5 \cdot 2 = 10$

43. $x+2=3\cdot 4$

44. $3+y=2+2$

45. $3y=1+2$

46. $2x=4\cdot 6$

47. $x-2=6$

48. $2-y=6$

49. $\frac{x}{2}=6$

50. $\frac{2}{x}=6$

51. $9-4+2=5+2=7$

52. $6-4+3=2+3=5$

53. $-6+4\cdot 3=-6+12=6$

54. $8-4\cdot 2=8-8=0$

55. $18-5\cdot 2=18-10=8$

56. $100-10\cdot 2=100-20=80$

57. $4+\frac{1}{3}=\frac{12+1}{3}=\frac{13}{3}$

58. $2-\frac{1}{2}=\frac{4-1}{2}=\frac{3}{2}$

59. $6-\left[3\cdot 5+2\cdot(3-2)\right]=6-\left[15+2\cdot(1)\right]$
$=6-17$
$=-11$

60. $2\cdot\left[8-3\cdot(4+2)\right]-3=2\cdot\left[8-3\cdot(6)\right]-3$
$=2\cdot\left[8-18\right]-3$
$=2\cdot\left[-10\right]-3$
$=-20-3$
$=-23$

61. $4\cdot(9+5)-6\cdot 7+3=4\cdot(14)-42+3$
$=56-42+3$
$=14+3$
$=17$

62. $1-(4\cdot 3-2+2)=1-(12-2+2)$
$=1-12$
$=-11$

63. $10-\left[6-2\cdot 2+(8-3)\right]\cdot 2=10-\left[6-4+5\right]\cdot 2$
$=10-\left[2+5\right]\cdot 2$
$=10-\left[7\right]\cdot 2$
$=10-14$
$=-4$

64. $2-5\cdot 4-\left[6\cdot(3-4)\right]=2-20-\left[6\cdot(-1)\right]$
$=-18-\left[-6\right]$
$=-18+6$
$=-12$

65. $(5-3)\frac{1}{2}=(2)\frac{1}{2}=1$

66. $(5+4)\frac{1}{3}=(9)\frac{1}{3}=3$

67. $\frac{4+8}{5-3}=\frac{12}{2}=6$

68. $\frac{2-4}{5-3}=\frac{-2}{2}=-1$

69. $\frac{3}{5}\cdot\frac{10}{21}=\frac{3\cdot 2\cdot 5}{5\cdot 3\cdot 7}=\frac{\cancel{3}\cdot 2\cdot\cancel{5}}{\cancel{5}\cdot\cancel{3}\cdot 7}=\frac{2}{7}$

70. $\frac{5}{9}\cdot\frac{3}{10}=\frac{5\cdot 3}{3\cdot 3\cdot 5\cdot 2}=\frac{\cancel{5}\cdot\cancel{3}}{\cancel{3}\cdot 3\cdot\cancel{5}\cdot 2}=\frac{1}{6}$

71. $\frac{6}{25}\cdot\frac{10}{27}=\frac{2\cdot 3\cdot 5\cdot 2}{5\cdot 5\cdot 3\cdot 9}=\frac{2\cdot\cancel{3}\cdot\cancel{5}\cdot 2}{\cancel{5}\cdot 5\cdot\cancel{3}\cdot 9}=\frac{4}{45}$

72. $\frac{21}{25}\cdot\frac{100}{3}=\frac{3\cdot 7\cdot 4\cdot 25}{25\cdot 3}=\frac{\cancel{3}\cdot 7\cdot 4\cdot\cancel{25}}{\cancel{25}\cdot\cancel{3}}=28$

73. $\frac{3}{4}+\frac{2}{5}=\frac{15+8}{20}=\frac{23}{20}$

74. $\frac{4}{3}+\frac{1}{2}=\frac{8+3}{6}=\frac{11}{6}$

75. $\frac{7}{8}+\frac{4}{7}=\frac{49+32}{56}=\frac{81}{56}$

76. $\frac{8}{9}+\frac{15}{2}=\frac{16+135}{18}=\frac{151}{18}$

77. $\frac{5}{18}+\frac{1}{12}=\frac{10+3}{36}=\frac{13}{36}$

78. $\frac{2}{15}+\frac{8}{9}=\frac{6+40}{45}=\frac{46}{45}$

79. $\frac{5}{24}-\frac{8}{15}=\frac{25-64}{120}=-\frac{39}{120}=-\frac{13}{40}$

80. $\frac{3}{14}-\frac{2}{21}=\frac{9-4}{42}=\frac{5}{42}$

81. $\frac{3}{20}-\frac{2}{15}=\frac{9-8}{60}=\frac{1}{60}$

82. $\frac{6}{35}-\frac{3}{14}=\frac{12-15}{70}=-\frac{3}{70}$

83. $\frac{\left(\frac{5}{18}\right)}{\left(\frac{11}{27}\right)}=\frac{5}{18}\cdot\frac{27}{11}=\frac{5\cdot 9\cdot 3}{9\cdot 2\cdot 11}=\frac{5\cdot\cancel{9}\cdot 3}{\cancel{9}\cdot 2\cdot 11}=\frac{15}{22}$

84. $\frac{\left(\frac{5}{21}\right)}{\left(\frac{2}{35}\right)}=\frac{5}{21}\cdot\frac{35}{2}=\frac{5\cdot 7\cdot 5}{7\cdot 3\cdot 2}=\frac{5\cdot\cancel{7}\cdot 5}{\cancel{7}\cdot 3\cdot 2}=\frac{25}{6}$

85. $\frac{1}{3}\cdot\frac{4}{7}+\frac{17}{21}=\frac{4}{21}+\frac{17}{21}=\frac{21}{21}=1$

86. $$\begin{aligned}\frac{2}{3}+\frac{4}{5}\cdot\frac{1}{6}&=\frac{2}{3}+\frac{2\cdot 2}{5\cdot 3\cdot 2}=\frac{2}{3}+\frac{2\cdot\cancel{2}}{5\cdot 3\cdot\cancel{2}}=\frac{2}{3}+\frac{2}{15}\\&=\frac{2}{3}\cdot\frac{5}{5}+\frac{2}{15}=\frac{10}{15}+\frac{2}{15}=\frac{10+2}{15}=\frac{12}{15}\\&=\frac{4\cdot 3}{5\cdot 3}=\frac{4\cdot\cancel{3}}{5\cdot\cancel{3}}=\frac{4}{5}\end{aligned}$$

87. $$\begin{aligned}2\cdot\frac{3}{4}+\frac{3}{8}&=\frac{2}{1}\cdot\frac{3}{4}+\frac{3}{8}=\frac{6}{4}+\frac{3}{8}=\frac{6}{4}\cdot\frac{2}{2}+\frac{3}{8}\\&=\frac{12}{8}+\frac{3}{8}=\frac{12+3}{8}=\frac{15}{8}\end{aligned}$$

88. $$\begin{aligned}3\cdot\frac{5}{6}-\frac{1}{2}&=\frac{3}{1}\cdot\frac{5}{6}-\frac{1}{2}=\frac{3\cdot 5}{3\cdot 2}-\frac{1}{2}=\frac{\cancel{3}\cdot 5}{\cancel{3}\cdot 2}-\frac{1}{2}\\&=\frac{5}{2}-\frac{1}{2}=\frac{5-1}{2}=\frac{4}{2}=2\end{aligned}$$

89. $6(x+4)=6x+24$

90. $4(2x-1)=8x-4$

91. $x(x-4)=x^2-4x$

92. $4x(x+3)=4x^2+12x$

93. $$\begin{aligned}2\left(\frac{3}{4}x-\frac{1}{2}\right)&=2\cdot\frac{3}{4}x-2\cdot\frac{1}{2}=\frac{2\cdot 3x}{2\cdot 2}-\frac{2}{2}\\&=\frac{\cancel{2}\cdot 3x}{\cancel{2}\cdot 2}-\frac{2}{2}=\frac{3}{2}x-1\end{aligned}$$

94. $$\begin{aligned}3\left(\frac{2}{3}x+\frac{1}{6}\right)&=3\cdot\frac{2}{3}x+3\cdot\frac{1}{6}=\frac{3\cdot 2x}{3}+\frac{3}{3\cdot 2}\\&=\frac{\cancel{3}\cdot 2x}{\cancel{3}}+\frac{\cancel{3}}{\cancel{3}\cdot 2}=2x+\frac{1}{2}\end{aligned}$$

95. $$\begin{aligned}(x+2)(x+4)&=x^2+4x+2x+8\\&=x^2+6x+8\end{aligned}$$

96. $$\begin{aligned}(x+5)(x+1)&=x^2+x+5x+5\\&=x^2+6x+5\end{aligned}$$

97. $$\begin{aligned}(x+9)(2x-7)&=2x^2-7x+18x-63\\&=2x^2+11x-63\end{aligned}$$

98. $$\begin{aligned}(3x-1)(x+5)&=3x^2+15x-x-5\\&=3x^2+14x-5\end{aligned}$$

99. $$\begin{aligned}(x-8)(x-2)&=x^2-2x-8x+16\\&=x^2-10x+16\end{aligned}$$

100. $$\begin{aligned}(x-4)(x-2)&=x^2-2x-4x+8\\&=x^2-6x+8\end{aligned}$$

101. $$\begin{aligned}3x(x-5k)&=3x^2-60x\\3x^2-15xk&=3x^2-60x\\-15xk&=-60x\\k&=4\end{aligned}$$

102. $$\begin{aligned}(x-k)(x+3k)&=x^2+4x-12\\x^2+3kx-kx-3k^2&=x^2+4x-12\\x^2+x(3k-k)-3k^2&=x^2+4x-12\\x^2+x(3k-k)-3k^2&=x^2+4x-12\\x^2+x(2k)-3k^2&=x^2+4x-12\\2k&=4\\k&=2\end{aligned}$$

103. $$\begin{aligned}2x+3x&=2\cdot x+3\cdot x\\&=(2+3)\cdot x\\&=(5)\cdot x\\&=5x\end{aligned}$$

104. $2+3\cdot 4=2+12=14$
since multiplication comes before addition in the order of operations for real numbers.

$(2+3)\cdot 4=5\cdot 4=20$
since operations inside parentheses come before multiplication in the order of operations for real numbers.

105. $2(3\cdot 4)=2(12)=24$
$(2\cdot 3)\cdot(2\cdot 4)=(6)(8)=48$

106. $\frac{4+3}{2+5}=\frac{7}{7}=1$, but
$\frac{4}{2}+\frac{3}{5}=\frac{4\cdot 5+3\cdot 2}{10}=\frac{20+6}{10}=\frac{26}{10}=\frac{13}{5}=2.6$

107. Subtraction is not commutative; for example: $2-3=-1\neq 1=3-2$.

108. Subtraction is not associative; for example: $(5-2)-1=2\neq 4=5-(2-1)$.

109. Division is not commutative; for example: $\frac{2}{3}\neq\frac{3}{2}$.

110. Division is not associative; for example: $(12\div 2)\div 2=6\div 2=3$, but $12\div(2\div 2)=12\div 1=12$.

111. The Symmetric Property implies that if 2 = x, then x = 2.

112. From the *principle of substitution,* if $x=5$, then

$$(x)(x)=(5)(5)$$
$$\Rightarrow x^2=25$$
$$\Rightarrow x^2+x=25+5$$
$$\Rightarrow x^2+x=30$$

113. There are no real numbers that are both rational and irrational, since an irrational number, by definition, is a number that cannot be expressed as the ratio of two integers; that is, not a rational number

Every real number is either a rational number or an irrational number, since the decimal form of a real number either involves an infinitely repeating pattern of digits or an infinite, non-repeating string of digits.

114. The sum of an irrational number and a rational number must be irrational. Otherwise, the irrational number would then be the difference of two rational numbers, and therefore would have to be rational.

115. Answers will vary.

116. Since 1 day = 24 hours, we compute $\frac{12997}{24}=541.541\overline{6}$.

Now we only need to consider the decimal part of the answer in terms of a 24 hour day. That is, $(0.541\overline{6})(24)\approx 13$ hours. So it must be 13 hours later than 12 noon, which makes the time 1 a.m. CST.

117. Answers will vary.

Section R.2

1. variable

2. origin

3. strict

4. base; exponent (or power)

5. 1.2345678×10^3

6. d

7. a

8. b

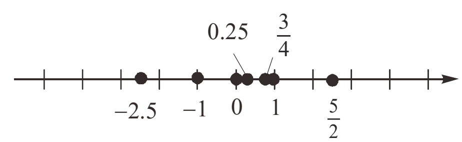
9. True

10. False; the absolute value of a real number is nonnegative. $|0|=0$ which is not a positive number.

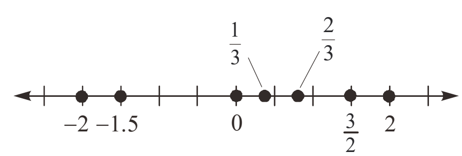
11. False; a number in scientific notation is expressed as the product of a number, x, $1\leq x<10$ or $-10<x\leq -1$, and a power of 10.

12. True

13.



14.



15. $\frac{1}{2}>0$

16. $5<6$

17. $-1>-2$

18. $-3<-\frac{5}{2}$

19. $\pi>3.14$

20. $\sqrt{2}>1.41$

21. $\frac{1}{2} = 0.5$

22. $\frac{1}{3} > 0.33$

23. $\frac{2}{3} < 0.67$

24. $\frac{1}{4} = 0.25$

25. $x > 0$

26. $z < 0$

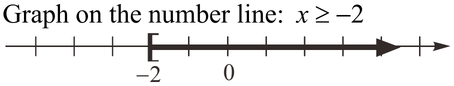
27. $x < 2$

28. $y > -5$

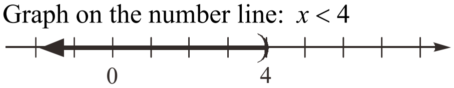
29. $x \le 1$

30. $x \ge 2$

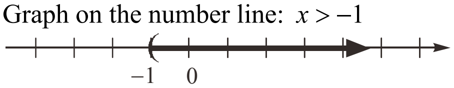
31. Graph on the number line: $x \ge -2$



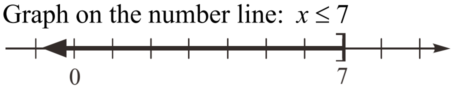
32. Graph on the number line: $x < 4$



33. Graph on the number line: $x > -1$



34. Graph on the number line: $x \le 7$



35. $d(C,D) = d(0,1) = |1-0| = |1| = 1$

36. $d(C,A) = d(0,-3) = |-3-0| = |-3| = 3$

37. $d(D,E) = d(1,3) = |3-1| = |2| = 2$

38. $d(C,E) = d(0,3) = |3-0| = |3| = 3$

39. $d(A,E) = d(-3,3) = |3-(-3)| = |6| = 6$

40. $d(D,B) = d(1,-1) = |-1-1| = |-2| = 2$

41. $x + 2y = -2 + 2\cdot 3 = -2 + 6 = 4$

42. $3x + y = 3(-2) + 3 = -6 + 3 = -3$

43. $5xy + 2 = 5(-2)(3) + 2 = -30 + 2 = -28$

44. $-2x + xy = -2(-2) + (-2)(3) = 4 - 6 = -2$

45. $\frac{2x}{x-y} = \frac{2(-2)}{-2-3} = \frac{-4}{-5} = \frac{4}{5}$

46. $\frac{x+y}{x-y} = \frac{-2+3}{-2-3} = \frac{1}{-5} = -\frac{1}{5}$

47. $\frac{3x+2y}{2+y} = \frac{3(-2)+2(3)}{2+3} = \frac{-6+6}{5} = \frac{0}{5} = 0$

48. $\frac{2x-3}{y} = \frac{2(-2)-3}{3} = \frac{-4-3}{3} = -\frac{7}{3}$

49. $|x+y| = |3+(-2)| = |1| = 1$

50. $|x-y| = |3-(-2)| = |5| = 5$

51. $|x| + |y| = |3| + |-2| = 3 + 2 = 5$

52. $|x| - |y| = |3| - |-2| = 3 - 2 = 1$

53. $\frac{|x|}{x} = \frac{|3|}{3} = \frac{3}{3} = 1$

54. $\frac{|y|}{y} = \frac{|-2|}{-2} = \frac{2}{-2} = -1$

55.
$$\begin{aligned} |4x-5y| &= |4(3)-5(-2)| \\ &= |12+10| \\ &= |22| \\ &= 22 \end{aligned}$$

56. $|3x+2y| = |3(3)+2(-2)| = |9-4| = |5| = 5$

57.
$$\begin{aligned} \big||4x|-|5y|\big| &= \big||4(3)|-|5(-2)|\big| \\ &= \big||12|-|-10|\big| \\ &= |12-10| \\ &= |2| \\ &= 2 \end{aligned}$$

58. $3|x|+2|y| = 3|3|+2|-2|$
$$= 3\cdot 3+2\cdot 2$$
$$= 9+4$$
$$= 13$$

59. $\dfrac{x^2-1}{x}$
Part (c) must be excluded. The value $x=0$ must be excluded from the domain because it causes division by 0.

60. $\dfrac{x^2+1}{x}$
Part (c) must be excluded. The value $x=0$ must be excluded from the domain because it causes division by 0.

61. $\dfrac{x}{x^2-9} = \dfrac{x}{(x-3)(x+3)}$
Part (a) , $x=3$, must be excluded because it causes the denominator to be 0.

62. $\dfrac{x}{x^2+9}$
None of the given values are excluded. The domain is all real numbers.

63. $\dfrac{x^2}{x^2+1}$
None of the given values are excluded. The domain is all real numbers.

64. $\dfrac{x^3}{x^2-1} = \dfrac{x^3}{(x-1)(x+1)}$
Parts (b) and (d) must be excluded. The values $x=1$, and $x=-1$ must be excluded from the domain because they cause division by 0.

65. $\dfrac{x^2+5x-10}{x^3-x} = \dfrac{x^2+5x-10}{x(x-1)(x+1)}$
Parts (b), (c), and (d) must be excluded. The values $x=0$, $x=1$, and $x=-1$ must be excluded from the domain because they cause division by 0.

66. $\dfrac{-9x^2-x+1}{x^3+x} = \dfrac{-9x^2-x+1}{x(x^2+1)}$
Part (c) must be excluded. The value $x=0$ must be excluded from the domain because it causes division by 0.

67. $\dfrac{4}{x-5}$
$x=5$ must be exluded because it makes the denominator equal 0.
Domain $= \{x \mid x \neq 5\}$

68. $\dfrac{-6}{x+4}$
$x=-4$ must be excluded sine it makes the denominator equal 0.
Domain $= \{x \mid x \neq -4\}$

69. $\dfrac{x}{x+4}$
$x=-4$ must be excluded sine it makes the denominator equal 0.
Domain $= \{x \mid x \neq -4\}$

70. $\dfrac{x-2}{x-6}$
$x=6$ must be excluded sine it makes the denominator equal 0.
Domain $= \{x \mid x \neq 6\}$

71. $C = \dfrac{5}{9}(F-32) = \dfrac{5}{9}(32-32) = \dfrac{5}{9}(0) = 0°\text{C}$

72. $C = \dfrac{5}{9}(F-32) = \dfrac{5}{9}(212-32) = \dfrac{5}{9}(180) = 100°\text{C}$

73. $C = \dfrac{5}{9}(F-32) = \dfrac{5}{9}(77-32) = \dfrac{5}{9}(45) = 25°\text{C}$

74. $C = \dfrac{5}{9}(F-32) = \dfrac{5}{9}(-4-32)$
$$= \frac{5}{9}(-36)$$
$$= -20°\text{C}$$

75. $(-9)^2 = (-9)(-9) = 81$

76. $-4^2 = -(4)^2 = -16$

77. $4^{-2} = \dfrac{1}{4^2} = \dfrac{1}{16}$

78. $-4^{-2}=-\frac{1}{4^2}=-\frac{1}{16}$

79. $3^{-6}\cdot 3^4=3^{-6+4}=3^{-2}=\frac{1}{3^2}=\frac{1}{9}$

80. $4^{-2}\cdot 4^3=4^{-2+3}=4^1=4$

81. $\left(4^{-3}\right)^{-1}=4^{(-3)(-1)}=4^3=64$

82. $\left(2^{-1}\right)^{-3}=2^{(-1)(-3)}=2^3=8$

83. $\sqrt{100}=\sqrt{10^2}=10$

84. $\sqrt{36}=\sqrt{6^2}=6$

85. $\sqrt{(-4)^2}=|-4|=4$

86. $\sqrt{(-3)^2}=|-3|=3$

87. $\left(9x^4\right)^2=9^2\left(x^4\right)^2=81x^8$

88. $\left(-4x^2\right)^{-1}=\frac{1}{-4x^2}=-\frac{1}{4x^2}$

89. $\left(x^2y^{-1}\right)^2=\left(x^2\right)^2\cdot\left(y^{-1}\right)^2=x^4y^{-2}=\frac{x^4}{y^2}$

90. $\left(x^{-1}y\right)^3=\left(x^{-1}\right)^3\cdot y^3=x^{-3}y^3=\frac{y^3}{x^3}$

91. $\frac{x^2y^5}{x^3y^4}=x^{2-3}y^{5-4}=x^{-1}y^1=\frac{y}{x}$

92. $\frac{x^{-2}y}{xy^2}=x^{-2-1}y^{1-2}=x^{-3}y^{-1}=\frac{1}{x^3y}$

93.
$$\begin{aligned}\frac{(-4)^2y^5(xz)^3}{(-3)^3xy^7z^2}&=\frac{16y^5x^3z^3}{-27xy^7z^2}\\&=-\frac{16}{27}x^{3-1}y^{5-7}z^{3-2}\\&=-\frac{16}{27}x^2y^{-2}z^1\\&=-\frac{16x^2z}{27y^2}\end{aligned}$$

94.
$$\begin{aligned}\frac{4x^{-2}(yz)^{-1}}{2^3x^4y}&=\frac{4x^{-2}y^{-1}z^{-1}}{8x^4y}\\&=\frac{4}{8}x^{-2-4}y^{-1-1}z^{-1}\\&=\frac{1}{2}x^{-6}y^{-2}z^{-1}\\&=\frac{1}{2x^6y^2z}\end{aligned}$$

95. $\left(\frac{2x^{-3}}{3y^{-1}}\right)^{-2}=\left(\frac{3y}{2x^3}\right)^{-2}=\left(\frac{3x^3}{2y}\right)^2=\frac{3^2x^6}{2^2y^2}=\frac{9x^6}{4y^2}$

96.
$$\begin{aligned}\left(\frac{5x^{-2}}{6y^{-2}}\right)^{-3}&=\left(\frac{5y^2}{6x^2}\right)^{-3}=\left(\frac{6x^2}{5y^2}\right)^3\\&=\frac{6^3\left(x^2\right)^3}{5^3\left(y^2\right)^3}=\frac{216x^6}{125y^6}\end{aligned}$$

97. $2xy^{-1}=\frac{2x}{y}=\frac{2(2)}{(-1)}=-4$

98. $-3x^{-1}y=\frac{-3y}{x}=\frac{-3(-1)}{(2)}=\frac{3}{2}$

99. $x^2+y^2=(2)^2+(-1)^2=4+1=5$

100. $x^2y^2=(2)^2(-1)^2=4\cdot 1=4$

101. $(xy)^2=(2\cdot(-1))^2=(-2)^2=4$

102. $(x+y)^2=(2+(-1))^2=(1)^2=1$

103. $\sqrt{x^2}=|x|=|2|=2$

104. $\left(\sqrt{x}\right)^2=x=2$

105. $\sqrt{x^2+y^2}=\sqrt{(2)^2+(-1)^2}=\sqrt{4+1}=\sqrt{5}$

106. $\sqrt{x^2}+\sqrt{y^2}=|x|+|y|=|2|+|-1|=2+1=3$

107. $x^y=2^{-1}=\frac{1}{2}$

108. $y^x = (-1)^2 = 1$

109. If $x = 2$,

$$\begin{aligned} 2x^3 - 3x^2 + 5x - 4 &= 2\cdot 2^3 - 3\cdot 2^2 + 5\cdot 2 - 4 \\ &= 16 - 12 + 10 - 4 \\ &= 10 \end{aligned}$$

If $x = 1$,

$$\begin{aligned} 2x^3 - 3x^2 + 5x - 4 &= 2\cdot 1^3 - 3\cdot 1^2 + 5\cdot 1 - 4 \\ &= 2 - 3 + 5 - 4 \\ &= 0 \end{aligned}$$

110. If $x = 1$,

$$\begin{aligned} 4x^3 + 3x^2 - x + 2 &= 4\cdot 1^3 + 3\cdot 1^2 - 1 + 2 \\ &= 4 + 3 - 1 + 2 \\ &= 8 \end{aligned}$$

If $x = 2$,

$$\begin{aligned} 4x^3 + 3x^2 - x + 2 &= 4\cdot 2^3 + 3\cdot 2^2 - 2 + 2 \\ &= 32 + 12 - 2 + 2 \\ &= 44 \end{aligned}$$

111. $\dfrac{(666)^4}{(222)^4} = \left(\dfrac{666}{222}\right)^4 = 3^4 = 81$

112.
$$\begin{aligned} (0.1)^3(20)^3 &= \left(\frac{1}{10}\right)^3 \cdot (2\cdot 10)^3 \\ &= \frac{1}{10^3}\cdot 2^3 \cdot 10^3 \\ &= 2^3 = 8 \end{aligned}$$

113. $(8.2)^6 \approx 304{,}006.671$

114. $(3.7)^5 \approx 693.440$

115. $(6.1)^{-3} \approx 0.004$

116. $(2.2)^{-5} \approx 0.019$

117. $(-2.8)^6 \approx 481.890$

118. $-(2.8)^6 \approx -481.890$

119. $(-8.11)^{-4} \approx 0.000$

120. $-(8.11)^{-4} \approx -0.000$

121. $454.2 = 4.542\times 10^2$

122. $32.14 = 3.214\times 10^1$

123. $0.013 = 1.3\times 10^{-2}$

124. $0.00421 = 4.21\times 10^{-3}$

125. $32{,}155 = 3.2155\times 10^4$

126. $21{,}210 = 2.121\times 10^4$

127. $0.000423 = 4.23\times 10^{-4}$

128. $0.0514 = 5.14\times 10^{-2}$

129. $6.15\times 10^4 = 61{,}500$

130. $9.7\times 10^3 = 9700$

131. $1.214\times 10^{-3} = 0.001214$

132. $9.88\times 10^{-4} = 0.000988$

133. $1.1\times 10^8 = 110{,}000{,}000$

134. $4.112\times 10^2 = 411.2$

135. $8.1\times 10^{-2} = 0.081$

136. $6.453\times 10^{-1} = 0.6453$

137. $A = lw$

138. $P = 2(l + w)$

139. $C = \pi d$

140. $A = \dfrac{1}{2}bh$

141. $A = \dfrac{\sqrt{3}}{4}x^2$

142. $P = 3x$

143. $V = \dfrac{4}{3}\pi r^3$

144. $S = 4\pi r^2$

145. $V = x^3$

146. $S = 6x^2$

147. **a.** If $x = 1000$,
$$\begin{aligned} C &= 4000 + 2x \\ &= 4000 + 2(1000) \\ &= 4000 + 2000 \\ &= \$6000 \end{aligned}$$
The cost of producing 1000 watches is \$6000.

b. If $x = 2000$,
$$\begin{aligned} C &= 4000 + 2x \\ &= 4000 + 2(2000) \\ &= 4000 + 4000 \\ &= \$8000 \end{aligned}$$
The cost of producing 2000 watches is \$8000.

148. $210 + 80 - 120 + 25 - 60 - 32 - 5 = \$98$
His balance at the end of the month was \$98.

149. We want the difference between x and 4 to be at least 6 units. Since we don't care whether the value for x is larger or smaller than 4, we take the absolute value of the difference. We want the inequality to be non-strict since we are dealing with an 'at least' situation. Thus, we have
$|x-4| \geq 6$

150. We want the difference between x and 2 to be more than 5 units. Since we don't care whether the value for x is larger or smaller than 2, we take the absolute value of the difference. We want the inequality to be strict since we are dealing with a 'more than' situation. Thus, we have
$|x-2| > 5$

151. **a.** $|x-110| = |108-110| = |-2| = 2 \leq 5$
108 volts is acceptable.

b. $|x-110| = |104-110| = |-6| = 6 > 5$
104 volts is *not* acceptable.

152. **a.** $|x-220| = |214-220| = |-6| = 6 \leq 8$
214 volts is acceptable.

b. $|x-220| = |209-220| = |-11| = 11 > 8$
209 volts is *not* acceptable.

153. **a.**
$$\begin{aligned} |x-3| &= |2.999-3| \\ &= |-0.001| \\ &= 0.001 \leq 0.01 \end{aligned}$$
A radius of 2.999 centimeters is acceptable.

b.
$$\begin{aligned} |x-3| &= |2.89-3| \\ &= |-0.11| \\ &= 0.11 \not\leq 0.01 \end{aligned}$$
A radius of 2.89 centimeters is *not* acceptable.

154. **a.**
$$\begin{aligned} |x-98.6| &= |97-98.6| \\ &= |-1.6| \\ &= 1.6 \geq 1.5 \end{aligned}$$
97˚F is unhealthy.

b.
$$\begin{aligned} |x-98.6| &= |100-98.6| \\ &= |1.4| \\ &= 1.4 < 1.5 \end{aligned}$$
100˚F is *not* unhealthy.

155. The distance from Earth to the Moon is about $4 \times 10^8 = 400{,}000{,}000$ meters.

156. The height of Mt. Everest is about $8848 = 8.848 \times 10^3$ meters.

157. The wavelength of visible light is about $5 \times 10^{-7} = 0.0000005$ meters.

158. The diameter of an atom is about $1 \times 10^{-10} = 0.0000000001$ meters.

159. The diameter is about $0.0403 = 4.03 \times 10^{-2}$ inches.

160. The tiniest motor is less than $0.00004 = 4 \times 10^{-5}$ millimeters tall.

161. $186,000 \cdot 60 \cdot 60 \cdot 24 \cdot 365$
$= \left(1.86 \times 10^5\right)\left(6 \times 10^1\right)^2 \left(2.4 \times 10^1\right)\left(3.65 \times 10^2\right)$
$= 586.5696 \times 10^{10} = 5.865696 \times 10^{12}$
There are about 5.9×10^{12} miles in one light-year.

162. $\dfrac{93,000,000}{186,000} = \dfrac{9.3 \times 10^7}{1.86 \times 10^5} = 5 \times 10^2$
$= 500$ seconds ≈ 8 min. 20 sec.
It takes about 8 minutes 20 seconds for a beam of light to reach Earth from the Sun.

163. $\frac{1}{3} = 0.333333 \ldots > 0.333$
$\frac{1}{3}$ is larger by approximately $0.0003333 \ldots$

164. $\frac{2}{3} = 0.666666 \ldots > 0.666$
$\frac{2}{3}$ is larger by approximately $0.000666 \ldots$

165. $\left(5.24 \times 10^6\right)\left(6.5 \times 10^{13}\right) = \left(5.24 \times 6.5\right)\left(10^6 \times 10^{13}\right)$
$34.06 \times 10^{19} = 3.406 \times 10^{20}$

166. $\dfrac{1.62 \times 10^{-4}}{4.5 \times 10^{-10}} = \dfrac{1.62}{4.5} \times \dfrac{10^{-4}}{10^{-10}} = 0.36 \times 10^6$
$= 3.6 \times 10^5$

167. No. For any positive number a, the value $\frac{a}{2}$ is smaller and therefore closer to 0.

168. We are given that $1 < x^2 < 10$. This implies that $1 < x < \sqrt{10}$. Since $x < \sqrt{10} \approx 3.162$ and $x > \pi \approx 3.142$, the number could be 3.15 or 3.16 (which are between 1 and 10 as required). The number could also be 3.14 since numbers such as 3.146 which lie between π and $\sqrt{10}$ would equal 3.14 when truncated to two decimal places.

169. Answers will vary.

170. Answers will vary.
5 < 8 is a true statement because 5 is further to the left than 8 on a real number line.

Section R.3

1. right; hypotenuse

2. $A = \frac{1}{2}bh$

3. $C = 2\pi r$

4. similar

5. c

6. b

7. True.

8. True. $6^2 + 8^2 = 36 + 64 = 100 = 10^2$

9. False; the surface area of a sphere of radius r is given by $V = 4\pi r^2$.

10. True. The lengths of the corresponding sides are equal.

11. True. Two corresponding angles are equal.

12. False. The sides are not proportional.

13. $a = 5,\ b = 12,$
$c^2 = a^2 + b^2$
$= 5^2 + 12^2$
$= 25 + 144$
$= 169 \Rightarrow c = 13$

14. $a = 6,\ b = 8,$
$c^2 = a^2 + b^2$
$= 6^2 + 8^2$
$= 36 + 64$
$= 100 \Rightarrow c = 10$

15. $a = 10,\ b = 24,$
$c^2 = a^2 + b^2$
$= 10^2 + 24^2$
$= 100 + 576$
$= 676 \Rightarrow c = 26$

16. $a = 4,\ b = 3,$
$c^2 = a^2 + b^2$
$= 4^2 + 3^2$
$= 16 + 9$
$= 25 \Rightarrow c = 5$

17. $a = 7,\ b = 24,$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 7^2 + 24^2 \\ &= 49 + 576 \\ &= 625 \ \Rightarrow\ c = 25 \end{aligned}$$

18. $a = 14,\ b = 48,$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 14^2 + 48^2 \\ &= 196 + 2304 \\ &= 2500 \ \Rightarrow\ c = 50 \end{aligned}$$

19. $5^2 = 3^2 + 4^2$

$25 = 9 + 16$

$25 = 25$

The given triangle is a right triangle. The hypotenuse is 5.

20. $10^2 = 6^2 + 8^2$

$100 = 36 + 64$

$100 = 100$

The given triangle is a right triangle. The hypotenuse is 10.

21. $6^2 = 4^2 + 5^2$

$36 = 16 + 25$

$36 = 41$ false

The given triangle is not a right triangle.

22. $3^2 = 2^2 + 2^2$

$9 = 4 + 4$

$9 = 8$ false

The given triangle is not a right triangle.

23. $25^2 = 7^2 + 24^2$

$625 = 49 + 576$

$625 = 625$

The given triangle is a right triangle. The hypotenuse is 25.

24. $26^2 = 10^2 + 24^2$

$676 = 100 + 576$

$676 = 676$

The given triangle is a right triangle. The hypotenuse is 26.

25. $6^2 = 3^2 + 4^2$

$36 = 9 + 16$

$36 = 25$ false

The given triangle is not a right triangle.

26. $7^2 = 5^2 + 4^2$

$49 = 25 + 16$

$49 = 41$ false

The given triangle is not a right triangle.

27. $A = l \cdot w = 6 \cdot 7 = 42 \text{ in}^2$

28. $A = l \cdot w = 9 \cdot 4 = 36 \text{ cm}^2$

29. $A = \frac{1}{2} b \cdot h = \frac{1}{2}(14)(4) = 28 \text{ in}^2$

30. $A = \frac{1}{2} b \cdot h = \frac{1}{2}(4)(9) = 18 \text{ cm}^2$

31. $A = \pi r^2 = \pi(5)^2 = 25\pi \text{ m}^2$

$C = 2\pi r = 2\pi(5) = 10\pi \text{ m}$

32. $A = \pi r^2 = \pi(2)^2 = 4\pi \text{ ft}^2$

$C = 2\pi r = 2\pi(2) = 4\pi \text{ ft}$

33. $V = lwh = 6 \cdot 8 \cdot 5 = 240 \text{ ft}^3$

$$\begin{aligned} S &= 2lw + 2lh + 2wh \\ &= 2(6)(8) + 2(6)(5) + 2(8)(5) \\ &= 96 + 60 + 80 \\ &= 236 \text{ ft}^2 \end{aligned}$$

34. $V = lwh = 9 \cdot 4 \cdot 8 = 288 \text{ in}^3$

$$\begin{aligned} S &= 2lw + 2lh + 2wh \\ &= 2(9)(4) + 2(9)(8) + 2(4)(8) \\ &= 72 + 144 + 64 \\ &= 280 \text{ in}^2 \end{aligned}$$

35. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 5^3 = \frac{500}{3}\pi \text{ cm}^3$

$S = 4\pi r^2 = 4\pi \cdot 5^2 = 100\pi \text{ cm}^2$

36. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 3^3 = 36\pi \text{ ft}^3$

$S = 4\pi r^2 = 4\pi \cdot 3^2 = 36\pi \text{ ft}^2$

37. $V = \pi r^2 h = \pi(9)^2(8) = 648\pi \text{ in}^3$

$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(9)^2 + 2\pi(9)(8) \\ &= 162\pi + 144\pi \\ &= 306\pi \text{ in}^2 \end{aligned}$$

38. $V = \pi r^2 h = \pi(8)^2(9) = 576\pi \text{ in}^3$

$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(8)^2 + 2\pi(8)(9) \\ &= 128\pi + 144\pi \\ &= 272\pi \text{ in}^2 \end{aligned}$$

39. The diameter of the circle is 2, so its radius is 1.
$A = \pi r^2 = \pi(1)^2 = \pi$ square units

40. The diameter of the circle is 2, so its radius is 1.
$A = 2^2 - \pi(1)^2 = 4 - \pi$ square units

41. The diameter of the circle is the length of the diagonal of the square.

$$\begin{aligned} d^2 &= 2^2 + 2^2 \\ &= 4 + 4 \\ &= 8 \end{aligned}$$

$$d = \sqrt{8} = 2\sqrt{2}$$

$$r = \frac{d}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

The area of the circle is:

$$A = \pi r^2 = \pi\left(\sqrt{2}\right)^2 = 2\pi \text{ square units}$$

42. The diameter of the circle is the length of the diagonal of the square.

$$\begin{aligned} d^2 &= 2^2 + 2^2 \\ &= 4 + 4 \\ &= 8 \end{aligned}$$

$$d = \sqrt{8} = 2\sqrt{2}$$

$$r = \frac{d}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

The area is:

$$A = \pi\left(\sqrt{2}\right)^2 - 2^2 = 2\pi - 4 \text{ square units}$$

43. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{8}{4} = \frac{x}{2}$$

$$\frac{8 \cdot 2}{4} = x$$

$$4 = x$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A = 90°$, $B = 60°$, and $C = 30°$.

44. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{6}{12} = \frac{x}{16}$$

$$\frac{6 \cdot 16}{12} = x$$

$$8 = x$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A = 30°$, $B = 75°$, and $C = 75°$.

45. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{30}{20} = \frac{x}{45}$$

$$\frac{30 \cdot 45}{20} = x$$

$$\frac{135}{2} = x \quad \text{or} \quad x = 67.5$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A = 60°$, $B = 95°$, and $C = 25°$.

46. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{8}{10} = \frac{x}{50}$$

$$\frac{8 \cdot 50}{10} = x$$

$$40 = x$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A = 50°$, $B = 125°$, and $C = 5°$.

47. The total distance traveled is 4 times the circumference of the wheel.

$$\text{Total Distance} = 4C = 4(\pi d) = 4\pi \cdot 16$$
$$= 64\pi \approx 201.1 \text{ inches} \approx 16.8 \text{ feet}$$

48. The distance traveled in one revolution is the circumference of the disk 4π.
The number of revolutions =

$$\frac{\text{dist. traveled}}{\text{circumference}} = \frac{20}{4\pi} = \frac{5}{\pi} \approx 1.6 \text{ revolutions}$$

49. Area of the border = area of EFGH – area of ABCD $= 10^2 - 6^2 = 100 - 36 = 64 \text{ ft}^2$

50. FG = 4 feet; BG = 4 feet and BC = 10 feet, so CG= 6 feet. The area of the triangle CGF is:

$$A = \frac{1}{2} \cdot (4)(6) = 12 \text{ ft}^2$$

51. Area of the window = area of the rectangle + area of the semicircle.

$$A = (6)(4) + \frac{1}{2} \cdot \pi \cdot 2^2 = 24 + 2\pi \approx 30.28 \text{ ft}^2$$

Perimeter of the window = 2 heights + width + one-half the circumference.

$$P = 2(6) + 4 + \frac{1}{2} \cdot \pi(4) = 12 + 4 + 2\pi$$
$$= 16 + 2\pi \approx 22.28 \text{ feet}$$

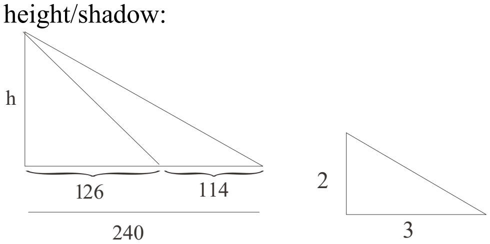
52. Area of the deck = area of the pool and deck – area of the pool.

$$A = \pi(13)^2 - \pi(10)^2 = 169\pi - 100\pi$$
$$= 69\pi \text{ ft}^2 \approx 216.77 \text{ ft}^2$$

The amount of fence is the circumference of the circle with radius 13 feet.

$$C = 2\pi(13) = 26\pi \text{ ft} \approx 81.68 \text{ ft}$$

53. We can form similar triangles using the Great Pyramid's height/shadow and Thales' height/shadow:



This allows us to write

$$\frac{h}{240} = \frac{2}{3}$$
$$h = \frac{2 \cdot 240}{3} = 160$$

The height of the Great Pyramid is 160 paces.

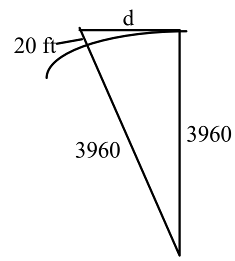
54. Let x = the approximate distance from San Juan to Hamilton and y = the approximate distance from Hamilton to Fort Lauderdale. Using similar triangles, we get

$$\frac{1046}{58} = \frac{x}{53.5} \qquad \frac{1046}{58} = \frac{y}{57}$$
$$\frac{1046 \cdot 53.5}{58} = x \qquad \frac{1046 \cdot 57}{58} = y$$
$$964.8 \approx x \qquad 1028.0 \approx y$$

The approximate distance between San Juan and Hamilton is 965 miles and the approximate distance between Hamilton and Fort Lauderdale is 1028 miles.

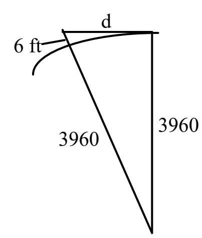
55. Convert 20 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$20 \text{ feet} = 20 \text{ feet} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.003788 \text{ miles}$$
$$d^2 = (3960 + 0.003788)^2 - 3960^2 = 30 \text{ sq. miles}$$
$$d \approx 5.477 \text{ miles}$$



56. Convert 6 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$6 \text{ feet} = 6 \text{ feet} \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.001136 \text{ miles}$$
$$d^2 = (3960 + 0.001136)^2 - 3960^2 = 9 \text{ sq. miles}$$
$$d \approx 3 \text{ miles}$$



57. Convert 100 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$100 \text{ feet } = 100 \text{ feet } \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.018939 \text{ miles}$$

$$d^2 = (3960 + 0.018939)^2 - 3960^2 \approx 150 \text{ sq. miles}$$

$$d \approx 12.2 \text{ miles}$$

Convert 150 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$150 \text{ feet } = 150 \text{ feet } \cdot \frac{1 \text{ mile}}{5280 \text{ feet}} = 0.028409 \text{ miles}$$

$$d^2 = (3960 + 0.028409)^2 - 3960^2 \approx 225 \text{ sq. miles}$$

$$d \approx 15.0 \text{ miles}$$

58. Given $m > 0$, $n > 0$ and $m > n$,

if $a = m^2 - n^2, b = 2mn$ and $c = m^2 + n^2$, then

$$\begin{aligned} a^2 + b^2 &= \left(m^2 - n^2\right)^2 + \left(2mn\right)^2 \\ &= m^4 - 2m^2n^2 + n^4 + 4m^2n^2 \\ &= m^4 + 2m^2n^2 + n^4 \end{aligned}$$

and $c^2 = \left(m^2 + n^2\right)^2 = m^4 + 2m^2n^2 + n^4$

$\therefore a^2 + b^2 = c^2 \rightarrow a, b$ and c represent the sides of a right triangle.

59.
$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(10)^2(4.5) \\ &= 450\pi \text{ ft}^3 \end{aligned}$$

So, $1 \text{ft}^3 \approx 7.48052$ gal so

$$\left(450\pi \text{ ft}^3\right)\left(7.48052 \text{ gal/ft}^3\right) \approx 10,575 \text{ gal}$$

60. $10000(5.61458) = 56145.8 \text{ ft}^3$

$$\begin{aligned} V &= \pi r^2 h \\ 56145.8 &= \pi(25)^2 h \\ h &= \frac{56145.8}{625\pi} = 28.6 \text{ ft} \end{aligned}$$

61.
$$\begin{aligned} A &= \pi r^2 \\ A_2 &= \pi(2r)^2 \\ &= \pi 4r^2 \\ &= 4\pi r^2 = 4A \end{aligned}$$

If you double the radius, the area is four times the original area.

62.
$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ V_2 &= \frac{4}{3}\pi(2r)^3 \\ &= \frac{4}{3}\pi \cdot 8r^3 \\ &= 8 \cdot \frac{4}{3}\pi r^3 = 8V \end{aligned}$$

If you double the radius the volume is 8 times the original volume.

63. Let l = length of the rectangle and w = width of the rectangle.
Notice that

$$\begin{aligned} &(l + w)^2 - (l - w)^2 \\ &= [(l + w) + (l - w)][(l + w) - (l - w)] \\ &= (2l)(2w) = 4lw = 4A \end{aligned}$$

So $A = \frac{1}{4}[(l + w)^2 - (l - w)^2]$

Since $(l - w)^2 \geq 0$, the largest area will occur when $l - w = 0$ or $l = w$; that is, when the rectangle is a square. But

$$\begin{aligned} 1000 &= 2l + 2w = 2(l + w) \\ 500 &= l + w = 2l \\ 250 &= l = w \end{aligned}$$

The largest possible area is $250^2 = 62500$ sq ft.
A circular pool with circumference = 1000 feet yields the equation: $2\pi r = 1000 \Rightarrow r = \dfrac{500}{\pi}$

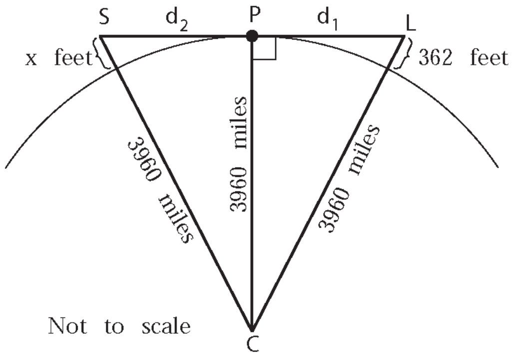
The area enclosed by the circular pool is:

$$A = \pi r^2 = \pi\left(\frac{500}{\pi}\right)^2 = \frac{500^2}{\pi} \approx 79577.47 \text{ ft}^2$$

Thus, a circular pool will enclose the most area.

64. Consider the diagram showing the lighthouse at point L, relative to the center of Earth, using the radius of Earth as 3960 miles. Let P refer to the furthest point on the horizon from which the light is visible. Note also that

$$362 \text{ feet} = \frac{362}{5280} \text{ miles.}$$



Apply the Pythagorean Theorem to ΔCPL :

$$(3960)^2 + (d_1)^2 = \left(3960 + \frac{362}{5280}\right)^2$$

$$(d_1)^2 = \left(3960 + \frac{362}{5280}\right)^2 - (3960)^2$$

$$d_1 = \sqrt{\left(3960 + \frac{362}{5280}\right)^2 - (3960)^2} \approx 23.30 \text{ mi.}$$

Therefore, the light from the lighthouse can be seen at point P on the horizon, where point P is approximately 23.30 miles away from the lighthouse. Brochure information is slightly overstated.

<u>Verify the ship information:</u>
Let S refer to the ship's location, and let x equal the height, in feet, of the ship.
We need $d_1 + d_2 \geq 40$.
Since $d_1 \approx 23.30$ miles we need
$d_2 \geq 40 - 23.30 = 16.70$ miles.

Apply the Pythagorean Theorem to ΔCPS :

$$(3960)^2 + (16.7)^2 = (3960 + x)^2$$

$$\sqrt{(3960)^2 + (16.7)^2} = 3960 + x$$

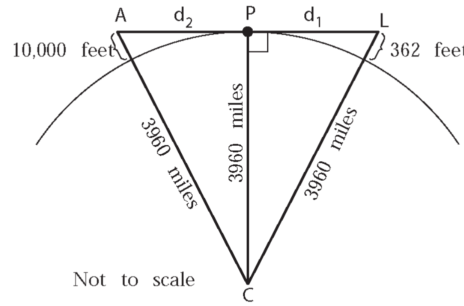
$$\sqrt{(3960)^2 + (16.7)^2} - 3960 = x$$

$$x \approx 0.035 \text{ miles}$$

$$x \approx 185.93 \text{ feet.}$$

The ship would have to be at least 186 feet tall to see the lighthouse from 40 miles away.

<u>Verify the airplane information:</u>



Let A refer to the airplane's location. The distance from the plane to point P is d_2.

We want to show that $d_1 + d_2 \geq 120$.

Assume the altitude of the airplane is

$10{,}000 \text{ feet} = \dfrac{10000}{5280}$ miles.

Apply the Pythagorean Theorem to ΔCPA :

$$(3960)^2 + (d_2)^2 = \left(3960 + \frac{10000}{5280}\right)^2$$

$$(d_2)^2 = \left(3960 + \frac{10000}{5280}\right)^2 - (3960)^2$$

$$d_2 = \sqrt{\left(3960 + \frac{10000}{5280}\right)^2 - (3960)^2} \text{ 61Therefo}$$

$$\approx 122.49 \text{ miles.}$$

re, $d_1 + d_2 \approx 23.30 + 122.49 = 145.79 \geq 120$.

The brochure information is slightly understated. Note that a plane at an altitude of 6233 feet could see the lighthouse from 120 miles away.

Section R.4

1. 4; 3
2. $x^4 - 16$
3. $x^3 - 8$
4. a
5. c
6. False; monomials cannot have negative degrees.
7. True
8. False; the dividend = (quotient)(divisor) + remainder

9. $2x^3$ Monomial; Variable: x; Coefficient: 2; Degree: 3

10. $-4x^2$ Monomial; Variable: x; Coefficient: –4; Degree: 2

11. $\frac{8}{x}=8x^{-1}$ Not a monomial; when written in the form ax^k, the variable has a negative exponent.

12. $-2x^{-3}$ Not a monomial; when written in the form ax^k, the variable has a negative exponent.

13. $-2xy^2$ Monomial; Variables: x, y; Coefficient: –2; Degree: 3

14. $5x^2y^3$ Monomial; Variables: x, y; Coefficient: 5; Degree: 5

15. $\frac{8x}{y}=8xy^{-1}$ Not a monomial; when written in the form $ax^n y^m$, the exponent on the variable y is negative.

16. $-\frac{2x^2}{y^3}=-2x^2y^{-3}$ Not a monomial; when written in the form $ax^n y^m$, the exponent on the variable y is negative.

17. x^2+y^2 Not a monomial; the expression contains more than one term. This expression is a binomial.

18. $3x^2+4$ Not a monomial; the expression contains more than one term. This expression is a binomial.

19. $3x^2-5$ Polynomial; Degree: 2

20. $1-4x$ Polynomial; Degree: 1

21. 5 Polynomial; Degree: 0

22. $-\pi$ Polynomial; Degree: 0

23. $3x^2-\frac{5}{x}$ Not a polynomial; the variable in the denominator results in an exponent that is not a nonnegative integer.

24. $\frac{3}{x}+2$ Not a polynomial; the variable in the denominator results in an exponent that is not a nonnegative integer.

25. $2y^3-\sqrt{2}$ Polynomial; Degree: 3

26. $10z^2+z$ Polynomial; Degree: 2

27. $\frac{x^2+5}{x^3-1}$ Not a polynomial; the polynomial in the denominator has a degree greater than 0.

28. $\frac{3x^3+2x-1}{x^2+x+1}$ Not a polynomial; the polynomial in the denominator has a degree greater than 0.

29.
$$\begin{aligned}&(x^2+6x+8)+(3x^2-4x+7)\\&=(x^2+3x^2)+(6x-4x)+(8+7)\\&=4x^2+2x+15\end{aligned}$$

30.
$$\begin{aligned}&(x^3+3x^2+2)+(x^2-4x+4)\\&=x^3+(3x^2+x^2)+(-4x)+(2+4)\\&=x^3+4x^2-4x+6\end{aligned}$$

31.
$$\begin{aligned}&(x^3-2x^2+5x+10)-(2x^2-4x+3)\\&=x^3-2x^2+5x+10-2x^2+4x-3\\&=x^3+(-2x^2-2x^2)+(5x+4x)+(10-3)\\&=x^3-4x^2+9x+7\end{aligned}$$

32.
$$\begin{aligned}&(x^2-3x-4)-(x^3-3x^2+x+5)\\&=x^2-3x-4-x^3+3x^2-x-5\\&=-x^3+(x^2+3x^2)+(-3x-x)+(-4-5)\\&=-x^3+4x^2-4x-9\end{aligned}$$

33.
$$\begin{aligned}&\left(6x^5+x^3+x\right)+\left(5x^4-x^3+3x^2\right)\\&=6x^5+5x^4+3x^2+x\end{aligned}$$

34.
$$\begin{aligned}&\left(10x^5-8x^2\right)+\left(3x^3-2x^2+6\right)\\&=10x^5+3x^3-10x^2+6\end{aligned}$$

35.
$$\begin{aligned}&(x^2-6x+4)+3(2x^2+x-5)\\&=x^2-6x+4+6x^2+3x-15\\&=7x^2-3x-11\end{aligned}$$

36. $-2(x^2+x+1)+(-5x^2-x+2)$
$=-2x^2-2x-2-5x^2-x+2$
$=-7x^2-3x$

37. $6(x^3+x^2-3)-4(2x^3-3x^2)$
$=6x^3+6x^2-18-8x^3+12x^2$
$=-2x^3+18x^2-18$

38. $8(4x^3-3x^2-1)-6(4x^3+8x-2)$
$=32x^3-24x^2-8-24x^3-48x+12$
$=8x^3-24x^2-48x+4$

39. $\left(x^2-x+2\right)+\left(2x^2-3x+5\right)-\left(x^2+1\right)$
$=x^2-x+2+2x^2-3x+5-x^2-1$
$=2x^2-4x+6$

40. $\left(x^2+1\right)-\left(4x^2+5\right)+\left(x^2+x-2\right)$
$=x^2+1-4x^2-5+x^2+x-2$
$=-2x^2+x-6$

41. $7\left(y^2-5y+3\right)-4\left(3-y^2\right)$
$=7y^2-35y+21-12+4y^2$
$=11y^2-35y+9$

42. $8\left(1-y^3\right)+4\left(1+y+y^2+y^3\right)$
$=8-8y^3+4+4y+4y^2+4y^3$
$=-4y^3+4y^2+4y+12$

43. $x^2(x^2+2x-5)=x^4+2x^3-5x^2$

44. $4x^2(x^3-x+2)=4x^5-4x^3+8x^2$

45. $-2x^2(4x^3+5)=-8x^5-10x^2$

46. $5x^3(3x-4)=15x^4-20x^3$

47. $(x+1)(x^2+2x-4)$
$=x(x^2+2x-4)+1(x^2+2x-4)$
$=x^3+2x^2-4x+x^2+2x-4$
$=x^3+3x^2-2x-4$

48. $(2x-3)(x^2+x+1)$
$=2x(x^2+x+1)-3(x^2+x+1)$
$=2x^3+2x^2+2x-3x^2-3x-3$
$=2x^3-x^2-x-3$

49. $(x+2)(x+4)=x^2+4x+2x+8$
$=x^2+6x+8$

50. $(x+3)(x+5)=x^2+5x+3x+15$
$=x^2+8x+15$

51. $(2x+7)(x+5)=2x^2+7x+10x+35$
$=2x^2+17x+35$

52. $(3x+1)(2x+1)=6x^2+3x+2x+1$
$=6x^2+5x+1$

53. $(x-4)(x+2)=x^2+2x-4x-8$
$=x^2-2x-8$

54. $(x+4)(x-2)=x^2-2x+4x-8$
$=x^2+2x-8$

55. $(x-6)(x-3)=x^2-6x-3x+18$
$=x^2-9x+18$

56. $(x-5)(x-1)=x^2-x-5x+5$
$=x^2-6x+5$

57. $(2x+3)(x-2)=2x^2-4x+3x-6$
$=2x^2-x-6$

58. $(2x-4)(3x+1)=6x^2+2x-12x-4$
$=6x^2-10x-4$

59. $(-3x+4)(x-2)=-3x^2+4x+6x-8$
$=-3x^2+10x-8$

60. $(-3x-1)(x+1)=-3x^2-3x-x-1$
$=-3x^2-4x-1$

61. $(-x-5)(-2x-7)=2x^2+10x+7x+35$
$=2x^2+17x+35$

62. $(-2x-3)(3-x) = -6x+2x^2-9+3x$
$= 2x^2-3x-9$

63. $(x-2y)(x+y) = x^2+xy-2xy-2y^2$
$= x^2-xy-2y^2$

64. $(2x+3y)(x-y) = 2x^2-2xy+3xy-3y^2$
$= 2x^2+xy-3y^2$

65. $(-2x-3y)(3x+2y) = -6x^2-4xy-9xy-6y^2$
$= -6x^2-13xy-6y^2$

66. $(x-3y)(-2x+y) = -2x^2+xy+6xy-3y^2$
$= -2x^2+7xy-3y^2$

67. $(x-7)(x+7) = x^2-7^2 = x^2-49$

68. $(x-1)(x+1) = x^2-1^2 = x^2-1$

69. $(2x+3)(2x-3) = (2x)^2-3^2 = 4x^2-9$

70. $(3x+2)(3x-2) = (3x)^2-2^2 = 9x^2-4$

71. $(x+4)^2 = x^2+2\cdot x\cdot 4+4^2 = x^2+8x+16$

72. $(x+5)^2 = x^2+2\cdot x\cdot 5+5^2 = x^2+10x+25$

73. $(x-4)^2 = x^2-2\cdot x\cdot 4+4^2 = x^2-8x+16$

74. $(x-5)^2 = x^2-2\cdot x\cdot 5+5^2 = x^2-10x+25$

75. $(3x+4)(3x-4) = (3x)^2-4^2 = 9x^2-16$

76. $(5x-3)(5x+3) = (5x)^2-3^2 = 25x^2-9$

77. $(2x-3)^2 = (2x)^2-2(2x)(3)+3^2$
$= 4x^2-12x+9$

78. $(3x-4)^2 = (3x)^2-2(3x)(4)+4^2$
$= 9x^2-24x+16$

79. $(x+y)(x-y) = (x)^2-(y)^2 = x^2-y^2$

80. $(x+3y)(x-3y) = (x)^2-(3y)^2 = x^2-9y^2$

81. $(3x+y)(3x-y) = (3x)^2-(y)^2 = 9x^2-y^2$

82. $(3x+4y)(3x-4y) = (3x)^2-(4y)^2 = 9x^2-16y^2$

83. $(x+y)^2 = x^2+2xy+y^2$

84. $(x-y)^2 = x^2-2xy+y^2$

85. $(x-2y)^2 = x^2+2\left(x\cdot(-2y)\right)+(2y)^2$
$= x^2-4xy+4y^2$

86. $(2x+3y)^2 = (2x)^2+2(2x\cdot 3y)+(3y)^2$
$= 4x^2+12xy+9y^2$

87. $(x-2)^3 = x^3-3\cdot x^2\cdot 2+3\cdot x\cdot 2^2-2^3$
$= x^3-6x^2+12x-8$

88. $(x+1)^3 = x^3+3\cdot x^2\cdot 1+3\cdot x\cdot 1^2+1^3$
$= x^3+3x^2+3x+1$

89. $(2x+1)^3 = (2x)^3+3(2x)^2(1)+3(2x)\cdot 1^2+1^3$
$= 8x^3+12x^2+6x+1$

90. $(3x-2)^3 = (3x)^3-3(3x)^2(2)+3(3x)\cdot 2^2-2^3$
$= 27x^3-54x^2+36x-8$

91.
$$
\begin{array}{r}
4x^2-11x+23 \\
x+2\overline{\big)\,4x^3-3x^2+x+1} \\
\underline{4x^3+8x^2}\phantom{+x+1} \\
-11x^2+x\phantom{+1} \\
\underline{-11x^2-22x}\phantom{+1} \\
23x+1 \\
\underline{23x+46} \\
-45
\end{array}
$$

Check:
$(x+2)(4x^2-11x+23)+(-45)$
$= 4x^3-11x^2+23x+8x^2-22x+46-45$
$= 4x^3-3x^2+x+1$

The quotient is $4x^2-11x+23$; the remainder is –45.

92.
$$\begin{array}{r} 3x^2-7x+15 \\ x+2\overline{\big)3x^3-x^2+x-2} \\ \underline{3x^3+6x^2} \\ -7x^2+x \\ \underline{-7x^2-14x} \\ 15x-2 \\ \underline{15x+30} \\ -32 \end{array}$$

Check:
$(x+2)(3x^2-7x+15)+(-32)$
$=3x^3-7x^2+15x+6x^2-14x+30-32$
$=3x^3-x^2+x-2$
The quotient is $3x^2-7x+15$; the remainder is –32.

93.
$$\begin{array}{r} 4x-3 \\ x^2\overline{\big)4x^3-3x^2+x+1} \\ \underline{4x^3} \\ -3x^2+x+1 \\ \underline{-3x^2} \\ x+1 \end{array}$$

Check:
$(x^2)(4x-3)+(x+1)=4x^3-3x^2+x+1$
The quotient is $4x-3$; the remainder is $x+1$.

94.
$$\begin{array}{r} 3x-1 \\ x^2\overline{\big)3x^3-x^2+x-2} \\ \underline{3x^3} \\ -x^2+x-2 \\ \underline{-x^2} \\ x-2 \end{array}$$

Check:
$(x^2)(3x-1)+(x-2)=3x^3-x^2+x-2$
The quotient is $3x-1$; the remainder is $x-2$.

95.
$$\begin{array}{r} 5x^2-13 \\ x^2+2\overline{\big)5x^4+0x^3-3x^2+x+1} \\ \underline{5x^4+10x^2} \\ -13x^2+x+1 \\ \underline{-13x^2-26} \\ x+27 \end{array}$$

Check:
$\left(x^2+2\right)\left(5x^2-13\right)+\left(x+27\right)$
$=5x^4+10x^2-13x^2-26+x+27$
$=5x^4-3x^2+x+1$
The quotient is $5x^2-13$; the remainder is $x+27$.

96.
$$\begin{array}{r} 5x^2-11 \\ x^2+2\overline{\big)5x^4+0x^3-x^2+x-2} \\ \underline{5x^4+10x^2} \\ -11x^2+x-2 \\ \underline{-11x^2-22} \\ x+20 \end{array}$$

Check:
$\left(x^2+2\right)\left(5x^2-11\right)+\left(x+20\right)$
$=5x^4+10x^2-11x^2-22+x+20$
$=5x^4-x^2+x-2$
The quotient is $5x^2-11$; the remainder is $x+20$.

97.
$$\begin{array}{r} 2x^2 \\ 2x^3-1\overline{\big)4x^5+0x^4+0x^3-3x^2+x+1} \\ \underline{4x^5-2x^2} \\ -x^2+x+1 \end{array}$$

Check:
$\left(2x^3-1\right)\left(2x^2\right)+\left(-x^2+x+1\right)$
$=4x^5-2x^2-x^2+x+1=4x^5-3x^2+x+1$
The quotient is $2x^2$; the remainder is $-x^2+x+1$.

98.
$$\begin{array}{r|l} & x^2 \\ \hline 3x^3-1 & 3x^5+0x^4+0x^3-x^2+x-2 \\ & \underline{3x^5 \qquad\qquad -x^2} \\ & \qquad\qquad\qquad x-2 \end{array}$$

Check:
$(3x^3-1)(x^2)+(x-2)=3x^5-x^2+x-2$

The quotient is x^2; the remainder is $x-2$.

99.
$$\begin{array}{r|l} & x^2-2x+\frac{1}{2} \\ \hline 2x^2+x+1 & 2x^4-3x^3+0x^2+x+1 \\ & \underline{2x^4+x^3+x^2} \\ & -4x^3-x^2+x \\ & \underline{-4x^3-2x^2-2x} \\ & x^2+3x+1 \\ & \underline{x^2+\frac{1}{2}x+\frac{1}{2}} \\ & \frac{5}{2}x+\frac{1}{2} \end{array}$$

Check:
$(2x^2+x+1)\left(x^2-2x+\frac{1}{2}\right)+\frac{5}{2}x+\frac{1}{2}$
$=2x^4-4x^3+x^2+x^3-2x^2+\frac{1}{2}x$
$\quad +x^2-2x+\frac{1}{2}+\frac{5}{2}x+\frac{1}{2}$
$=2x^4-3x^3+x+1$

The quotient is $x^2-2x+\frac{1}{2}$; the remainder is $\frac{5}{2}x+\frac{1}{2}$.

100.
$$\begin{array}{r|l} & x^2-\frac{2}{3}x-\frac{1}{9} \\ \hline 3x^2+x+1 & 3x^4-x^3+0x^2+x-2 \\ & \underline{3x^4+x^3+x^2} \\ & -2x^3-x^2+x \\ & \underline{-2x^3-\frac{2}{3}x^2-\frac{2}{3}x} \\ & -\frac{1}{3}x^2+\frac{5}{3}x-2 \\ & \underline{-\frac{1}{3}x^2-\frac{1}{9}x-\frac{1}{9}} \\ & \frac{16}{9}x-\frac{17}{9} \end{array}$$

Check:
$(3x^2+x+1)\left(x^2-\frac{2}{3}x-\frac{1}{9}\right)+\left(\frac{16}{9}x-\frac{17}{9}\right)$
$=3x^4+x^3+x^2-2x^3-\frac{2}{3}x^2-\frac{2}{3}x$
$\quad -\frac{1}{3}x^2-\frac{1}{9}x-\frac{1}{9}+\frac{16}{9}x-\frac{17}{9}$
$=3x^4-x^3+x-2$

The quotient is $x^2-\frac{2}{3}x-\frac{1}{9}$; the remainder is $\frac{16}{9}x-\frac{17}{9}$.

101.
$$\begin{array}{r|l} & -4x^2-3x-3 \\ \hline x-1 & -4x^3+x^2+0x-4 \\ & \underline{-4x^3+4x^2} \\ & -3x^2 \\ & \underline{-3x^2+3x} \\ & -3x-4 \\ & \underline{-3x+3} \\ & -7 \end{array}$$

Check:
$(x-1)(-4x^2-3x-3)+(-7)$
$=-4x^3-3x^2-3x+4x^2+3x+3-7$
$=-4x^3+x^2-4$

The quotient is $-4x^2-3x-3$; the remainder is -7.

102.
$$\begin{array}{r|l} & -3x^3-3x^2-3x-5 \\ \hline x-1 & -3x^4+0x^3+0x^2-2x-1 \\ & \underline{-3x^4+3x^3} \\ & -3x^3 \\ & \underline{-3x^3+3x^2} \\ & -3x^2-2x \\ & \underline{-3x^2+3x} \\ & -5x-1 \\ & \underline{-5x+5} \\ & -6 \end{array}$$

Check:

$(x-1)(-3x^3-3x^2-3x-5)+(-6)$

$=-3x^4-3x^3-3x^2-5x+3x^3+3x^2$

$\quad +3x+5-6$

$=-3x^4-2x-1$

The quotient is $-3x^3-3x^2-3x-5$; the remainder is –6.

103.

$$
\begin{array}{rl}
 & x^2-x-1 \\
x^2+x+1\,\big) & x^4+0x^3-x^2+0x+1 \\
 & \underline{x^4+x^3+x^2} \\
 & -x^3-2x^2 \\
 & \underline{-x^3-x^2-x} \\
 & -x^2+x+1 \\
 & \underline{-x^2-x-1} \\
 & 2x+2
\end{array}
$$

Check:

$(x^2+x+1)(x^2-x-1)+2x+2$

$=x^4+x^3+x^2-x^3-x^2-x-x^2-x$

$\quad -1+2x+2$

$=x^4-x^2+1$

The quotient is x^2-x-1; the remainder is $2x+2$.

104.

$$
\begin{array}{rl}
 & x^2+x-1 \\
x^2-x+1\,\big) & x^4+0x^3-x^2+0x+1 \\
 & \underline{x^4-x^3+x^2} \\
 & x^3-2x^2 \\
 & \underline{x^3-x^2+x} \\
 & -x^2-x+1 \\
 & \underline{-x^2+x-1} \\
 & -2x+2
\end{array}
$$

Check:

$(x^2-x+1)(x^2+x-1)+(-2x+2)$

$=x^4+x^3-x^2-x^3-x^2+x+x^2+x$

$\quad -1-2x+2$

$=x^4-x^2+1$

The quotient is x^2+x-1; the remainder is $-2x+2$.

105.

$$
\begin{array}{rl}
 & x^2+ax+a^2 \\
x-a\,\big) & x^3+0x^2+0x-a^3 \\
 & \underline{x^3-ax^2} \\
 & ax^2 \\
 & \underline{ax^2-a^2x} \\
 & a^2x-a^3 \\
 & \underline{a^2x-a^3} \\
 & 0
\end{array}
$$

Check:

$(x-a)(x^2+ax+a^2)+0$

$=x^3+ax^2+a^2x-ax^2-a^2x-a^3$

$=x^3-a^3$

The quotient is x^2+ax+a^2; the remainder is 0.

106.

$$
\begin{array}{rl}
 & x^4+ax^3+a^2x^2+a^3x+a^4 \\
x-a\,\big) & x^5+0x^4+0x^3+0x^2+0x-a^5 \\
 & \underline{x^5-ax^4} \\
 & ax^4 \\
 & \underline{ax^4-a^2x^3} \\
 & a^2x^3 \\
 & \underline{a^2x^3-a^3x^2} \\
 & a^3x^2 \\
 & \underline{a^3x^2-a^4x} \\
 & a^4x-a^5 \\
 & \underline{a^4x-a^5} \\
 & 0
\end{array}
$$

Check:

$(x-a)(x^4+ax^3+a^2x^2+a^3x+a^4)+0$
$=x^5+ax^4+a^2x^3+a^3x^2+a^4x-ax^4$
$\quad -a^2x^3-a^3x^2-a^4x-a^5$
$=x^5-a^5$

The quotient is $x^4+ax^3+a^2x^2+a^3x+a^4$; the remainder is 0.

107.
$$(3x-2k)(4x+3k)=12x^2+kx-96$$
$$12x^2-8kx+9kx-6k^2=12x^2+kx-96$$
$$kx-6k^2=kx-96$$
$$-6k^2=-96$$
$$k^2=16$$
$$k=\pm 4$$

108. The products $(x+y)(x-y)$ and $(z+w)(z-w)$ will each result in a binomial that is the difference of squares. The product of those resulting binomials will have 4 terms.

109. When we multiply polynomials $p_1(x)$ and $p_2(x)$, each term of $p_1(x)$ will be multiplied by each term of $p_2(x)$. So when the highest-powered term of $p_1(x)$ multiplies by the highest powered term of $p_2(x)$, the exponents on the variables in those terms will add according to the basic rules of exponents. Therefore, the highest powered term of the product polynomial will have degree equal to the sum of the degrees of $p_1(x)$ and $p_2(x)$.

110. When we add two polynomials $p_1(x)$ and $p_2(x)$, where the degree of $p_1(x)\neq$ the degree of $p_2(x)$, each term of $p_1(x)$ will be added to each term of $p_2(x)$. Since only the terms with equal degrees will combine via addition, the degree of the sum polynomial will be the degree of the highest powered term overall, that is, the degree of the polynomial that had the higher degree.

111. When we add two polynomials $p_1(x)$ and $p_2(x)$, where the degree of $p_1(x)=$ the degree of $p_2(x)$, the new polynomial will have degree $\leq$ the degree of $p_1(x)$ and $p_2(x)$.

112. Answers will vary.

113. Answers will vary.

Section R.5

1. $3x(x-2)(x+2)$

2. prime

3. c

4. b

5. d

6. c

7. True; x^2+4 is prime over the set of real numbers.

8. False; $3x^3-2x^2-6x+4=(3x-2)(x^2-2)$

9. $3x+6=3(x+2)$

10. $7x-14=7(x-2)$

11. $ax^2+a=a(x^2+1)$

12. $ax-a=a(x-1)$

13. $x^3+x^2+x=x(x^2+x+1)$

14. $x^3-x^2+x=x(x^2-x+1)$

15. $2x^2-2x=2x(x-1)$

16. $3x^2-3x=3x(x-1)$

17. $3x^2y-6xy^2+12xy=3xy(x-2y+4)$

18. $60x^2y-48xy^2+72x^3y=12xy(5x-4y+6x^2)$

19. $x^2-1=x^2-1^2=(x-1)(x+1)$

20. $x^2-4=x^2-2^2=(x-2)(x+2)$

21. $4x^2-1=(2x)^2-1^2=(2x-1)(2x+1)$

22. $9x^2-1=(3x)^2-1^2=(3x-1)(3x+1)$

23. $x^2-16=x^2-4^2=(x-4)(x+4)$

24. $x^2-25=x^2-5^2=(x-5)(x+5)$

25. $25x^2-4=(5x-2)(5x+2)$

26. $36x^2-9=9\left(4x^2-1\right)=9(2x-1)(2x+1)$

27. $x^2+2x+1=(x+1)^2$

28. $x^2-4x+4=(x-2)^2$

29. $x^2+4x+4=(x+2)^2$

30. $x^2-2x+1=(x-1)^2$

31. $x^2-10x+25=(x-5)^2$

32. $x^2+10x+25=(x+5)^2$

33. $4x^2+4x+1=(2x+1)^2$

34. $9x^2+6x+1=(3x+1)^2$

35. $16x^2+8x+1=(4x+1)^2$

36. $25x^2+10x+1=(5x+1)^2$

37. $x^3-27=x^3-3^3=(x-3)(x^2+3x+9)$

38. $x^3+125=x^3+5^3=(x+5)(x^2-5x+25)$

39. $x^3+27=x^3+3^3=(x+3)(x^2-3x+9)$

40. $$\begin{aligned}27-8x^3&=3^3-(2x)^3\\&=(3-2x)(9+6x+4x^2)\\&=-\left(2x-3\right)\left(4x^2+6x+9\right)\end{aligned}$$

41. $$\begin{aligned}8x^3+27&=(2x)^3+3^3\\&=(2x+3)(4x^2-6x+9)\end{aligned}$$

42. $$\begin{aligned}64-27x^3&=4^3-(3x)^3\\&=(4-3x)(16+12x+9x^2)\\&=-\left(3x-4\right)\left(9x^2+12x+16\right)\end{aligned}$$

43. $x^2+5x+6=(x+2)(x+3)$

44. $x^2+6x+8=(x+2)(x+4)$

45. $x^2+7x+6=(x+6)(x+1)$

46. $x^2+9x+8=(x+8)(x+1)$

47. $x^2+7x+10=(x+2)(x+5)$

48. $x^2+11x+10=(x+10)(x+1)$

49. $x^2-10x+16=(x-2)(x-8)$

50. $x^2-17x+16=(x-16)(x-1)$

51. $x^2-7x-8=(x+1)(x-8)$

52. $x^2-2x-8=(x+2)(x-4)$

53. $x^2+7x-8=(x+8)(x-1)$

54. $x^2+2x-8=(x+4)(x-2)$

55. $$\begin{aligned}2x^2+4x+3x+6&=2x(x+2)+3(x+2)\\&=(x+2)(2x+3)\end{aligned}$$

56. $$\begin{aligned}3x^2-3x+2x-2&=3x(x-1)+2(x-1)\\&=(x-1)(3x+2)\end{aligned}$$

57. $$\begin{aligned}5x^2-15x+x-3&=5x(x-3)+1(x-3)\\&=(x-3)(5x+1)\end{aligned}$$

58. $$\begin{aligned}3x^2+6x-x-2&=3x(x+2)-1(x+2)\\&=(x+2)(3x-1)\end{aligned}$$

59. $6x^2+21x+8x+28=3x(2x+7)+4(2x+7)$
$=(2x+7)(3x+4)$

60. $9x^2-6x+3x-2=3x(3x-2)+1(3x-2)$
$=(3x-2)(3x+1)$

61. $3x^2+4x+1=(3x+1)(x+1)$

62. $2x^2+3x+1=(2x+1)(x+1)$

63. $2z^2+9z+7=(2z+7)(z+1)$

64. $6z^2+5z+1=(3z+1)(2z+1)$

65. $5x^2+6x-8=(5x-4)(x+2)$

66. $3x^2+10x+8=(3x+4)(x+2)$

67. $5x^2-6x-8=(5x+4)(x-2)$

68. $3x^2-10x+8=(3x-4)(x-2)$

69. $5x^2+22x+8=(5x+2)(x+4)$

70. $3x^2-14x+8=(3x-2)(x-4)$

71. $5x^2+18x-8=(5x-2)(x+4)$

72. $3x^2-10x-8=(3x+2)(x-4)$

73. Since b is 10 then we need half of 10 squared to be the last term in our trinomial. Thus
$\frac{1}{2}(10)=5;\ (5)^2=25$
$x^2+10x+25=(x+5)^2$

74. Since b is 14 then we need half of 14 squared to be the last term in our trinomial. Thus
$\frac{1}{2}(14)=7;\ (7)^2=49$
$p^2+14p+49=(p+7)^2$

75. Since b is -6 then we need half of -6 squared to be the last term in our trinomial. Thus
$\frac{1}{2}(-6)=-3;\ (-3)^2=9$
$y^2-6y+9=(y-3)^2$

76. Since b is -4 then we need half of -4 squared to be the last term in our trinomial. Thus
$\frac{1}{2}(-4)=-2;\ (-2)^2=4$
$x^2-4x+4=(x-2)^2$

77. Since b is $-\frac{1}{2}$ then we need half of $-\frac{1}{2}$ squared to be the last term in our trinomial. Thus
$\frac{1}{2}(-\frac{1}{2})=-\frac{1}{4};\ (-\frac{1}{4})^2=\frac{1}{16}$
$x^2-\frac{1}{2}x+\frac{1}{16}=(x-\frac{1}{4})^2$

78. Since b is $\frac{1}{3}$ then we need half of $\frac{1}{3}$ squared to be the last term in our trinomial. Thus
$\frac{1}{2}(\frac{1}{3})=\frac{1}{6};\ (\frac{1}{6})^2=\frac{1}{36}$
$x^2+\frac{1}{3}x+\frac{1}{36}=(x+\frac{1}{6})^2$

79. $x^2-36=(x-6)(x+6)$

80. $x^2-9=(x-3)(x+3)$

81. $2-8x^2=2(1-4x^2)=2(1-2x)(1+2x)$

82. $3-27x^2=3(1-9x^2)=3(1-3x)(1+3x)$

83. $x^2+11x+10=(x+1)(x+10)$

84. $x^2+5x+4=(x+4)(x+1)$

85. $x^2-10x+21=(x-7)(x-3)$

86. $x^2-6x+8=(x-2)(x-4)$

87. $4x^2-8x+32=4(x^2-2x+8)$

88. $3x^2-12x+15=3(x^2-4x+5)$

89. $x^2+4x+16$ is prime over the reals because there are no factors of 16 whose sum is 4.

90. $x^2+12x+36=(x+6)^2$

91. $15+2x-x^2=-(x^2-2x-15)=-(x-5)(x+3)$

92. $14+6x-x^2=-(x^2-6x-14)$ is prime over the integers because there are no factors of –14 whose sum is –6.

93. $3x^2-12x-36=3(x^2-4x-12)$
$=3(x-6)(x+2)$

94. $x^3+8x^2-20x=x(x^2+8x-20)$
$=x(x+10)(x-2)$

95. $y^4+11y^3+30y^2=y^2(y^2+11y+30)$
$=y^2(y+5)(y+6)$

96. $3y^3-18y^2-48y=3y(y^2-6y-16)$
$=3y(y+2)(y-8)$

97. $4x^2+12x+9=(2x+3)^2$

98. $9x^2-12x+4=(3x-2)^2$

99. $6x^2+8x+2=2\left(3x^2+4x+1\right)$
$=2\left(3x+1\right)\left(x+1\right)$

100. $8x^2+6x-2=2\left(4x^2+3x-1\right)$
$=2\left(4x-1\right)\left(x+1\right)$

101. $x^4-81=\left(x^2\right)^2-9^2=(x^2-9)(x^2+9)$
$=(x-3)(x+3)(x^2+9)$

102. $x^4-1=\left(x^2\right)^2-1^2=(x^2-1)(x^2+1)$
$=(x-1)(x+1)(x^2+1)$

103. $x^6-2x^3+1=(x^3-1)^2$
$=\left[(x-1)(x^2+x+1)\right]^2$
$=(x-1)^2(x^2+x+1)^2$

104. $x^6+2x^3+1=(x^3+1)^2$
$=\left[(x+1)(x^2-x+1)\right]^2$
$=(x+1)^2(x^2-x+1)^2$

105. $x^7-x^5=x^5(x^2-1)=x^5(x-1)(x+1)$

106. $x^8-x^5=x^5(x^3-1)=x^5(x-1)(x^2+x+1)$

107. $16x^2+24x+9=\left(4x+3\right)^2$

108. $9x^2-24x+16=\left(3x-4\right)^2$

109. $5+16x-16x^2=-(16x^2-16x-5)$
$=-(4x-5)(4x+1)$

110. $5+11x-16x^2=-(16x^2-11x-5)$
$=-(16x+5)(x-1)$

111. $4y^2-16y+15=(2y-5)(2y-3)$

112. $9y^2+9y-4=(3y+4)(3y-1)$

113. $1-8x^2-9x^4=-(9x^4+8x^2-1)$
$=-(9x^2-1)(x^2+1)$
$=-(3x-1)(3x+1)(x^2+1)$

114. $4-14x^2-8x^4=-2(4x^4+7x^2-2)$
$=-2(4x^2-1)(x^2+2)$
$=-2(2x-1)(2x+1)(x^2+2)$

115. $x(x+3)-6(x+3)=(x+3)(x-6)$

116. $5(3x-7)+x(3x-7)=(3x-7)(x+5)$

117. $(x+2)^2-5(x+2)=(x+2)\left[(x+2)-5\right]$
$=(x+2)(x-3)$

118. $(x-1)^2-2(x-1)=(x-1)\left[(x-1)-2\right]$
$=(x-1)(x-3)$

119. $\left(3x-2\right)^3-27$
$=\left(3x-2\right)^3-3^3$
$=\left[\left(3x-2\right)-3\right]\left[\left(3x-2\right)^2+3\left(3x-2\right)+9\right]$
$=\left(3x-5\right)\left(9x^2-12x+4+9x-6+9\right)$
$=\left(3x-5\right)\left(9x^2-3x+7\right)$

120. $(5x+1)^3-1$
$=(5x+1)^3-1^3$
$=\left[(5x+1)-1\right]\left[(5x+1)^2+(1)(5x+1)+1\right]$
$=5x\left(25x^2+10x+1+5x+1+1\right)$
$=5x\left(25x^2+15x+3\right)$

121. $3\left(x^2+10x+25\right)-4(x+5)$
$=3(x+5)^2-4(x+5)$
$=(x+5)\left[3(x+5)-4\right]$
$=(x+5)(3x+15-4)$
$=(x+5)(3x+11)$

122. $7\left(x^2-6x+9\right)+5(x-3)$
$=7(x-3)^2+5(x-3)$
$=(x-3)\left[7(x-3)+5\right]$
$=(x-3)(7x-21+5)$
$=(x-3)(7x-16)$

123. $x^3+2x^2-x-2=x^2(x+2)-1(x+2)$
$=(x+2)(x^2-1)$
$=(x+2)(x-1)(x+1)$

124. $x^3-3x^2-x+3=x^2(x-3)-1(x-3)$
$=(x-3)(x^2-1)$
$=(x-3)(x-1)(x+1)$

125. $x^4-x^3+x-1=x^3(x-1)+1(x-1)$
$=(x-1)(x^3+1)$
$=(x-1)(x+1)(x^2-x+1)$

126. $x^4+x^3+x+1=x^3(x+1)+1(x+1)$
$=(x+1)(x^3+1)$
$=(x+1)(x+1)(x^2-x+1)$
$=(x+1)^2(x^2-x+1)$

127. $2(3x+4)^2+(2x+3)\cdot 2(3x+4)\cdot 3$
$=2(3x+4)\left((3x+4)+(2x+3)\cdot 3\right)$
$=2(3x+4)(3x+4+6x+9)$
$=2(3x+4)(9x+13)$

128. $5(2x+1)^2+(5x-6)\cdot 2(2x+1)\cdot 2$
$=(2x+1)\left(5(2x+1)+(5x-6)\cdot 4\right)$
$=(2x+1)(10x+5+20x-24)$
$=(2x+1)(30x-19)$

129. $2x(2x+5)+x^2\cdot 2=2x\left((2x+5)+x\right)$
$=2x(2x+5+x)$
$=2x(3x+5)$

130. $3x^2(8x-3)+x^3\cdot 8=x^2\left(3(8x-3)+8x\right)$
$=x^2(24x-9+8x)$
$=x^2(32x-9)$

131. $2(x+3)(x-2)^3+(x+3)^2\cdot 3(x-2)^2$
$=(x+3)(x-2)^2\left(2(x-2)+(x+3)\cdot 3\right)$
$=(x+3)(x-2)^2(2x-4+3x+9)$
$=(x+3)(x-2)^2(5x+5)$
$=5(x+3)(x-2)^2(x+1)$

132. $4(x+5)^3(x-1)^2+(x+5)^4\cdot 2(x-1)$
$=2(x+5)^3(x-1)\left(2(x-1)+(x+5)\right)$
$=2(x+5)^3(x-1)(2x-2+x+5)$
$=2(x+5)^3(x-1)(3x+3)$
$=2\cdot 3(x+5)^3(x-1)(x+1)$
$=6(x+5)^3(x-1)(x+1)$

133. $(4x-3)^2+x\cdot 2(4x-3)\cdot 4$
$=(4x-3)\left((4x-3)+8x\right)$
$=(4x-3)(4x-3+8x)$
$=(4x-3)(12x-3)$
$=3(4x-3)(4x-1)$

134. $3x^2(3x+4)^2 + x^3 \cdot 2(3x+4)\cdot 3$
$$\begin{aligned}&= 3x^2(3x+4)\big((3x+4)+2x\big)\\ &= 3x^2(3x+4)(3x+4+2x)\\ &= 3x^2(3x+4)(5x+4)\end{aligned}$$

135. $2(3x-5)\cdot 3(2x+1)^3 + (3x-5)^2 \cdot 3(2x+1)^2 \cdot 2$
$$\begin{aligned}&= 6(3x-5)(2x+1)^2\big((2x+1)+(3x-5)\big)\\ &= 6(3x-5)(2x+1)^2(2x+1+3x-5)\\ &= 6(3x-5)(2x+1)^2(5x-4)\end{aligned}$$

136. $3(4x+5)^2 \cdot 4(5x+1)^2 + (4x+5)^3 \cdot 2(5x+1)\cdot 5$
$$\begin{aligned}&= 2(4x+5)^2(5x+1)\big(6(5x+1)+5(4x+5)\big)\\ &= 2(4x+5)^2(5x+1)(30x+6+20x+25)\\ &= 2(4x+5)^2(5x+1)(50x+31)\end{aligned}$$

137. $x^{-4} + x^{-2} = x^{-4}(1+x^{-2})$

138. $2x^{-5} - 6x^{-4} + 8x^{-3} = 2x^{-5}(1-3x+4x^2)$

139.
$$\begin{aligned}x^{-2}(x+1) + x^{-1}(x-1) &= x^{-2}[(x+1)+x(x-1)]\\ &= x^{-2}(x+1+x^2-x)\\ &= x^{-2}(x^2+1)\end{aligned}$$

140.
$$\begin{aligned}x(x+3)^{-1} + 4x^2(x+3)^{-2} &= x(x+3)^{-2}[(x+3)+4x]\\ &= x(x+3)^{-2}(5x+3)\end{aligned}$$

141. The possible factorizations are $(x\pm 1)(x\pm 4) = x^2 \pm 5x + 4$ or $(x\pm 2)(x\pm 2) = x^2 \pm 4x + 4$, none of which equals x^2+4.

142. The possibile factorizations are $(x\pm 1)^2 = x^2 \pm 2x + 1$, neither of which equals x^2+x+1.

143. Answers will vary.

144. Answers will vary.

Section R.6

1. quotient; divisor; remainder

2. $-3\overline{)2\quad 0\quad -5\quad 1}$

3. d

4. a

5. True

6. True

7.
$$\begin{array}{r|rrrr} 2 & 1 & -7 & 5 & 10\\ & & 2 & -10 & -10\\ \hline & 1 & -5 & -5 & 0\end{array}$$

Quotient: x^2-5x-5
Remainder: 0

8.
$$\begin{array}{r|rrrr} -1 & 1 & 2 & -3 & 1\\ & & -1 & -1 & 4\\ \hline & 1 & 1 & -4 & 5\end{array}$$

Quotient: x^2+x-4
Remainder: 5

9.
$$\begin{array}{r|rrrr} 3 & 3 & 2 & -1 & 3\\ & & 9 & 33 & 96\\ \hline & 3 & 11 & 32 & 99\end{array}$$

Quotient: $3x^2+11x+32$
Remainder: 99

10.
$$\begin{array}{r|rrrr} -2 & -4 & 2 & -1 & 1\\ & & 8 & -20 & 42\\ \hline & -4 & 10 & -21 & 43\end{array}$$

Quotient: $-4x^2+10x-21$
Remainder: 43

11.
$$\begin{array}{r|rrrrrr} -3 & 1 & 0 & -4 & 0 & 1 & 0\\ & & -3 & 9 & -15 & 45 & -138\\ \hline & 1 & -3 & 5 & -15 & 46 & -138\end{array}$$

Quotient: $x^4-3x^3+5x^2-15x+46$
Remainder: -138

12. $$\begin{array}{r|rrrrr} 2 & 1 & 0 & 1 & 0 & 2 \\ & & 2 & 4 & 10 & 20 \\ \hline & 1 & 2 & 5 & 10 & 22 \end{array}$$

Quotient: $x^3+2x^2+5x+10$
Remainder: 22

13. $$\begin{array}{r|rrrrrrr} 1 & 4 & 0 & -3 & 0 & 1 & 0 & 5 \\ & & 4 & 4 & 1 & 1 & 2 & 2 \\ \hline & 4 & 4 & 1 & 1 & 2 & 2 & 7 \end{array}$$

Quotient: $4x^5+4x^4+x^3+x^2+2x+2$
Remainder: 7

14. $$\begin{array}{r|rrrrrr} -1 & 1 & 0 & 5 & 0 & 0 & -10 \\ & & -1 & 1 & -6 & 6 & -6 \\ \hline & 1 & -1 & 6 & -6 & 6 & -16 \end{array}$$

Quotient: $x^4-x^3+6x^2-6x+6$
Remainder: –16

15. $$\begin{array}{r|rrrr} -1.1 & 0.1 & 0 & 0.2 & 0 \\ & & -0.11 & 0.121 & -0.3531 \\ \hline & 0.1 & -0.11 & 0.321 & -0.3531 \end{array}$$

Quotient: $0.1x^2-0.11x+0.321$
Remainder: –0.3531

16. $$\begin{array}{r|rrr} -2.1 & 0.1 & 0 & -0.2 \\ & & -0.21 & 0.441 \\ \hline & 0.1 & -0.21 & 0.241 \end{array}$$

Quotient: $0.1x-0.21$
Remainder: 0.241

17. $$\begin{array}{r|rrrrrr} 2 & 1 & 0 & 0 & 0 & 0 & -32 \\ & & 2 & 4 & 8 & 16 & 32 \\ \hline & 1 & 2 & 4 & 8 & 16 & 0 \end{array}$$

Quotient: $x^4+2x^3+4x^2+8x+16$
Remainder: 0

18. $$\begin{array}{r|rrrrrr} -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ & & -1 & 1 & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & -1 & 1 & 0 \end{array}$$

Quotient: $x^4-x^3+x^2-x+1$
Remainder: 0

19. $$\begin{array}{r|rrrr} 2 & 4 & -3 & -8 & 4 \\ & & 8 & 10 & 4 \\ \hline & 4 & 5 & 2 & 8 \end{array}$$

Remainder $= 8 \neq 0$. Therefore, $x-2$ is not a factor of $4x^3-3x^2-8x+4$.

20. $$\begin{array}{r|rrrr} -3 & -4 & 5 & 0 & 8 \\ & & 12 & -51 & 153 \\ \hline & -4 & 17 & -51 & 161 \end{array}$$

Remainder $= 161 \neq 0$. Therefore, $x+3$ is not a factor of $-4x^3+5x^2+8$.

21. $$\begin{array}{r|rrrrr} 3 & 2 & -6 & 0 & -7 & 21 \\ & & 6 & 0 & 0 & -21 \\ \hline & 2 & 0 & 0 & -7 & 0 \end{array}$$

Remainder $= 0$. Therefore, $x-3$ is a factor of $2x^4-6x^3-7x+21$.

22. $$\begin{array}{r|rrrrr} 2 & 4 & 0 & -15 & 0 & -4 \\ & & 8 & 16 & 2 & 4 \\ \hline & 4 & 8 & 1 & 2 & 0 \end{array}$$

Remainder $= 0$. Therefore, $x-2$ is a factor of $4x^4-15x^2-4$.

23. $$\begin{array}{r|rrrrrrr} -2 & 5 & 0 & 0 & 43 & 0 & 0 & 24 \\ & & -10 & 20 & -40 & -6 & 12 & -24 \\ \hline & 3 & -10 & 20 & 3 & -6 & 12 & 0 \end{array}$$

Remainder $= 0$. Therefore, $x+3$ is a factor of $5x^6+43x^3+24$.

24. $$\begin{array}{r|rrrrrrr} -3 & 2 & 0 & -18 & 0 & 1 & 0 & -9 \\ & & -6 & 18 & 0 & 0 & -3 & 9 \\ \hline & 2 & -6 & 0 & 0 & 1 & -3 & 0 \end{array}$$

Remainder $= 0$. Therefore, $x+2$ is a factor of $2x^6-18x^4+x^2-9$.

25. $$\begin{array}{r|rrrrrr} -4 & 1 & 0 & -16 & -1 & 0 & 19 \\ & & -4 & 16 & 0 & 4 & -16 \\ \hline & 1 & -4 & 0 & -1 & 4 & 3 \end{array}$$
Remainder $= 1 \neq 0$. Therefore, $x+4$ is not a factor of $x^5 - 16x^3 - x^2 + 19$.

26. $$\begin{array}{r|rrrrrrr} -4 & 1 & 0 & -16 & 0 & 1 & 0 & -16 \\ & & -4 & 16 & 0 & 0 & -4 & 16 \\ \hline & 1 & -4 & 0 & 0 & 1 & -4 & 0 \end{array}$$
Remainder $= 0$. Therefore, $x+4$ is a factor $x^6 - 16x^4 + x^2 - 16$.

27. $$\begin{array}{r|rrrrr} \frac{1}{3} & 3 & -1 & 0 & 6 & -2 \\ & & 1 & 0 & 0 & 2 \\ \hline & 3 & 0 & 0 & 6 & 0 \end{array}$$
Remainder $= 0$; therefore $x - \frac{1}{3}$ is a factor of $3x^4 - x^3 + 6x - 2$.

28. $$\begin{array}{r|rrrrr} -\frac{1}{3} & 3 & 1 & 0 & -3 & 1 \\ & & -1 & 0 & 0 & 1 \\ \hline & 3 & 0 & 0 & -3 & 2 \end{array}$$
Remainder $= 2 \neq 0$; therefore $x + \frac{1}{3}$ is not a factor of $3x^4 + x^3 - 3x + 1$.

29. $$\begin{array}{r|rrrr} -2 & 1 & -2 & 3 & 5 \\ & & -2 & 8 & -22 \\ \hline & 1 & -4 & 11 & -17 \end{array}$$
$$\frac{x^3 - 2x^2 + 3x + 5}{x+2} = x^2 - 4x + 11 + \frac{-17}{x+2}$$
$$a + b + c + d = 1 - 4 + 11 - 17 = -9$$

30. $$\begin{array}{r|rrrrr} h & 1 & 3-h & -2h & -2h^2 & h^3 \\ & & h & 3h & h^2 & -h^3 \\ \hline & 1 & 3 & h & -h^2 & 0 \end{array}$$

$x^3 + 3x^2 + hx - h^2$ is the quotient and 0 is the remainder.

31. $$\begin{array}{r|rrrrr} -y & 1 & 3y & -3y^2 & -y^3 & 4y^4 \\ & & -y & -2y^2 & 5y^3 & 4y^3 \\ \hline & 1 & 2y & -5y^2 & 4y^3 & 0 \end{array}$$

Yes, $x + y$ is a factor of $x^4 + 3x^3y - 3x^2y^2 - xy^3 + 4y^4$.

32. Answers will vary.

Section R.7

1. lowest terms
2. Least Common Multiple
3. d
4. a
5. True; $$\frac{\frac{1}{3}+\frac{1}{x}}{\frac{1}{5}-\frac{1}{x}} = \frac{\frac{x+3}{3x}}{\frac{x-5}{5x}} = \frac{x+3}{3x}\cdot\frac{5x}{x-5} = \frac{5(x+3)}{3(x-5)}$$
6. False;
$$2x^3 + 6x^2 = 2x^2(x+3)$$
$$6x^4 + 4x^3 = 2x^3(3x+2)$$
$$LCM = 2x^3(x+3)(3x+2)$$
7. $$\frac{3x+9}{x^2-9} = \frac{3(x+3)}{(x-3)(x+3)} = \frac{3}{x-3}$$
8. $$\frac{4x^2+8x}{12x+24} = \frac{4x(x+2)}{12(x+2)} = \frac{x}{3}$$
9. $$\frac{x^2-2x}{3x-6} = \frac{x(x-2)}{3(x-2)} = \frac{x}{3}$$
10. $$\frac{15x^2+24x}{3x^2} = \frac{3x(5x+8)}{3x^2} = \frac{5x+8}{x}$$
11. $$\frac{24x^2}{12x^2-6x} = \frac{24x^2}{6x(2x-1)} = \frac{4x}{2x-1}$$
12. $$\frac{x^2+4x+4}{x^2-4} = \frac{(x+2)(x+2)}{(x-2)(x+2)} = \frac{x+2}{x-2}$$

13. $\dfrac{y^2-49}{3y^2-18y-21}=\dfrac{(y+7)(y-7)}{3(y^2-6y-7)}$

$=\dfrac{(y+7)(y-7)}{3(y-7)(y+1)}$

$=\dfrac{y+7}{3(y+1)}$

14. $\dfrac{3y^2-y-2}{3y^2+5y+2}=\dfrac{(3y+2)(y-1)}{(3y+2)(y+1)}=\dfrac{y-1}{y+1}$

15. $\dfrac{x^2+4x-12}{x^2-4x+4}=\dfrac{(x+6)(x-2)}{(x-2)(x-2)}=\dfrac{x+6}{x-2}$

16. $\dfrac{x-x^2}{x^2+x-2}=\dfrac{-x(x-1)}{(x+2)(x-1)}=\dfrac{-x}{x+2}=-\dfrac{x}{x+2}$

17. $\dfrac{x^2+x-20}{4-x}=\dfrac{(x+5)(x-4)}{2-x}$

$=\dfrac{(x+5)(x-4)}{-1(x-4)}$

$=-(x+5)$

18. $\dfrac{2x^2+5x-3}{1-2x}=\dfrac{(2x-1)(x+3)}{-1(2x-1)}=-(x+3)=-x-3$

19. $\dfrac{3x+6}{5x^2}\cdot\dfrac{x}{x^2-4}=\dfrac{3(x+2)}{5x^2}\cdot\dfrac{x}{(x-2)(x+2)}$

$=\dfrac{3}{5x(x-2)}$

20. $\dfrac{3}{2x}\cdot\dfrac{x^2}{6x+10}=\dfrac{3}{2}\cdot\dfrac{x}{2(3x+5)}=\dfrac{3x}{4(3x+5)}$

21. $\dfrac{4x^2}{x^2-16}\cdot\dfrac{x^3-64}{2x}$

$=\dfrac{4x^2}{(x-4)(x+4)}\cdot\dfrac{(x-4)(x^2+4x+16)}{2x}$

$=\dfrac{2x\cdot 2x(x-4)(x^2+4x+16)}{2x(x-4)(x+4)}$

$=\dfrac{2x(x^2+4x+16)}{x+4}$

22. $\dfrac{12}{x^2+x}\cdot\dfrac{x^3+1}{4x-2}=\dfrac{12}{x(x+1)}\cdot\dfrac{(x+1)(x^2-x+1)}{2(2x-1)}$

$=\dfrac{2\cdot 6(x+1)(x^2-x+1)}{2x(x+1)(2x-1)}$

$=\dfrac{6(x^2-x+1)}{x(2x-1)}$

23. $\dfrac{4x-8}{-3x}\cdot\dfrac{12}{12-6x}=\dfrac{4(x-2)}{-3x}\cdot\dfrac{12}{6(2-x)}$

$=\dfrac{4(x-2)}{-3x}\cdot\dfrac{2}{(-1)(x-2)}$

$=\dfrac{8}{3x}$

24. $\dfrac{6x-27}{5x}\cdot\dfrac{2}{4x-18}=\dfrac{3(2x-9)}{5x}\cdot\dfrac{2}{2(2x-9)}=\dfrac{3}{5x}$

25. $\dfrac{x^2-4x-12}{x^2+2x-48}\cdot\dfrac{x^2+4x-32}{x^2+10x+16}$

$=\dfrac{(x-6)(x+2)}{(x+8)(x-6)}\cdot\dfrac{(x+8)(x-4)}{(x+8)(x+2)}$

$=\dfrac{x-4}{x+8}$

26. $\dfrac{x^2+x-6}{x^2+4x-5}\cdot\dfrac{x^2-25}{x^2+2x-15}$

$=\dfrac{(x-2)(x+3)}{(x+5)(x-1)}\cdot\dfrac{(x+5)(x-5)}{(x+5)(x-3)}$

$=\dfrac{(x-2)(x+3)(x-5)}{(x+5)(x-1)(x-3)}$

27. $\dfrac{\dfrac{6x}{x^2-4}}{\dfrac{3x-9}{2x+4}}=\dfrac{6x}{x^2-4}\cdot\dfrac{2x+4}{3x-9}$

$=\dfrac{6x}{(x-2)(x+2)}\cdot\dfrac{2(x+2)}{3(x-3)}$

$=\dfrac{4x}{(x-2)(x-3)}$

28. $$\frac{\frac{12x}{5x+20}}{\frac{4x^2}{x^2-16}}=\frac{12x}{5x+20}\cdot\frac{x^2-16}{4x^2}$$
$$=\frac{12x}{5(x+4)}\cdot\frac{(x+4)(x-4)}{4x^2}$$
$$=\frac{3(x-4)}{5x}$$

29. $$\frac{\frac{8x}{x^2-1}}{\frac{10x}{x+1}}=\frac{8x}{x^2-1}\cdot\frac{x+1}{10x}$$
$$=\frac{8x}{(x-1)(x+1)}\cdot\frac{x+1}{10x}$$
$$=\frac{4}{5(x-1)}$$

30. $$\frac{\frac{x-2}{4x}}{\frac{x^2-4x+4}{12x}}=\frac{x-2}{4x}\cdot\frac{12x}{x^2-4x+4}$$
$$=\frac{x-2}{4x}\cdot\frac{12x}{(x-2)(x-2)}$$
$$=\frac{3}{x-2}$$

31. $$\frac{\frac{4-x}{4+x}}{\frac{4x}{x^2-16}}=\frac{4-x}{4+x}\cdot\frac{x^2-16}{4x}$$
$$=\frac{4-x}{4+x}\cdot\frac{(x+4)(x-4)}{4x}$$
$$=\frac{(4-x)(x-4)}{4x}$$
$$=-\frac{(x-4)^2}{4x}$$

32. $$\frac{\frac{3+x}{3-x}}{\frac{x^2-9}{9x^3}}=\frac{3+x}{3-x}\cdot\frac{9x^3}{x^2-9}$$
$$=\frac{3+x}{3-x}\cdot\frac{9x^3}{(x+3)(x-3)}$$
$$=\frac{9x^3}{(3-x)(x-3)}$$
$$=\frac{9x^3}{-(x-3)^2}$$
$$=-\frac{9x^3}{(x-3)^2}$$

33. $$\frac{\frac{x^2+7x+12}{x^2-7x+12}}{\frac{x^2+x-12}{x^2-x-12}}=\frac{x^2+7x+12}{x^2-7x+12}\cdot\frac{x^2-x-12}{x^2+x-12}$$
$$=\frac{(x+3)(x+4)}{(x-3)(x-4)}\cdot\frac{(x-4)(x+3)}{(x+4)(x-3)}$$
$$=\frac{(x+3)^2}{(x-3)^2}$$

34. $$\frac{\frac{x^2+7x+6}{x^2+x-6}}{\frac{x^2+5x-6}{x^2+5x+6}}=\frac{x^2+7x+6}{x^2+x-6}\cdot\frac{x^2+5x+6}{x^2+5x-6}$$
$$=\frac{(x+6)(x+1)}{(x+3)(x-2)}\cdot\frac{(x+2)(x+3)}{(x+6)(x-1)}$$
$$=\frac{(x+1)(x+2)}{(x-2)(x-1)}$$

35. $$\frac{\frac{5x^2-7x-6}{2x^2+3x-5}}{\frac{15x^2+14x+3}{2x^2+13x+20}}=\frac{5x^2-7x-6}{2x^2+3x-5}\cdot\frac{2x^2+13x+20}{15x^2+14x+3}$$
$$=\frac{(5x+3)(x-2)}{(x-1)(2x+5)}\cdot\frac{(2x+5)(x+4)}{(5x+3)(3x+1)}$$
$$=\frac{(x-2)(x+4)}{(x-1)(3x+1)}$$

36. $\dfrac{\dfrac{9x^2+3x-2}{12x^2+5x-2}}{\dfrac{9x^2-6x+1}{8x^2-10x-3}} = \dfrac{9x^2+3x-2}{12x^2+5x-2}\cdot\dfrac{8x^2-10x-3}{9x^2-6x+1}$

$= \dfrac{(3x+2)(3x-1)}{(3x+2)(4x-1)}\cdot\dfrac{(4x+1)(2x-3)}{(3x-1)(3x-1)}$

$= \dfrac{(4x+1)(2x-3)}{(4x-1)(3x-1)}$

37. $\dfrac{x}{2}+\dfrac{5}{2}=\dfrac{x+5}{2}$

38. $\dfrac{3}{x}-\dfrac{6}{x}=\dfrac{3-6}{x}=\dfrac{-3}{x}=-\dfrac{3}{x}$

39. $\dfrac{x^2}{2x-3}-\dfrac{4}{2x-3}=\dfrac{x^2-4}{2x-3}=\dfrac{(x+2)(x-2)}{2x-3}$

40. $\dfrac{3x^2}{2x-1}-\dfrac{9}{2x-1}=\dfrac{3x^2-9}{2x-1}=\dfrac{3\left(x^2-3\right)}{2x-1}$

41. $\dfrac{x+5}{x-4}+\dfrac{3x-2}{x-4}=\dfrac{x+5+3x-2}{x-4}=\dfrac{4x+3}{x-4}$

42. $\dfrac{2x-5}{3x+2}+\dfrac{x+4}{3x+2}=\dfrac{2x-5+x+4}{3x+2}=\dfrac{3x-1}{3x+2}$

43. $\dfrac{3x+5}{2x-1}-\dfrac{2x-4}{2x-1}=\dfrac{(3x+5)-(2x-4)}{2x-1}$

$=\dfrac{3x+5-2x+4}{2x-1}$

$=\dfrac{x+9}{2x-1}$

44. $\dfrac{5x-4}{3x+4}-\dfrac{x+1}{3x+4}=\dfrac{(5x-4)-(x+1)}{3x+4}$

$=\dfrac{5x-4-x-1}{3x+4}$

$=\dfrac{4x-5}{3x+4}$

45. $\dfrac{4}{x-2}+\dfrac{x}{2-x}=\dfrac{4}{x-2}-\dfrac{x}{x-2}=\dfrac{4-x}{x-2}$

46. $\dfrac{6}{x-1}-\dfrac{x}{1-x}=\dfrac{6}{x-1}+\dfrac{x}{x-1}=\dfrac{x+6}{x-1}$

47. $\dfrac{7}{x-3}-\dfrac{3}{x+1}=\dfrac{7(x+1)}{(x-3)(x+1)}-\dfrac{3(x-3)}{(x+1)(x-3)}$

$=\dfrac{7x+7-3x+9}{(x+1)(x-3)}$

$=\dfrac{4x+16}{(x+1)(x-3)}$

$=\dfrac{4(x+4)}{(x+1)(x-3)}$

48. $\dfrac{2}{x+5}-\dfrac{5}{x-5}=\dfrac{2(x-5)}{(x+5)(x-5)}-\dfrac{5(x+5)}{(x+5)(x-5)}$

$=\dfrac{2x-10-5x-25}{(x+5)(x-5)}$

$=\dfrac{-3x-35}{(x+5)(x-5)}$

$=-\dfrac{3x+35}{(x+5)(x-5)}$

49. $\dfrac{x}{x+1}+\dfrac{2x-3}{x-1}=\dfrac{x(x-1)}{(x+1)(x-1)}+\dfrac{(2x-3)(x+1)}{(x-1)(x+1)}$

$=\dfrac{x^2-x+2x^2-x-3}{(x-1)(x+1)}$

$=\dfrac{3x^2-2x-3}{(x-1)(x+1)}$

50. $\dfrac{3x}{x-4}+\dfrac{2x}{x+3}=\dfrac{3x(x+3)}{(x-4)(x+3)}+\dfrac{2x(x-4)}{(x-4)(x+3)}$

$=\dfrac{3x^2+9x+2x^2-8x}{(x-4)(x+3)}$

$=\dfrac{5x^2+x}{(x-4)(x+3)}$

$=\dfrac{x(5x+1)}{(x-4)(x+3)}$

51. $\dfrac{x-3}{x+2}-\dfrac{x+4}{x-2}=\dfrac{(x-3)(x-2)}{(x+2)(x-2)}-\dfrac{(x+4)(x+2)}{(x-2)(x+2)}$

$=\dfrac{x^2-5x+6-(x^2+6x+8)}{(x+2)(x-2)}$

$=\dfrac{x^2-5x+6-x^2-6x-8}{(x+2)(x-2)}$

$=\dfrac{-11x-2}{(x+2)(x-2)}$ or $\dfrac{-(11x+2)}{(x+2)(x-2)}$

52. $\dfrac{2x-3}{x-1}-\dfrac{2x+1}{x+1}=\dfrac{(2x-3)(x+1)}{(x-1)(x+1)}-\dfrac{(2x+1)(x-1)}{(x+1)(x-1)}$

$=\dfrac{2x^2-x-3-(2x^2-x-1)}{(x+1)(x-1)}$

$=\dfrac{2x^2-x-3-2x^2+x+1}{(x+1)(x-1)}$

$=\dfrac{-2}{(x+1)(x-1)}$

$=-\dfrac{2}{(x+1)(x-1)}$

53. $\dfrac{x}{x^2-4}+\dfrac{1}{x}=\dfrac{x^2+x^2-4}{x\left(x^2-4\right)}$

$=\dfrac{2x^2-4}{x\left(x^2-4\right)}$

$=\dfrac{2\left(x^2-2\right)}{x(x-2)(x+2)}$

54. $\dfrac{x-1}{x^3}+\dfrac{x}{x^2+1}=\dfrac{(x-1)\left(x^2+1\right)+x^4}{x^3\left(x^2+1\right)}$

$=\dfrac{x^3-x^2+x-1+x^4}{x^3\left(x^2+1\right)}$

$=\dfrac{x^4+x^3-x^2+x-1}{x^3\left(x^2+1\right)}$

55. $x^2-4=(x+2)(x-2)$

$x^2-x-2=(x+1)(x-2)$

Therefore, $\text{LCM}=(x+2)(x-2)(x+1)$.

56. $x^2-x-12=(x+3)(x-4)$

$x^2-8x+16=(x-4)(x-4)$

Therefore, $\text{LCM}=(x+3)(x-4)^2$.

57. $x^3-x=x\left(x^2-1\right)=x(x+1)(x-1)$

$x^2-x=x(x-1)$

Therefore, $\text{LCM}=x(x+1)(x-1)$.

58. $3x^2-27=3\left(x^2-9\right)=3(x+3)(x-3)$

$2x^2-x-15=(2x+5)(x-3)$

Therefore, $\text{LCM}=3(2x+5)(x-3)(x+3)$.

59. $4x^3-4x^2+x=x\left(4x^2-4x+1\right)$

$=x(2x-1)(2x-1)$

$2x^3-x^2=x^2(2x-1)$

x^3

Therefore, $\text{LCM}=x^3(2x-1)^2$.

60. $x-3$

$x^2+3x=x(x+3)$

$x^3-9x=x\left(x^2-9\right)=x(x+3)(x-3)$

Therefore, $\text{LCM}=x(x+3)(x-3)$.

61. $x^3-x=x\left(x^2-1\right)=x(x+1)(x-1)$

$x^3-2x^2+x=x\left(x^2-2x+1\right)=x(x-1)^2$

$x^3-1=(x-1)\left(x^2+x+1\right)$

Therefore, $\text{LCM}=x(x+1)(x-1)^2\left(x^2+x+1\right)$.

62. $x^2+4x+4=(x+2)^2$

$x^3+2x^2=x^2(x+2)$

$(x+2)^3$

Therefore, $\text{LCM}=x^2(x+2)^3$.

63. $\dfrac{x}{x^2-7x+6}-\dfrac{x}{x^2-2x-24}$

$=\dfrac{x}{(x-6)(x-1)}-\dfrac{x}{(x-6)(x+4)}$

$=\dfrac{x(x+4)}{(x-6)(x-1)(x+4)}-\dfrac{x(x-1)}{(x-6)(x+4)(x-1)}$

$=\dfrac{x^2+4x-x^2+x}{(x-6)(x+4)(x-1)}=\dfrac{5x}{(x-6)(x+4)(x-1)}$

64. $\dfrac{x}{x-3}-\dfrac{x+1}{x^2+5x-24}$

$=\dfrac{x}{(x-3)}-\dfrac{x+1}{(x-3)(x+8)}$

$=\dfrac{x(x+8)}{(x-3)(x+8)}-\dfrac{x+1}{(x-3)(x+8)}$

$=\dfrac{x^2+8x-x-1}{(x-3)(x+8)}=\dfrac{x^2+7x-1}{(x-3)(x+8)}$

65. $\dfrac{4x}{x^2-4}-\dfrac{2}{x^2+x-6}$
$=\dfrac{4x}{(x-2)(x+2)}-\dfrac{2}{(x+3)(x-2)}$
$=\dfrac{4x(x+3)}{(x-2)(x+2)(x+3)}-\dfrac{2(x+2)}{(x+3)(x-2)(x+2)}$
$=\dfrac{4x^2+12x-2x-4}{(x-2)(x+2)(x+3)}$
$=\dfrac{4x^2+10x-4}{(x-2)(x+2)(x+3)}$
$=\dfrac{2(2x^2+5x-2)}{(x-2)(x+2)(x+3)}$

66. $\dfrac{3x}{x-1}-\dfrac{x-4}{x^2-2x+1}=\dfrac{3x}{(x-1)}-\dfrac{x-4}{(x-1)^2}$
$=\dfrac{3x(x-1)}{(x-1)(x-1)}-\dfrac{x-4}{(x-1)^2}$
$=\dfrac{3x^2-3x-x+4}{(x-1)^2}$
$=\dfrac{3x^2-4x+4}{(x-1)^2}$

67. $\dfrac{3}{(x-1)^2(x+1)}+\dfrac{2}{(x-1)(x+1)^2}$
$=\dfrac{3(x+1)+2(x-1)}{(x-1)^2(x+1)^2}$
$=\dfrac{3x+3+2x-2}{(x-1)^2(x+1)^2}$
$=\dfrac{5x+1}{(x-1)^2(x+1)^2}$

68. $\dfrac{2}{(x+2)^2(x-1)}-\dfrac{6}{(x+2)(x-1)^2}$
$=\dfrac{2(x-1)-6(x+2)}{(x+2)^2(x-1)^2}$
$=\dfrac{2x-2-6x-12}{(x+2)^2(x-1)^2}$
$=\dfrac{-4x-14}{(x+2)^2(x-1)^2}$
$=\dfrac{-2(2x+7)}{(x+2)^2(x-1)^2}$

69. $\dfrac{x+4}{x^2-x-2}-\dfrac{2x+3}{x^2+2x-8}$
$=\dfrac{x+4}{(x-2)(x+1)}-\dfrac{2x+3}{(x+4)(x-2)}$
$=\dfrac{(x+4)(x+4)}{(x-2)(x+1)(x+4)}-\dfrac{(2x+3)(x+1)}{(x+4)(x-2)(x+1)}$
$=\dfrac{x^2+8x+16-(2x^2+5x+3)}{(x-2)(x+1)(x+4)}$
$=\dfrac{-x^2+3x+13}{(x-2)(x+1)(x+4)}$

70. $\dfrac{2x-3}{x^2+8x+7}-\dfrac{x-2}{(x+1)^2}$
$=\dfrac{2x-3}{(x+1)(x+7)}-\dfrac{x-2}{(x+1)^2}$
$=\dfrac{(2x-3)(x+1)}{(x+1)(x+7)(x+1)}-\dfrac{(x-2)(x+7)}{(x+1)^2(x+7)}$
$=\dfrac{2x^2-x-3-(x^2+5x-14)}{(x+1)^2(x+7)}$
$=\dfrac{x^2-6x+11}{(x+1)^2(x+7)}$

71. $\dfrac{1}{x}-\dfrac{2}{x^2+x}+\dfrac{3}{x^3-x^2}$
$=\dfrac{1}{x}-\dfrac{2}{x(x+1)}+\dfrac{3}{x^2(x-1)}$
$=\dfrac{x(x+1)(x-1)-2x(x-1)+3(x+1)}{x^2(x+1)(x-1)}$
$=\dfrac{x(x^2-1)-2x^2+2x+3x+3}{x^2(x+1)(x-1)}$
$=\dfrac{x^3-x-2x^2+5x+3}{x^2(x+1)(x-1)}$
$=\dfrac{x^3-2x^2+4x+3}{x^2(x+1)(x-1)}$

72. $\dfrac{x}{(x-1)^2}+\dfrac{2}{x}-\dfrac{x+1}{x^3-x^2}$

$=\dfrac{x}{(x-1)^2}+\dfrac{2}{x}-\dfrac{x+1}{x^2(x-1)}$

$=\dfrac{x^3+2x(x-1)^2-(x+1)(x-1)}{x^2(x-1)^2}$

$=\dfrac{x^3+2x(x^2-2x+1)-(x^2-1)}{x^2(x-1)^2}$

$=\dfrac{x^3+2x^3-4x^2+2x-x^2+1}{x^2(x-1)^2}$

$=\dfrac{3x^3-5x^2+2x+1}{x^2(x-1)^2}$

73. $\dfrac{1}{h}\left(\dfrac{1}{x+h}-\dfrac{1}{x}\right)=\dfrac{1}{h}\left(\dfrac{1\cdot x}{(x+h)x}-\dfrac{1(x+h)}{x(x+h)}\right)$

$=\dfrac{1}{h}\left(\dfrac{x-x-h}{x(x+h)}\right)$

$=\dfrac{-h}{hx(x+h)}$

$=\dfrac{-1}{x(x+h)}$

74. $\dfrac{1}{h}\left(\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}\right)$

$=\dfrac{1}{h}\left(\dfrac{1\cdot x^2}{(x+h)^2x^2}-\dfrac{1(x+h)^2}{x^2(x+h)^2}\right)$

$=\dfrac{1}{h}\left(\dfrac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2}\right)$

$=\dfrac{-2xh-h^2}{hx^2(x+h)^2}$

$=\dfrac{h(-2x-h)}{hx^2(x+h)^2}$

$=\dfrac{-2x-h}{x^2(x+h)^2}$

$=-\dfrac{2x+h}{x^2(x+h)^2}$

75. $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x}}=\dfrac{\left(\frac{x}{x}+\frac{1}{x}\right)}{\left(\frac{x}{x}-\frac{1}{x}\right)}=\dfrac{\left(\frac{x+1}{x}\right)}{\left(\frac{x-1}{x}\right)}=\dfrac{x+1}{x}\cdot\dfrac{x}{x-1}=\dfrac{x+1}{x-1}$

76. $\dfrac{4+\frac{1}{x^2}}{3-\frac{1}{x^2}}=\dfrac{\left(\frac{4x^2}{x^2}+\frac{1}{x^2}\right)}{\left(\frac{3x^2}{x^2}-\frac{1}{x^2}\right)}=\dfrac{\left(\frac{4x^2+1}{x^2}\right)}{\left(\frac{3x^2-1}{x^2}\right)}$

$=\dfrac{4x^2+1}{x^2}\cdot\dfrac{x^2}{3x^2-1}$

$=\dfrac{4x^2+1}{3x^2-1}$

77. $\dfrac{2-\frac{x+1}{x}}{3+\frac{x-1}{x+1}}=\dfrac{\frac{2x}{x}-\frac{x+1}{x}}{\frac{3(x+1)}{x+1}+\frac{x-1}{x+1}}=\dfrac{\frac{2x-x-1}{x}}{\frac{3x+3+x-1}{x+1}}$

$=\dfrac{\frac{x-1}{x}}{\frac{4x+2}{x+1}}=\dfrac{x-1}{x}\cdot\dfrac{x+1}{2(2x+1)}$

$=\dfrac{(x-1)(x+1)}{2x(2x+1)}$

78. $\dfrac{1-\frac{x}{x+1}}{2-\frac{x-1}{x}}=\dfrac{\left(\frac{x+1}{x+1}-\frac{x}{x+1}\right)}{\left(\frac{2x}{x}-\frac{x-1}{x}\right)}=\dfrac{\left(\frac{1}{x+1}\right)}{\left(\frac{x+1}{x}\right)}$

$=\dfrac{1}{x+1}\cdot\dfrac{x}{x+1}$

$=\dfrac{x}{(x+1)^2}$

79. $\dfrac{\frac{x+4}{x-2}-\frac{x-3}{x+1}}{x+1}$

$=\dfrac{\left(\frac{(x+4)(x+1)}{(x-2)(x+1)}-\frac{(x-3)(x-2)}{(x+1)(x-2)}\right)}{x+1}$

$=\dfrac{\left(\frac{x^2+5x+4-(x^2-5x+6)}{(x-2)(x+1)}\right)}{x+1}$

$=\dfrac{10x-2}{(x-2)(x+1)}\cdot\dfrac{1}{x+1}$

$=\dfrac{2(5x-1)}{(x-2)(x+1)^2}$

80. $$\frac{\frac{x-2}{x+1}-\frac{x}{x-2}}{x+3}$$
$$=\frac{\left(\frac{(x-2)(x-2)}{(x+1)(x-2)}-\frac{x(x+1)}{(x-2)(x+1)}\right)}{x+3}$$
$$=\frac{\left(\frac{x^2-4x+4-(x^2+x)}{(x-2)(x+1)}\right)}{x+3}$$
$$=\frac{-5x+4}{(x-2)(x+1)}\cdot\frac{1}{x+3}$$
$$=\frac{-5x+4}{(x-2)(x+1)(x+3)}$$
$$=\frac{-(5x-4)}{(x-2)(x+1)(x+3)}$$

81. $$\frac{\frac{x-2}{x+2}+\frac{x-1}{x+1}}{\frac{x}{x+1}-\frac{2x-3}{x}}$$
$$=\frac{\left(\frac{(x-2)(x+1)}{(x+2)(x+1)}+\frac{(x-1)(x+2)}{(x+1)(x+2)}\right)}{\left(\frac{x^2}{(x+1)(x)}-\frac{(2x-3)(x+1)}{x(x+1)}\right)}$$
$$=\frac{\left(\frac{x^2-x-2+x^2+x-2}{(x+2)(x+1)}\right)}{\left(\frac{x^2-(2x^2-x-3)}{x(x+1)}\right)}$$
$$=\frac{\left(\frac{2x^2-4}{(x+2)(x+1)}\right)}{\left(\frac{-x^2+x+3}{x(x+1)}\right)}$$
$$=\frac{2(x^2-2)}{(x+2)(x+1)}\cdot\frac{x(x+1)}{-(x^2-x-3)}$$
$$=\frac{2x(x^2-2)}{-(x+2)(x^2-x-3)}$$
$$=\frac{-2x(x^2-2)}{(x+2)(x^2-x-3)}$$

82. $$\frac{\frac{2x+5}{x}-\frac{x}{x-3}}{\frac{x^2}{x-3}-\frac{(x+1)^2}{x+3}}$$
$$=\frac{\left(\frac{(2x+5)(x-3)}{x(x-3)}-\frac{x(x)}{x(x-3)}\right)}{\left(\frac{x^2(x+3)}{(x-3)(x+3)}-\frac{(x-3)(x+1)^2}{(x-3)(x+3)}\right)}$$
$$=\frac{\left(\frac{2x^2-x-15-x^2}{x(x-3)}\right)}{\left(\frac{x^3+3x^2-(x^3-x^2-5x-3)}{(x-3)(x+3)}\right)}$$
$$=\frac{\left(\frac{x^2-x-15}{x(x-3)}\right)}{\left(\frac{4x^2+5x+3}{(x-3)(x+3)}\right)}$$
$$=\frac{x^2-x-15}{x(x-3)}\cdot\frac{(x-3)(x+3)}{4x^2+5x+3}$$
$$=\frac{(x^2-x-15)(x+3)}{x(4x^2+5x+3)}$$

83. $$1-\frac{1}{1-\frac{1}{x}}=1-\frac{1}{\frac{x-1}{x}}$$
$$=1-\frac{x}{x-1}$$
$$=\frac{x-1-x}{x-1}$$
$$=\frac{-1}{x-1}$$

84. $$1-\frac{1}{1-\frac{1}{1-x}}=1-\frac{1}{\frac{1-x-1}{1-x}}=1-\frac{1}{\frac{-x}{1-x}}$$
$$=1-\frac{1-x}{-x}=1+\frac{1-x}{x}$$
$$=\frac{x+1-x}{x}$$
$$=\frac{1}{x}$$

85. $$\frac{2(x-1)^{-1}+3}{3(x-1)^{-1}+2}=\frac{\frac{2}{x-1}+3}{\frac{3}{x-1}+2}=\frac{\frac{2}{x-1}+\frac{3(x-1)}{x-1}}{\frac{3}{x-1}+\frac{2(x-1)}{x-1}}$$
$$=\frac{\frac{2+3(x-1)}{x-1}}{\frac{3+2(x-1)}{x-1}}$$
$$=\frac{2+3(x-1)}{x-1}\cdot\frac{x-1}{3+2(x-1)}$$
$$=\frac{2+3(x-1)}{3+2(x-1)}=\frac{2+3x-3}{3+2x-2}$$
$$=\frac{3x-1}{2x+1}$$

86. $$\frac{4(x+2)^{-1}-3}{3(x+2)^{-1}-1}=\frac{\frac{4}{x+2}-3}{\frac{3}{x+2}-1}=\frac{\frac{4}{x+2}-\frac{3(x+2)}{x+2}}{\frac{3}{x+2}-\frac{1(x+2)}{x+2}}$$
$$=\frac{\frac{4-3(x+2)}{x+2}}{\frac{3-(x+2)}{x+2}}$$
$$=\frac{4-3(x+2)}{x+2}\cdot\frac{x+2}{3-(x+2)}$$
$$=\frac{4-3(x+2)}{3-(x+2)}=\frac{4-3x-6}{3-x-2}$$
$$=\frac{-3x-2}{-x+1}=\frac{3x+2}{x-1}$$

87. $$\frac{(2x+3)\cdot 3-(3x-5)\cdot 2}{(3x-5)^2}=\frac{6x+9-6x+10}{(3x-5)^2}$$
$$=\frac{19}{(3x-5)^2}$$

88. $$\frac{(4x+1)\cdot 5-(5x-2)\cdot 4}{(5x-2)^2}=\frac{20x+5-20x+8}{(5x-2)^2}$$
$$=\frac{13}{(5x-2)^2}$$

89. $$\frac{x\cdot 2x-(x^2+1)\cdot 1}{(x^2+1)^2}=\frac{2x^2-x^2-1}{(x^2+1)^2}$$
$$=\frac{x^2-1}{(x^2+1)^2}$$
$$=\frac{(x-1)(x+1)}{(x^2+1)^2}$$

90. $$\frac{x\cdot 2x-(x^2-4)\cdot 1}{(x^2-4)^2}=\frac{2x^2-x^2+4}{(x^2-4)^2}=\frac{x^2+4}{(x^2-4)^2}$$
$$=\frac{x^2+4}{(x+2)^2(x-2)^2}$$

91. $$\frac{(3x+1)\cdot 2x-x^2\cdot 3}{(3x+1)^2}=\frac{6x^2+2x-3x^2}{(3x+1)^2}$$
$$=\frac{3x^2+2x}{(3x+1)^2}$$
$$=\frac{x(3x+2)}{(3x+1)^2}$$

92. $$\frac{(2x-5)\cdot 3x^2-x^3\cdot 2}{(2x-5)^2}=\frac{6x^3-15x^2-2x^3}{(2x-5)^2}$$
$$=\frac{4x^3-15x^2}{(2x-5)^2}$$
$$=\frac{x^2(4x-15)}{(2x-5)^2}$$

93. $$\frac{(x^2+1)\cdot 3-(3x+4)\cdot 2x}{(x^2+1)^2}=\frac{3x^2+3-6x^2-8x}{(x^2+1)^2}$$
$$=\frac{-3x^2-8x+3}{(x^2+1)^2}$$
$$=\frac{-(3x^2+8x-3)}{(x^2+1)^2}$$
$$=-\frac{(3x-1)(x+3)}{(x^2+1)^2}$$

94. $$\frac{\left(x^2+9\right)\cdot 2-\left(2x-5\right)\cdot 2x}{\left(x^2+9\right)^2}=\frac{2x^2+18-4x^2+10x}{\left(x^2+9\right)^2}$$
$$=\frac{-2x^2+10x+18}{\left(x^2+9\right)^2}$$
$$=\frac{-2\left(x^2-5x-9\right)}{\left(x^2+9\right)^2}$$

95. $$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$
$$\frac{1}{f}=(n-1)\left(\frac{R_2+R_1}{R_1\cdot R_2}\right)$$
$$\frac{R_1\cdot R_2}{f}=(n-1)\left(R_2+R_1\right)$$
$$\frac{f}{R_1\cdot R_2}=\frac{1}{(n-1)\left(R_2+R_1\right)}$$
$$f=\frac{R_1\cdot R_2}{(n-1)\left(R_2+R_1\right)}$$
$$f=\frac{0.1(0.2)}{(1.5-1)(0.2+0.1)}$$
$$=\frac{0.02}{0.5(0.3)}=\frac{0.02}{0.15}=\frac{2}{15}\text{ meters}$$

96. $$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}=\frac{R_2R_3+R_1R_3+R_1R_2}{R_1R_2R_3}$$
$$R=\frac{R_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2}$$
$$=\frac{5\cdot 4\cdot 10}{4\cdot 10+5\cdot 10+5\cdot 4}$$
$$=\frac{200}{110}=\frac{20}{11}\text{ ohms}$$

97. $$\frac{x^2+3xk-3x-9k}{x^2+2x-15}=\frac{x\left(x+3k\right)-3\left(x+3k\right)}{(x-3)(x+5)}$$
$$\frac{\left(x-3\right)\left(x+3k\right)}{(x-3)(x+5)}=\frac{\left(x+3k\right)}{(x+5)}$$
$$=\frac{x+12}{x+5}$$
$$x+3k=x+12$$
$$3k=12$$
$$k=4$$

98. $$\frac{A}{x+2}+\frac{7}{x-3}=\frac{12x-1}{x^2-x+6}$$
$$A(x-3)+7(x+2)=12x-1$$
$$Ax-3A+7x+14=12x-1$$
$$x(A+7)-3A+14=12x-1$$
$$so$$
$$A+7=12$$
$$A=5$$

99. $$1+\frac{1}{x}=\frac{x+1}{x}\Rightarrow a=1,b=1,c=0$$
$$1+\frac{1}{1+\frac{1}{x}}=1+\frac{1}{\left(\frac{x+1}{x}\right)}=1+\frac{x}{x+1}$$
$$=\frac{x+1+x}{x+1}=\frac{2x+1}{x+1}$$
$$\Rightarrow a=2,b=1,c=1$$

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}=1+\frac{1}{\left(\frac{2x+1}{x+1}\right)}=1+\frac{x+1}{2x+1}$$
$$=\frac{2x+1+x+1}{2x+1}=\frac{3x+2}{2x+1}$$
$$\Rightarrow a=3,b=2,c=1$$

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}=1+\frac{1}{\left(\frac{3x+2}{2x+1}\right)}=1+\frac{2x+1}{3x+2}$$
$$=\frac{3x+2+2x+1}{3x+2}=\frac{5x+3}{3x+2}$$
$$\Rightarrow a=5,b=3,c=2$$

If we continue this process, the values of a, b and c produce the following sequences:

$a:1,2,3,5,8,13,21,....$

$b:1,1,2,3,5,8,13,21,.....$

$c:0,1,1,2,3,5,8,13,21,.....$

In each case we have a *Fibonacci Sequence*, where the next value in the list is obtained from the sum of the previous 2 values in the list.

100. Answers will vary.

101. Answers will vary.

Section R.8

1. 9; –9

2. 4; $|-4| = 4$

3. index

4. cube root

5. b

6. d

7. c

8. c

9. true

10. False; $\sqrt[4]{(-3)^4} = |-3| = 3$

11. $\sqrt[3]{27} = \sqrt[3]{3^3} = 3$

12. $\sqrt[4]{16} = \sqrt[4]{2^4} = 2$

13. $\sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$

14. $\sqrt[3]{-1} = \sqrt[3]{(-1)^3} = -1$

15. $\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

16. $\sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3}$

17. $\sqrt{700} = \sqrt{100 \cdot 7} = 10\sqrt{7}$

18. $\sqrt{45x^3} = \sqrt{9 \cdot 5 \cdot x^2 \cdot x} = 3x\sqrt{5x}$

19. $\sqrt[3]{32} = \sqrt[3]{8 \cdot 4} = 2\sqrt[3]{4}$

20. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = 3\sqrt[3]{2}$

21. $\sqrt[3]{-8x^4} = \sqrt[3]{-8x^3 \cdot x} = -2x\sqrt[3]{x}$

22. $\sqrt[3]{192x^5} = \sqrt[3]{64 \cdot 3x^3 \cdot x^2} = 4x\sqrt[3]{3x^2}$

23. $\sqrt[4]{243} = \sqrt[4]{81 \cdot 3} = 3\sqrt[4]{3}$

24. $\sqrt[4]{48x^5} = \sqrt[4]{16x^4 \cdot 3x} = 2x\sqrt[4]{3x}$

25. $\sqrt[4]{x^{12}y^8} = \sqrt[4]{\left(x^3\right)^4\left(y^2\right)^4} = x^3y^2$

26. $\sqrt[5]{x^{10}y^5} = \sqrt[5]{\left(x^2\right)^5 y^5} = x^2y$

27. $\sqrt[4]{\dfrac{x^9y^7}{xy^3}} = \sqrt[4]{x^8y^4} = x^2y$

28. $\sqrt[3]{\dfrac{3xy^2}{81x^4y^2}} = \sqrt[3]{\dfrac{1}{27x^3}} = \dfrac{\sqrt[3]{1}}{\sqrt[3]{27x^3}} = \dfrac{1}{3x}$

29. $\sqrt{64x} = 8\sqrt{x}$

30. $\sqrt{9x^5} = 3\sqrt{x^4 \cdot x} = 3x^2\sqrt{x}$

31. $\sqrt[4]{162x^9y^{12}} = \sqrt[4]{2(3)^4 x\left(x^2\right)^4\left(y^3\right)^4}$
$= 3x^2y^3\sqrt[4]{2x}$

32. $\sqrt[3]{-40x^{14}y^{10}} = \sqrt[3]{5(-2)^3x^2\left(x^4\right)^3 y\left(y^3\right)^3}$
$= -2x^4y^3\sqrt[3]{5x^2y}$

33. $\sqrt{15x^2}\sqrt{5x} = \sqrt{75x^2 \cdot x} = \sqrt{25 \cdot 3 \cdot x^2 \cdot x} = 5x\sqrt{3x}$

34. $\sqrt{5x}\sqrt{20x^3} = \sqrt{100x^4} = 10x^2$

35. $\left(\sqrt{5}\sqrt[3]{9}\right)^2 = \left(\sqrt{5}\right)^2\left(\sqrt[3]{9}\right)^2$
$= 5 \cdot \sqrt[3]{9^2} = 5\sqrt[3]{81} = 5 \cdot 3\sqrt[3]{3} = 15\sqrt[3]{3}$

36. $\left(\sqrt[3]{3}\sqrt{10}\right)^4 = \left(\sqrt[3]{3}\right)^4\left(\sqrt{10}\right)^4$
$= \sqrt[3]{3^4} \cdot 10^2 = 3\sqrt[3]{3} \cdot 100 = 300\sqrt[3]{3}$

37. $\left(3\sqrt{6}\right)\left(2\sqrt{2}\right) = 6\sqrt{12} = 6\sqrt{4 \cdot 3} = 12\sqrt{3}$

38. $\left(5\sqrt{8}\right)\left(-3\sqrt{3}\right)=-15\sqrt{24}=-30\sqrt{6}$

39. $3\sqrt{2}+4\sqrt{2}=(3+4)\sqrt{2}=7\sqrt{2}$

40. $6\sqrt{5}-4\sqrt{5}=(6-4)\sqrt{5}=2\sqrt{5}$

41. $$\begin{aligned}-\sqrt{48}+5\sqrt{12}&=-\sqrt{16\cdot 3}+5\sqrt{4\cdot 3}\\&=-4\sqrt{3}+5\cdot 2\sqrt{3}\\&=(-4+10)\sqrt{3}\\&=6\sqrt{3}\end{aligned}$$

42. $$\begin{aligned}2\sqrt{12}-3\sqrt{27}&=2\sqrt{4\cdot 3}-3\sqrt{9\cdot 3}\\&=4\sqrt{3}-9\sqrt{3}\\&=(4-9)\sqrt{3}\\&=-5\sqrt{3}\end{aligned}$$

43. $$\begin{aligned}\left(\sqrt{3}+3\right)\left(\sqrt{3}-1\right)&=\left(\sqrt{3}\right)^2+3\sqrt{3}-\sqrt{3}-3\\&=3+2\sqrt{3}-3\\&=2\sqrt{3}\end{aligned}$$

44. $$\begin{aligned}\left(\sqrt{5}-2\right)\left(\sqrt{5}+3\right)&=\left(\sqrt{5}\right)^2-2\sqrt{5}+3\sqrt{5}-6\\&=5+\sqrt{5}-6\\&=\sqrt{5}-1\end{aligned}$$

45. $$\begin{aligned}5\sqrt[3]{2}-2\sqrt[3]{54}&=5\sqrt[3]{2}-2\cdot 3\sqrt[3]{2}\\&=5\sqrt[3]{2}-6\sqrt[3]{2}\\&=(5-6)\sqrt[3]{2}\\&=-\sqrt[3]{2}\end{aligned}$$

46. $$\begin{aligned}9\sqrt[3]{24}-\sqrt[3]{81}&=9\cdot 2\sqrt[3]{3}-3\sqrt[3]{3}\\&=18\sqrt[3]{3}-3\sqrt[3]{3}\\&=(18-3)\sqrt[3]{3}\\&=15\sqrt[3]{3}\end{aligned}$$

47. $$\begin{aligned}\left(\sqrt{x}-1\right)^2&=\left(\sqrt{x}\right)^2-2\sqrt{x}+1\\&=x-2\sqrt{x}+1\end{aligned}$$

48. $$\begin{aligned}\left(\sqrt{x}+\sqrt{5}\right)^2&=\left(\sqrt{x}\right)^2+2\left(\sqrt{x}\right)\left(\sqrt{5}\right)+\left(\sqrt{5}\right)^2\\&=x+2\sqrt{5x}+5\end{aligned}$$

49. $$\begin{aligned}\sqrt[3]{16x^4}-\sqrt[3]{2x}&=\sqrt[3]{8x^3\cdot 2x}-\sqrt[3]{2x}\\&=2x\sqrt[3]{2x}-\sqrt[3]{2x}\\&=(2x-1)\sqrt[3]{2x}\end{aligned}$$

50. $$\begin{aligned}\sqrt[4]{32x}+\sqrt[4]{2x^5}&=\sqrt[4]{16\cdot 2x}+\sqrt[4]{x^4\cdot 2x}\\&=2\sqrt[4]{2x}+x\sqrt[4]{2x}\\&=(2+x)\sqrt[4]{2x}\ \text{ or }\ (x+2)\sqrt[4]{2x}\end{aligned}$$

51. $$\begin{aligned}\sqrt{8x^3}-3\sqrt{50x}&=\sqrt{4x^2\cdot 2x}-3\sqrt{25\cdot 2x}\\&=2x\sqrt{2x}-15\sqrt{2x}\\&=(2x-15)\sqrt{2x}\end{aligned}$$

52. $$\begin{aligned}3x\sqrt{9y}+4\sqrt{25y}&=9x\sqrt{y}+20\sqrt{y}\\&=(9x+20)\sqrt{y}\end{aligned}$$

53. $$\begin{aligned}&\sqrt[3]{16x^4y}-3x\sqrt[3]{2xy}+5\sqrt[3]{-2xy^4}\\&=\sqrt[3]{8x^3\cdot 2xy}-3x\sqrt[3]{2xy}+5\sqrt[3]{-y^3\cdot 2xy}\\&=2x\sqrt[3]{2xy}-3x\sqrt[3]{2xy}-5y\sqrt[3]{2xy}\\&=(2x-3x-5y)\sqrt[3]{2xy}\\&=(-x-5y)\sqrt[3]{2xy}\ \text{ or }\ -(x+5y)\sqrt[3]{2xy}\end{aligned}$$

54. $$\begin{aligned}8xy-\sqrt{25x^2y^2}+\sqrt[3]{8x^3y^3}&=8xy-5xy+2xy\\&=(8-5+2)xy\\&=5xy\end{aligned}$$

55. $\dfrac{1}{\sqrt{2}}=\dfrac{1}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}$

56. $\dfrac{2}{\sqrt{3}}=\dfrac{2}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{2\sqrt{3}}{3}$

57. $\dfrac{-\sqrt{3}}{\sqrt{5}}=\dfrac{-\sqrt{3}}{\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}}=\dfrac{-\sqrt{15}}{5}$

58. $\dfrac{-\sqrt{3}}{\sqrt{8}}=\dfrac{-\sqrt{3}}{2\sqrt{2}}=\dfrac{-\sqrt{3}}{2\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{-\sqrt{6}}{2\cdot 2}=\dfrac{-\sqrt{6}}{4}$

59. $\dfrac{\sqrt{3}}{5-\sqrt{2}}=\dfrac{\sqrt{3}}{5-\sqrt{2}}\cdot\dfrac{5+\sqrt{2}}{5+\sqrt{2}}$
$=\dfrac{\sqrt{3}\left(5+\sqrt{2}\right)}{25-2}$
$=\dfrac{\sqrt{3}\left(5+\sqrt{2}\right)}{23}$ or $\dfrac{5\sqrt{3}+\sqrt{6}}{23}$

60. $\dfrac{\sqrt{2}}{\sqrt{7}+2}=\dfrac{\sqrt{2}}{\sqrt{7}+2}\cdot\dfrac{\sqrt{7}-2}{\sqrt{7}-2}$
$=\dfrac{\sqrt{2}\left(\sqrt{7}-2\right)}{7-4}$
$=\dfrac{\sqrt{2}\left(\sqrt{7}-2\right)}{3}$ or $\dfrac{\sqrt{14}-2\sqrt{2}}{3}$

61. $\dfrac{2-\sqrt{5}}{2+3\sqrt{5}}=\dfrac{2-\sqrt{5}}{2+3\sqrt{5}}\cdot\dfrac{2-3\sqrt{5}}{2-3\sqrt{5}}$
$=\dfrac{4-2\sqrt{5}-6\sqrt{5}+15}{4-45}$
$=\dfrac{19-8\sqrt{5}}{-41}=\dfrac{8\sqrt{5}-19}{41}$

62. $\dfrac{\sqrt{3}-1}{2\sqrt{3}+3}=\dfrac{\sqrt{3}-1}{2\sqrt{3}+3}\cdot\dfrac{2\sqrt{3}-3}{2\sqrt{3}-3}$
$=\dfrac{6-2\sqrt{3}-3\sqrt{3}+3}{12-9}=\dfrac{9-5\sqrt{3}}{3}$

63. $\dfrac{5}{\sqrt{2}-1}=\dfrac{5}{\sqrt{2}-1}\cdot\dfrac{\sqrt{2}+1}{\sqrt{2}+1}$
$=\dfrac{5\sqrt{2}+5}{2-1}=5\sqrt{2}+5$

64. $\dfrac{-3}{\sqrt{5}+4}=\dfrac{-3}{\sqrt{5}+4}\cdot\dfrac{\sqrt{5}-4}{\sqrt{5}-4}$
$=\dfrac{-3\sqrt{5}+12}{5-16}=\dfrac{-3\sqrt{5}+12}{-11}$
$=\dfrac{3\sqrt{5}-12}{11}$

65. $\dfrac{5}{\sqrt[3]{2}}=\dfrac{5}{\sqrt[3]{2}}\cdot\dfrac{\sqrt[3]{4}}{\sqrt[3]{4}}=\dfrac{5\sqrt[3]{4}}{2}$

66. $\dfrac{-2}{\sqrt[3]{9}}=\dfrac{-2}{\sqrt[3]{9}}\cdot\dfrac{\sqrt[3]{3}}{\sqrt[3]{3}}=\dfrac{-2\sqrt[3]{3}}{3}$

67. $\dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=\dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\cdot\dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}-\sqrt{x}}$
$=\dfrac{(x+h)-2\sqrt{x(x+h)}+x}{(x+h)-x}$
$=\dfrac{x+h-2\sqrt{x^2+xh}+x}{x+h-x}$
$=\dfrac{2x+h-2\sqrt{x^2+xh}}{h}$

68. $\dfrac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}-\sqrt{x-h}}$
$=\dfrac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}-\sqrt{x-h}}\cdot\dfrac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}+\sqrt{x-h}}$
$=\dfrac{(x+h)+2\sqrt{(x-h)(x+h)}+(x-h)}{(x+h)-(x-h)}$
$=\dfrac{x+h+2\sqrt{x^2-h^2}+x-h}{x+h-x+h}$
$=\dfrac{2x+2\sqrt{x^2-h^2}}{2h}$
$=\dfrac{x+\sqrt{x^2-h^2}}{h}$

69. $\dfrac{\sqrt{11}+1}{2}=\dfrac{\sqrt{11}+1}{2}\cdot\dfrac{\sqrt{11}-1}{\sqrt{11}-1}$
$=\dfrac{11-1}{2\left(\sqrt{11}-1\right)}=\dfrac{10}{2\left(\sqrt{11}-1\right)}$
$=\dfrac{5}{\sqrt{11}-1}$

70. $\dfrac{5-\sqrt{43}}{3}=\dfrac{5-\sqrt{43}}{3}\cdot\dfrac{5+\sqrt{43}}{5+\sqrt{43}}=\dfrac{25-43}{3\left(5+\sqrt{43}\right)}$
$=\dfrac{-18}{3\left(5+\sqrt{43}\right)}=\dfrac{-6}{5+\sqrt{43}}=-\dfrac{6}{5+\sqrt{43}}$

71. $\dfrac{\sqrt{6}-\sqrt{15}}{\sqrt{15}}=\dfrac{\sqrt{6}-\sqrt{15}}{\sqrt{15}}\cdot\dfrac{\sqrt{6}+\sqrt{15}}{\sqrt{6}+\sqrt{15}}$
$=\dfrac{6-15}{\sqrt{90}-15}=\dfrac{-9}{3\sqrt{10}-15}$
$=\dfrac{-9}{3\left(\sqrt{10}-5\right)}=-\dfrac{3}{\sqrt{10}-5}$

72. $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}}=\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}}\cdot\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$
$=\dfrac{5-3}{5-\sqrt{15}}=\dfrac{2}{5-\sqrt{15}}$

73. $\dfrac{\sqrt{x}-\sqrt{c}}{x-c}=\dfrac{\sqrt{x}-\sqrt{c}}{x-c}\cdot\dfrac{\sqrt{x}+\sqrt{c}}{\sqrt{x}+\sqrt{c}}$
$=\dfrac{x-c}{(x-c)\left(\sqrt{x}+\sqrt{c}\right)}=\dfrac{1}{\sqrt{x}+\sqrt{c}}$

74. $\dfrac{\sqrt{x}-2}{x-4}=\dfrac{\sqrt{x}-2}{x-4}\cdot\dfrac{\sqrt{x}+2}{\sqrt{x}+2}$
$=\dfrac{x-4}{(x-4)\left(\sqrt{x}+2\right)}=\dfrac{1}{\sqrt{x}+2}$

75. $\dfrac{\sqrt{x-7}-1}{x-8}=\dfrac{\sqrt{x-7}-1}{x-8}\cdot\dfrac{\sqrt{x-7}+1}{\sqrt{x-7}+1}$
$=\dfrac{x-7-1}{(x-8)\left(\sqrt{x-7}+1\right)}$
$=\dfrac{x-8}{(x-8)\left(\sqrt{x-7}+1\right)}$
$=\dfrac{1}{\sqrt{x-7}+1}$

76. $\dfrac{4-\sqrt{x-9}}{x-25}=\dfrac{4-\sqrt{x-9}}{x-25}\cdot\dfrac{4+\sqrt{x-9}}{4+\sqrt{x-9}}$
$=\dfrac{16-(x-9)}{(x-25)\left(4+\sqrt{x-9}\right)}$
$=\dfrac{25-x}{(x-25)\left(4+\sqrt{x-9}\right)}$
$=-\dfrac{x-25}{(x-25)\left(4+\sqrt{x-9}\right)}$
$=-\dfrac{1}{4+\sqrt{x-9}}$

77. $8^{2/3}=\left(\sqrt[3]{8}\right)^2=2^2=4$

78. $4^{3/2}=\left(\sqrt{4}\right)^3=2^3=8$

79. $(-64)^{1/3}=\sqrt[3]{-64}=-4$

80. $16^{3/4}=\left(\sqrt[4]{16}\right)^3=2^3=8$

81. $100^{3/2}=\left(\sqrt{100}\right)^3=10^3=1000$

82. $25^{3/2}=\left(\sqrt{25}\right)^3=5^3=125$

83. $4^{-3/2}=\dfrac{1}{4^{3/2}}=\dfrac{1}{\left(\sqrt{4}\right)^3}=\dfrac{1}{2^3}=\dfrac{1}{8}$

84. $16^{-3/2}=\dfrac{1}{16^{3/2}}=\dfrac{1}{\left(\sqrt{16}\right)^3}=\dfrac{1}{4^3}=\dfrac{1}{64}$

85. $\left(\dfrac{9}{8}\right)^{3/2}=\left(\sqrt{\dfrac{9}{8}}\right)^3=\left(\dfrac{3}{2\sqrt{2}}\right)^3=\dfrac{3^3}{2^3\left(\sqrt{2}\right)^3}$
$=\dfrac{27}{8\cdot 2\sqrt{2}}=\dfrac{27}{16\sqrt{2}}=\dfrac{27}{16\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}$
$=\dfrac{27\sqrt{2}}{32}$

86. $\left(\dfrac{27}{8}\right)^{2/3}=\left(\sqrt[3]{\dfrac{27}{8}}\right)^2=\left(\dfrac{3}{2}\right)^2=\dfrac{9}{4}$

87. $\left(\dfrac{8}{9}\right)^{-3/2}=\left(\dfrac{9}{8}\right)^{3/2}=\left(\sqrt{\dfrac{9}{8}}\right)^3=\left(\dfrac{3}{2\sqrt{2}}\right)^3$
$=\dfrac{3^3}{2^3\left(\sqrt{2}\right)^3}=\dfrac{27}{8\cdot 2\sqrt{2}}=\dfrac{27}{16\sqrt{2}}$
$=\dfrac{27}{16\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{27\sqrt{2}}{32}$

88. $\left(\dfrac{8}{27}\right)^{-2/3}=\left(\dfrac{27}{8}\right)^{2/3}=\left(\sqrt[3]{\dfrac{27}{8}}\right)^2=\left(\dfrac{3}{2}\right)^2=\dfrac{9}{4}$

89. $(-1000)^{-1/3}=\left(-\dfrac{1}{1000}\right)^{1/3}=\sqrt[3]{-\dfrac{1}{1000}}=-\dfrac{1}{10}$

90. $-25^{-1/2}=-\left(\dfrac{1}{25}\right)^{1/2}=-\sqrt{\dfrac{1}{25}}=-\dfrac{1}{5}$

91. $\left(-\dfrac{64}{125}\right)^{-2/3}=\left(-\dfrac{125}{64}\right)^{2/3}=\left(\sqrt[3]{-\dfrac{125}{64}}\right)^{2}$
$=\left(-\dfrac{5}{4}\right)^{2}=\dfrac{25}{16}$

92. $-81^{-3/4}=-\left(\dfrac{1}{81}\right)^{3/4}=-\left(\sqrt[4]{\dfrac{1}{81}}\right)^{3}$
$=-\left(\dfrac{1}{3}\right)^{3}=-\dfrac{1}{27}$

93. $x^{3/4}x^{1/3}x^{-1/2}=x^{3/4+1/3-1/2}=x^{7/12}$

94. $x^{2/3}x^{1/2}x^{-1/4}=x^{2/3+1/2-1/4}=x^{11/12}$

95. $\left(x^3y^6\right)^{1/3}=\left(x^3\right)^{1/3}\left(y^6\right)^{1/3}=xy^2$

96. $\left(x^4y^8\right)^{3/4}=\left(x^4\right)^{3/4}\left(y^8\right)^{3/4}=x^3y^6$

97. $\dfrac{\left(x^2y\right)^{1/3}\left(xy^2\right)^{2/3}}{x^{2/3}y^{2/3}}=\dfrac{\left(x^2\right)^{1/3}(y)^{1/3}(x)^{2/3}\left(y^2\right)^{2/3}}{x^{2/3}y^{2/3}}$
$=\dfrac{x^{2/3}y^{1/3}x^{2/3}y^{4/3}}{x^{2/3}y^{2/3}}$
$=x^{2/3+2/3-2/3}y^{1/3+4/3-2/3}$
$=x^{2/3}y^{1}=x^{2/3}y$

98. $\dfrac{(xy)^{1/4}\left(x^2y^2\right)^{1/2}}{\left(x^2y\right)^{3/4}}=\dfrac{x^{1/4}y^{1/4}\left(x^2\right)^{1/2}\left(y^2\right)^{1/2}}{\left(x^2\right)^{3/4}y^{3/4}}$
$=\dfrac{x^{1/4}y^{1/4}xy}{x^{3/2}y^{3/4}}$
$=x^{1/4+1-3/2}y^{1/4+1-3/4}$
$=x^{-1/4}y^{1/2}=\dfrac{y^{1/2}}{x^{1/4}}$

99. $\dfrac{\left(16x^2y^{-1/3}\right)^{3/4}}{\left(xy^2\right)^{1/4}}=\dfrac{16^{3/4}\left(x^2\right)^{3/4}\left(y^{-1/3}\right)^{3/4}}{x^{1/4}\left(y^2\right)^{1/4}}$
$=\dfrac{\left(\sqrt[4]{16}\right)^3x^{3/2}y^{-1/4}}{x^{1/4}y^{1/2}}$
$=2^3x^{3/2-1/4}y^{-1/4-1/2}$
$=8x^{5/4}y^{-3/4}$
$=\dfrac{8x^{5/4}}{y^{3/4}}$

100. $\dfrac{\left(4x^{-1}y^{1/3}\right)^{3/2}}{(xy)^{3/2}}=\dfrac{4^{3/2}\left(x^{-1}\right)^{3/2}\left(y^{1/3}\right)^{3/2}}{x^{3/2}y^{3/2}}$
$=\dfrac{\left(\sqrt{4}\right)^3x^{-3/2}y^{1/2}}{x^{3/2}y^{3/2}}$
$=2^3x^{-3/2-3/2}y^{1/2-3/2}$
$=8x^{-3}y^{-1}$
$=\dfrac{8}{x^3y}$

101. $\dfrac{x}{(1+x)^{1/2}}+2(1+x)^{1/2}=\dfrac{x+2(1+x)^{1/2}(1+x)^{1/2}}{(1+x)^{1/2}}$
$=\dfrac{x+2(1+x)}{(1+x)^{1/2}}$
$=\dfrac{x+2+2x}{(1+x)^{1/2}}$
$=\dfrac{3x+2}{(1+x)^{1/2}}$

102. $\dfrac{1+x}{2x^{1/2}}+x^{1/2}=\dfrac{1+x+x^{1/2}\cdot 2x^{1/2}}{2x^{1/2}}$
$=\dfrac{1+x+2x}{2x^{1/2}}=\dfrac{3x+1}{2x^{1/2}}$

103. $2x\left(x^2+1\right)^{1/2}+x^2\cdot\frac{1}{2}\left(x^2+1\right)^{-1/2}\cdot 2x$

$=2x\left(x^2+1\right)^{1/2}+\frac{x^3}{\left(x^2+1\right)^{1/2}}$

$=\frac{2x\left(x^2+1\right)^{1/2}\cdot\left(x^2+1\right)^{1/2}+x^3}{\left(x^2+1\right)^{1/2}}$

$=\frac{2x\left(x^2+1\right)^{1/2+1/2}+x^3}{\left(x^2+1\right)^{1/2}}=\frac{2x\left(x^2+1\right)^{1}+x^3}{\left(x^2+1\right)^{1/2}}$

$=\frac{2x^3+2x+x^3}{\left(x^2+1\right)^{1/2}}=\frac{3x^3+2x}{\left(x^2+1\right)^{1/2}}$

$=\frac{x\left(3x^2+2\right)}{\left(x^2+1\right)^{1/2}}$

104. $(x+1)^{1/3}+x\cdot\frac{1}{3}(x+1)^{-2/3},\ x\neq -1$

$=(x+1)^{1/3}+\frac{x}{3(x+1)^{2/3}}$

$=\frac{3(x+1)^{2/3}(x+1)^{1/3}+x}{3(x+1)^{2/3}}$

$=\frac{3(x+1)^{2/3+1/3}+x}{3(x+1)^{2/3}}=\frac{3(x+1)^{1}+x}{3(x+1)^{2/3}}$

$=\frac{3x+3+x}{3(x+1)^{2/3}}=\frac{4x+3}{3(x+1)^{2/3}}$

105. $\sqrt{4x+3}\cdot\frac{1}{2\sqrt{x-5}}+\sqrt{x-5}\cdot\frac{1}{5\sqrt{4x+3}},\ x>5$

$=\frac{\sqrt{4x+3}}{2\sqrt{x-5}}+\frac{\sqrt{x-5}}{5\sqrt{4x+3}}$

$=\frac{\sqrt{4x+3}\cdot 5\cdot\sqrt{4x+3}+\sqrt{x-5}\cdot 2\cdot\sqrt{x-5}}{10\sqrt{x-5}\sqrt{4x+3}}$

$=\frac{5(4x+3)+2(x-5)}{10\sqrt{(x-5)(4x+3)}}$

$=\frac{20x+15+2x-10}{10\sqrt{(x-5)(4x+3)}}$

$=\frac{22x+5}{10\sqrt{(x-5)(4x+3)}}$

106. $\frac{\sqrt[3]{8x+1}}{3\sqrt[3]{(x-2)^2}}+\frac{\sqrt[3]{x-2}}{24\sqrt[3]{(8x+1)^2}},\ x\neq 2, x\neq -\frac{1}{8}$

$=\frac{8\sqrt[3]{8x+1}\cdot\sqrt[3]{(8x+1)^2}+\sqrt[3]{x-2}\cdot\sqrt[3]{(x-2)^2}}{24\sqrt[3]{(x-2)^2}\cdot\sqrt[3]{(8x+1)^2}}$

$=\frac{8\sqrt[3]{(8x+1)^3}+\sqrt[3]{(x-2)^3}}{24\sqrt[3]{(x-2)^2}\cdot\sqrt[3]{(8x+1)^2}}$

$=\frac{8(8x+1)+x-2}{24\sqrt[3]{(x-2)^2(8x+1)^2}}$

$=\frac{64x+8+x-2}{24\sqrt[3]{(x-2)^2(8x+1)^2}}$

$=\frac{65x+6}{24\sqrt[3]{(x-2)^2(8x+1)^2}}$

107. $\frac{\left(\sqrt{1+x}-x\cdot\frac{1}{2\sqrt{1+x}}\right)}{1+x}=\frac{\left(\sqrt{1+x}-\frac{x}{2\sqrt{1+x}}\right)}{1+x}$

$=\frac{\left(\frac{2\sqrt{1+x}\sqrt{1+x}-x}{2\sqrt{1+x}}\right)}{1+x}$

$=\frac{2(1+x)-x}{2(1+x)^{1/2}}\cdot\frac{1}{1+x}$

$=\frac{2+x}{2(1+x)^{3/2}}$

108. $\frac{\left(\sqrt{x^2+1}-x\cdot\frac{2x}{2\sqrt{x^2+1}}\right)}{x^2+1}$

$=\frac{\left(\sqrt{x^2+1}-\frac{x^2}{\sqrt{x^2+1}}\right)}{x^2+1}$

$=\frac{\left(\sqrt{x^2+1}\cdot\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}-\frac{x^2}{\sqrt{x^2+1}}\right)}{x^2+1}$

$=\frac{\left(\frac{x^2+1-x^2}{\sqrt{x^2+1}}\right)}{x^2+1}=\frac{1}{\sqrt{x^2+1}}\cdot\frac{1}{x^2+1}$

$=\frac{1}{\left(x^2+1\right)^{3/2}}$

109. $$\frac{(x+4)^{1/2}-2x(x+4)^{-1/2}}{x+4}$$

$$=\frac{\left((x+4)^{1/2}-\frac{2x}{(x+4)^{1/2}}\right)}{x+4}$$

$$=\frac{\left((x+4)^{1/2}\cdot\frac{(x+4)^{1/2}}{(x+4)^{1/2}}-\frac{2x}{(x+4)^{1/2}}\right)}{x+4}$$

$$=\frac{\left(\frac{x+4-2x}{(x+4)^{1/2}}\right)}{x+4}$$

$$=\frac{-x+4}{(x+4)^{1/2}}\cdot\frac{1}{x+4}$$

$$=\frac{-x+4}{(x+4)^{3/2}}$$

$$=\frac{4-x}{(x+4)^{3/2}}$$

110. $$\frac{(9-x^2)^{1/2}+x^2(9-x^2)^{-1/2}}{9-x^2}, -3<x<3$$

$$\frac{\left((9-x^2)^{1/2}+\frac{x^2}{(9-x^2)^{1/2}}\right)}{9-x^2}$$

$$=\frac{\left(\frac{(9-x^2)^{1/2}\cdot(9-x^2)^{1/2}+x^2}{(9-x^2)^{1/2}}\right)}{9-x^2}$$

$$=\frac{(9-x^2)^{1/2}\cdot(9-x^2)^{1/2}+x^2}{(9-x^2)^{1/2}}\cdot\frac{1}{9-x^2}$$

$$=\frac{9-x^2+x^2}{(9-x^2)^{1/2}}\cdot\frac{1}{9-x^2}$$

$$=\frac{9}{(9-x^2)^{3/2}}$$

111. $$\frac{\frac{x^2}{(x^2-1)^{1/2}}-(x^2-1)^{1/2}}{x^2}, x<-1 \text{ or } x>1$$

$$=\frac{\left(\frac{x^2-(x^2-1)^{1/2}\cdot(x^2-1)^{1/2}}{(x^2-1)^{1/2}}\right)}{x^2}$$

$$=\frac{x^2-(x^2-1)^{1/2}\cdot(x^2-1)^{1/2}}{(x^2-1)^{1/2}}\cdot\frac{1}{x^2}$$

$$=\frac{x^2-(x^2-1)}{(x^2-1)^{1/2}}\cdot\frac{1}{x^2}$$

$$=\frac{x^2-x^2+1}{(x^2-1)^{1/2}}\cdot\frac{1}{x^2}$$

$$=\frac{1}{x^2(x^2-1)^{1/2}}$$

112. $$\frac{(x^2+4)^{1/2}-x^2(x^2+4)^{-1/2}}{x^2+4}$$

$$=\frac{\left((x^2+4)^{1/2}-\frac{x^2}{(x^2+4)^{1/2}}\right)}{x^2+4}$$

$$=\frac{\left(\frac{(x^2+4)^{1/2}\cdot(x^2+4)^{1/2}-x^2}{(x^2+4)^{1/2}}\right)}{x^2+4}$$

$$=\frac{(x^2+4)^{1/2}\cdot(x^2+4)^{1/2}-x^2}{(x^2+4)^{1/2}}\cdot\frac{1}{x^2+4}$$

$$=\frac{x^2+4-x^2}{(x^2+4)^{1/2}}\cdot\frac{1}{x^2+4}=\frac{4}{(x^2+4)^{3/2}}$$

113. $\dfrac{\dfrac{1+x^2}{2\sqrt{x}}-2x\sqrt{x}}{\left(1+x^2\right)^2}, x>0$

$$=\frac{\left(\dfrac{1+x^2-\left(2\sqrt{x}\right)\left(2x\sqrt{x}\right)}{2\sqrt{x}}\right)}{\left(1+x^2\right)^2}$$

$$=\frac{1+x^2-\left(2\sqrt{x}\right)\left(2x\sqrt{x}\right)}{2\sqrt{x}}\cdot\frac{1}{\left(1+x^2\right)^2}$$

$$=\frac{1+x^2-4x^2}{2\sqrt{x}}\cdot\frac{1}{\left(1+x^2\right)^2}=\frac{1-3x^2}{2\sqrt{x}\left(1+x^2\right)^2}$$

114. $\dfrac{2x\left(1-x^2\right)^{1/3}+\dfrac{2}{3}x^3\left(1-x^2\right)^{-2/3}}{\left(1-x^2\right)^{2/3}}, x\neq -1, x\neq 1$

$$=\frac{\left(2x\left(1-x^2\right)^{1/3}+\dfrac{2x^3}{3\left(1-x^2\right)^{2/3}}\right)}{\left(1-x^2\right)^{2/3}}$$

$$=\frac{\left(\dfrac{2x\left(1-x^2\right)^{1/3}3\left(1-x^2\right)^{2/3}+2x^3}{3\left(1-x^2\right)^{2/3}}\right)}{\left(1-x^2\right)^{2/3}}$$

$$=\frac{6x\left(1-x^2\right)^{1/3+2/3}+2x^3}{3\left(1-x^2\right)^{2/3}}\cdot\frac{1}{\left(1-x^2\right)^{2/3}}$$

$$=\frac{6x\left(1-x^2\right)+2x^3}{3\left(1-x^2\right)^{2/3+2/3}}=\frac{6x-6x^3+2x^3}{3\left(1-x^2\right)^{4/3}}$$

$$=\frac{6x-4x^3}{3\left(1-x^2\right)^{4/3}}=\frac{2x\left(3-2x^2\right)}{3\left(1-x^2\right)^{4/3}}$$

115. $(x+1)^{3/2}+x\cdot\dfrac{3}{2}(x+1)^{1/2}$

$$=(x+1)^{1/2}\left(x+1+\frac{3}{2}x\right)$$

$$=(x+1)^{1/2}\left(\frac{5}{2}x+1\right)$$

$$=\frac{1}{2}(x+1)^{1/2}\left(5x+2\right)$$

116. $(x^2+4)^{4/3}+x\cdot\dfrac{4}{3}(x^2+4)^{1/3}\cdot 2x$

$$=(x^2+4)^{1/3}\left(x^2+4+\frac{8}{3}x^2\right)$$

$$=(x^2+4)^{1/3}\left(\frac{11}{3}x^2+4\right)$$

$$=\frac{1}{3}\left(x^2+4\right)^{1/3}\left(11x^2+12\right)$$

117. $6x^{1/2}\left(x^2+x\right)-8x^{3/2}-8x^{1/2}$

$$=2x^{1/2}\left(3(x^2+x)-4x-4\right)$$

$$=2x^{1/2}\left(3x^2-x-4\right)$$

$$=2x^{1/2}(3x-4)(x+1)$$

118. $6x^{1/2}\left(2x+3\right)+x^{3/2}\cdot 8$

$$=2x^{1/2}\left(3(2x+3)+4x\right)$$

$$=2x^{1/2}\left(10x+9\right)$$

119. $3\left(x^2+4\right)^{4/3}+x\cdot 4\left(x^2+4\right)^{1/3}\cdot 2x$

$$=\left(x^2+4\right)^{1/3}\left[3\left(x^2+4\right)+8x^2\right]$$

$$=\left(x^2+4\right)^{1/3}\left[3x^2+12+8x^2\right]$$

$$=\left(x^2+4\right)^{1/3}\left(11x^2+12\right)$$

120. $2x\left(3x+4\right)^{4/3}+x^2\cdot 4\left(3x+4\right)^{1/3}$

$$=2x\left(3x+4\right)^{1/3}\left[\left(3x+4\right)+2x\right]$$

$$=2x\left(3x+4\right)^{1/3}\left(5x+4\right)$$

121. $4\left(3x+5\right)^{1/3}\left(2x+3\right)^{3/2}+3\left(3x+5\right)^{4/3}\left(2x+3\right)^{1/2}$

$$=\left(3x+5\right)^{1/3}\left(2x+3\right)^{1/2}\left[4\left(2x+3\right)+3\left(3x+5\right)\right]$$

$$=\left(3x+5\right)^{1/3}\left(2x+3\right)^{1/2}\left(8x+12+9x+15\right)$$

$$=\left(3x+5\right)^{1/3}\left(2x+3\right)^{1/2}\left(17x+27\right)$$

where $x\geq -\dfrac{3}{2}$.

122. $6(6x+1)^{1/3}(4x-3)^{3/2}+6(6x+1)^{4/3}(4x-3)^{1/2}$

$$=6(6x+1)^{1/3}(4x-3)^{1/2}\left[(4x-3)+(6x+1)\right]$$
$$=6(6x+1)^{1/3}(4x-3)^{1/2}(10x-2)$$
$$=6(6x+1)^{1/3}(4x-3)^{1/2}(2)(5x-1)$$
$$=12(6x+1)^{1/3}(4x-3)^{1/2}(5x-1)$$

where $x \ge \frac{3}{4}$.

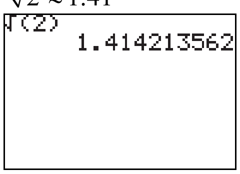
123. $3x^{-1/2}+\frac{3}{2}x^{1/2}, x>0$

$$=\frac{3}{x^{1/2}}+\frac{3}{2}x^{1/2}$$
$$=\frac{3\cdot 2+3x^{1/2}\cdot x^{1/2}}{2x^{1/2}}=\frac{6+3x}{2x^{1/2}}=\frac{3(x+2)}{2x^{1/2}}$$

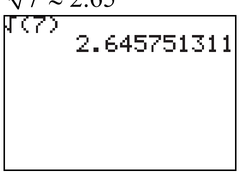
124. $8x^{1/3}-4x^{-2/3}, x\ne 0$

$$=8x^{1/3}-\frac{4}{x^{2/3}}$$
$$=\frac{8x^{1/3}\cdot x^{2/3}-4}{x^{2/3}}=\frac{8x-4}{x^{2/3}}=\frac{4(2x-1)}{x^{2/3}}$$

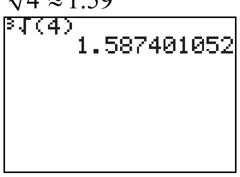
125. $\sqrt{2}\approx 1.41$



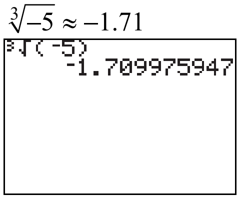
126. $\sqrt{7}\approx 2.65$



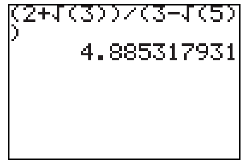
127. $\sqrt[3]{4}\approx 1.59$



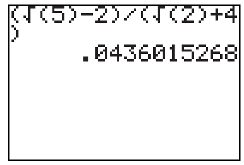
128. $\sqrt[3]{-5}\approx -1.71$



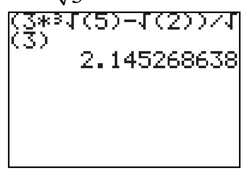
129. $\frac{2+\sqrt{3}}{3-\sqrt{5}}\approx 4.89$



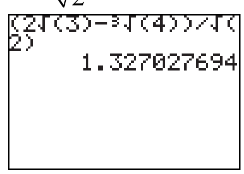
130. $\frac{\sqrt{5}-2}{\sqrt{2}+4}\approx 0.04$



131. $\frac{3\sqrt[3]{5}-\sqrt{2}}{\sqrt{3}}\approx 2.15$



132. $\frac{2\sqrt{3}-\sqrt[3]{4}}{\sqrt{2}}\approx 1.33$



133. **a.** $V=40(12)^2\sqrt{\frac{96}{12}-0.608}$

$\approx 15{,}660.4$ gallons

b. $V=40(1)^2\sqrt{\frac{96}{1}-0.608}\approx 390.7$ gallons

134. **a.** $v=\sqrt{64\cdot 4+0^2}=\sqrt{256}$

$=16$ feet per second

b. $v=\sqrt{64\cdot 16+0^2}=\sqrt{1024}$

$=32$ feet per second

c. $v=\sqrt{64\cdot 2+4^2}=\sqrt{144}$

$=12$ feet per second

135. $T=2\pi\sqrt{\frac{64}{32}}=2\pi\sqrt{2}\approx 8.89$ seconds

136. $T = 2\pi\sqrt{\frac{16}{32}} = 2\pi\sqrt{\frac{1}{2}} = \frac{2\pi}{\sqrt{2}}$

$= \pi\sqrt{2} \approx 4.44$ seconds

137.
$$\begin{array}{r|rrrr} \sqrt{3} & 1 & -4 & -3 & 13 \\ & & \sqrt{3} & 3-4\sqrt{3} & -12 \\ \hline & 1 & \sqrt{3}-4 & -4\sqrt{3} & 1 \end{array}$$

The quotient is $x^2 + (\sqrt{3}-4)x - 4\sqrt{3}$

The remainder is 1

138.
$$\begin{array}{r|rrrr} 1+\sqrt{2} & 1 & -9 & 13 & 7 \\ & & 1+\sqrt{2} & -6-7\sqrt{2} & -7 \\ \hline & 1 & \sqrt{3}-8 & -7-7\sqrt{2} & 0 \end{array}$$

Yes, $1+\sqrt{2}$ is a factor of $x^3 - 9x^2 + 13x + 7$.

139. Answers may vary. One possibility follows: If $a = -5$, then $\sqrt{a^2} = \sqrt{(-5)^2} = \sqrt{25} = 5 \neq a$.

Since we use the principal square root, which is always non-negative,

$$\sqrt{a^2} = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

which is the definition of $|a|$, so $\sqrt{a^2} = |a|$.

Chapter 1

Equations and Inequalities

Section 1.1

1. Distributive

2. Zero-Product

3. $\{x \mid x \neq 4\}$

4. False. Multiplying both sides of an equation by zero will not result in an equivalent equation.

5. identity

6. linear; first-degree

7. False. The solution is $\frac{8}{3}$.

8. True

9. b

10. d

11. $$\begin{aligned} 7x &= 21 \\ \frac{7x}{7} &= \frac{21}{7} \\ x &= 3 \end{aligned}$$
The solution set is $\{3\}$.

12. $$\begin{aligned} 6x &= -24 \\ \frac{6x}{6} &= \frac{-24}{6} \\ x &= -4 \end{aligned}$$
The solution set is $\{-4\}$.

13. $$\begin{aligned} 3x+15 &= 0 \\ 3x+15-15 &= 0-15 \\ 3x &= -15 \\ \frac{3x}{3} &= \frac{-15}{3} \\ x &= -5 \end{aligned}$$
The solution set is $\{-5\}$.

14. $$\begin{aligned} 6x+18 &= 0 \\ 6x+18-18 &= 0-18 \\ 6x &= -18 \\ \frac{6x}{6} &= \frac{-18}{6} \\ x &= -3 \end{aligned}$$
The solution set is $\{-3\}$.

15. $$\begin{aligned} 2x-3 &= 0 \\ 2x-3+3 &= 0+3 \\ 2x &= 3 \\ \frac{2x}{2} &= \frac{3}{2} \\ x &= \frac{3}{2} \end{aligned}$$
The solution set is $\left\{\frac{3}{2}\right\}$.

16. $$\begin{aligned} 3x+4 &= 0 \\ 3x+4-4 &= 0-4 \\ 3x &= -4 \\ \frac{3x}{3} &= \frac{-4}{3} \\ x &= -\frac{4}{3} \end{aligned}$$
The solution set is $\left\{-\frac{4}{3}\right\}$.

17. $$\begin{aligned} \frac{1}{4}x &= \frac{7}{20} \\ 4\left(\frac{1}{4}x\right) &= 4\left(\frac{7}{20}\right) \\ x &= \frac{28}{20} = \frac{7}{5} \end{aligned}$$
The solution set is $\left\{\frac{7}{5}\right\}$.

18.
$$\begin{aligned}\frac{2}{3}x &= \frac{9}{2}\\ 6\left(\frac{2}{3}x\right) &= 6\left(\frac{9}{2}\right)\\ 4x &= 27\\ \frac{4x}{4} &= \frac{27}{4}\\ x &= \frac{27}{4}\end{aligned}$$
The solution set is $\left\{\frac{27}{4}\right\}$.

19.
$$\begin{aligned}3x+4 &= x\\ 3x+4-4 &= x-4\\ 3x &= x-4\\ 3x-x &= x-4-x\\ 2x &= -4\\ \frac{2x}{2} &= \frac{-4}{2}\\ x &= -2\end{aligned}$$
The solution set is $\{-2\}$.

20.
$$\begin{aligned}2x+9 &= 5x\\ 2x+9-9 &= 5x-9\\ 2x &= 5x-9\\ 2x-5x &= 5x-9-5x\\ -3x &= -9\\ \frac{-3x}{-3} &= \frac{-9}{-3}\\ x &= 3\end{aligned}$$
The solution set is $\{3\}$.

21.
$$\begin{aligned}2t-6 &= 3-t\\ 2t-6+6 &= 3-t+6\\ 2t &= 9-t\\ 2t+t &= 9-t+t\\ 3t &= 9\\ \frac{3t}{3} &= \frac{9}{3}\\ t &= 3\end{aligned}$$
The solution set is $\{3\}$.

22.
$$\begin{aligned}5y+6 &= -18-y\\ 5y+6-6 &= -18-y-6\\ 5y &= -y-24\\ 5y+y &= -y-24+y\\ 6y &= -24\\ \frac{6y}{6} &= \frac{-24}{6}\\ y &= -4\end{aligned}$$
The solution set is $\{-4\}$.

23.
$$\begin{aligned}6-x &= 2x+9\\ 6-x-6 &= 2x+9-6\\ -x &= 2x+3\\ -x-2x &= 2x+3-2x\\ -3x &= 3\\ \frac{-3x}{-3} &= \frac{3}{-3}\\ x &= -1\end{aligned}$$
The solution set is $\{-1\}$.

24.
$$\begin{aligned}3-2x &= 2-x\\ 3-2x-3 &= 2-x-3\\ -2x &= -x-1\\ -2x+x &= -x-1+x\\ -x &= -1\\ \frac{-x}{-1} &= \frac{-1}{-1}\\ x &= 1\end{aligned}$$
The solution set is $\{1\}$.

25.
$$\begin{aligned}3+2n &= 4n+7\\ 3+2n-3 &= 4n+7-3\\ 2n &= 4n+4\\ 2n-4n &= 4n+4-4n\\ -2n &= 4\\ \frac{-2n}{-2} &= \frac{4}{-2}\\ n &= -2\end{aligned}$$
The solution set is $\{-2\}$.

26.
$$\begin{aligned} 6-2m &= 3m+1 \\ 6-2m-6 &= 3m+1-6 \\ -2m &= 3m-5 \\ -2m-3m &= 3m-5-3m \\ -5m &= -5 \\ \frac{-5m}{-5} &= \frac{-5}{-5} \\ m &= 1 \end{aligned}$$
The solution set is $\{1\}$.

27.
$$\begin{aligned} 3(5+3x) &= 8(x-1) \\ 15+9x &= 8x-8 \\ 9x-8x+15 &= 8x-8x-8 \\ x+15-15 &= -8-15 \\ x &= -23 \end{aligned}$$
The solution set is $\{-23\}$.

28.
$$\begin{aligned} 3(2-x) &= 2x-1 \\ 6-3x &= 2x-1 \\ 6-3x-6 &= 2x-1-6 \\ -3x &= 2x-7 \\ -3x-2x &= 2x-7-2x \\ -5x &= -7 \\ \frac{-5x}{-5} &= \frac{-7}{-5} \\ x &= \frac{7}{5} \end{aligned}$$
The solution set is $\left\{\frac{7}{5}\right\}$.

29.
$$\begin{aligned} 8x-(3x+2) &= 3x-10 \\ 8x-3x-2 &= 3x-10 \\ 5x-2 &= 3x-10 \\ 5x-2+2 &= 3x-10+2 \\ 5x &= 3x-8 \\ 5x-3x &= 3x-8-3x \\ 2x &= -8 \\ \frac{2x}{2} &= \frac{-8}{2} \\ x &= -4 \end{aligned}$$
The solution set is $\{-4\}$.

30.
$$\begin{aligned} 7-(2x-1) &= 10 \\ 7-2x+1 &= 10 \\ 8-2x &= 10 \\ 8-2x-8 &= 10-8 \\ -2x &= 2 \\ \frac{-2x}{-2} &= \frac{2}{-2} \\ x &= -1 \end{aligned}$$
The solution set is $\{-1\}$.

31.
$$\begin{aligned} \frac{3}{2}x+2 &= \frac{1}{2}-\frac{1}{2}x \\ 2\left(\frac{3}{2}x+2\right) &= 2\left(\frac{1}{2}-\frac{1}{2}x\right) \\ 3x+4 &= 1-x \\ 3x+4-4 &= 1-x-4 \\ 3x &= -3-x \\ 3x+x &= -3-x+x \\ 4x &= -3 \\ \frac{4x}{4} &= \frac{-3}{4} \\ x &= -\frac{3}{4} \end{aligned}$$
The solution set is $\left\{-\frac{3}{4}\right\}$.

32.
$$\begin{aligned} \frac{1}{3}x &= 2-\frac{2}{3}x \\ 3\left(\frac{1}{3}x\right) &= 3\left(2-\frac{2}{3}x\right) \\ x &= 6-2x \\ x+2x &= 6-2x+2x \\ 3x &= 6 \\ \frac{3x}{3} &= \frac{6}{3} \\ x &= 2 \end{aligned}$$
The solution set is $\{2\}$.

33.
$$\frac{1}{2}x-5=\frac{3}{4}x$$
$$4\left(\frac{1}{2}x-5\right)=4\left(\frac{3}{4}x\right)$$
$$2x-20=3x$$
$$2x-20-2x=3x-2x$$
$$-20=x$$
$$x=-20$$
The solution set is $\{-20\}$.

34.
$$1-\frac{1}{2}x=6$$
$$2\left(1-\frac{1}{2}x\right)=2(6)$$
$$2-x=12$$
$$2-x-2=12-2$$
$$-x=10$$
$$\frac{-x}{-1}=\frac{10}{-1}$$
$$x=-10$$
The solution set is $\{-10\}$.

35.
$$\frac{2}{3}p=\frac{1}{2}p+\frac{1}{3}$$
$$6\left(\frac{2}{3}p\right)=6\left(\frac{1}{2}p+\frac{1}{3}\right)$$
$$4p=3p+2$$
$$4p-3p=3p+2-3p$$
$$p=2$$
The solution set is $\{2\}$.

36.
$$\frac{1}{2}-\frac{1}{3}p=\frac{4}{3}$$
$$6\left(\frac{1}{2}-\frac{1}{3}p\right)=6\left(\frac{4}{3}\right)$$
$$3-2p=8$$
$$3-2p-3=8-3$$
$$-2p=5$$
$$\frac{-2p}{-2}=\frac{5}{-2}$$
$$p=-\frac{5}{2}$$
The solution set is $\left\{-\frac{5}{2}\right\}$.

37.
$$0.2m=0.9+0.5m$$
$$0.2m-0.5m=0.9+0.5m-0.5m$$
$$-0.3m=0.9$$
$$\frac{-0.3m}{-0.3}=\frac{0.9}{-0.3}$$
$$m=-3$$
The solution set is $\{-3\}$.

38.
$$0.9t=1+t$$
$$0.9t-t=1+t-t$$
$$-0.1t=1$$
$$\frac{-0.1t}{-0.1}=\frac{1}{-0.1}$$
$$t=-10$$
The solution set is $\{-10\}$.

39.
$$\frac{x+1}{3}+\frac{x+2}{7}=2$$
$$21\left(\frac{x+1}{3}+\frac{x+2}{7}\right)=21(2)$$
$$7(x+1)+(3)(x+2)=42$$
$$7x+7+3x+6=42$$
$$10x+13=42$$
$$10x+13-13=42-13$$
$$10x=29$$
$$\frac{10x}{10}=\frac{29}{10}$$
$$x=\frac{29}{10}$$
The solution set is $\left\{\frac{29}{10}\right\}$.

40.
$$\frac{2x+1}{3}+16=3x$$
$$3\left(\frac{2x+1}{3}+16\right)=3(3x)$$
$$2x+1+48=9x$$
$$2x+49=9x$$
$$2x+49-2x=9x-2x$$
$$49=7x$$
$$\frac{49}{7}=\frac{7x}{7}$$
$$x=7$$
The solution set is $\{7\}$.

41. $$\frac{5}{8}(p+3)-2=\frac{1}{4}(2p-3)+\frac{11}{16}$$
$$10(p+3)-32=4(2p-3)+11$$
$$10p+30-32=8p-12+11$$
$$10p-2=8p-1$$
$$10p-8p-2=8p-8p-1$$
$$2p-2+2=-1+2$$
$$2p=1$$
$$\frac{2}{2}p=\frac{1}{2}$$
$$p=\frac{1}{2}$$
The solution set is $\left\{\frac{1}{2}\right\}$.

42. $$\tfrac{1}{3}(w+1)-3=\tfrac{2}{5}(w-4)-\tfrac{2}{15}$$
$$5(w+1)-45=6(w-4)-2$$
$$5w+5-45=6w-24-2$$
$$5w-40=6w-26$$
$$5w-6w-40=6w-6w-26$$
$$-w-40+40=-26+40$$
$$-w=14$$
$$\frac{-1}{-1}w=14$$
$$w=-14$$
The solution set is $\{-14\}$.

43. $$\frac{2}{y}+\frac{4}{y}=3$$
$$y\left(\frac{2}{y}+\frac{4}{y}\right)=y(3)$$
$$2+4=3y$$
$$6=3y$$
$$\frac{6}{3}=\frac{3y}{3}$$
$$2=y$$
Since $y=2$ does not cause a denominator to equal zero, the solution set is $\{2\}$.

44. $$\frac{4}{y}-5=\frac{5}{2y}$$
$$2y\left(\frac{4}{y}-5\right)=2y\left(\frac{5}{2y}\right)$$
$$8-10y=5$$
$$8-10y-8=5-8$$
$$-10y=-3$$
$$\frac{-10y}{-10}=\frac{-3}{-10}$$
$$y=\frac{3}{10}$$
Since $y=\frac{3}{10}$ does not cause a denominator to equal zero, the solution set is $\left\{\frac{3}{10}\right\}$.

45. $$\frac{1}{2}+\frac{2}{x}=\frac{3}{4}$$
$$4x\left(\frac{1}{2}+\frac{2}{x}\right)=4x\left(\frac{3}{4}\right)$$
$$2x+8=3x$$
$$2x+8-2x=3x-2x$$
$$8=x$$
Since $x=8$ does not cause any denominator to equal zero, the solution set is $\{8\}$.

46. $$\frac{3}{x}-\frac{1}{3}=\frac{1}{6}$$
$$6x\left(\frac{3}{x}-\frac{1}{3}\right)=6x\left(\frac{1}{6}\right)$$
$$18-2x=x$$
$$18-2x+2x=x+2x$$
$$18=3x$$
$$\frac{18}{3}=\frac{3x}{3}$$
$$6=x$$
Since $x=6$ does not cause a denominator to equal zero, the solution set is $\{6\}$.

47. $$(x+7)(x-1)=(x+1)^2$$
$$x^2+6x-7=x^2+2x+1$$
$$x^2+6x-7-x^2=x^2+2x+1-x^2$$
$$6x-7=2x+1$$
$$6x-7+7=2x+1+7$$
$$6x=2x+8$$
$$6x-2x=2x+8-2x$$
$$4x=8$$
$$\frac{4x}{4}=\frac{8}{4}$$
$$x=2$$
The solution set is $\{2\}$.

48. $$(x+2)(x-3)=(x+3)^2$$
$$x^2-x-6=x^2+6x+9$$
$$x^2-x-6-x^2=x^2+6x+9-x^2$$
$$-x-6=6x+9$$
$$-x-6+6=6x+9+6$$
$$-x=6x+15$$
$$-x-6x=6x+15-6x$$
$$-7x=15$$
$$\frac{-7x}{-7}=\frac{15}{-7}$$
$$x=-\frac{15}{7}$$
The solution set is $\left\{-\frac{15}{7}\right\}$.

49. $$x(2x-3)=(2x+1)(x-4)$$
$$2x^2-3x=2x^2-7x-4$$
$$2x^2-3x-2x^2=2x^2-7x-4-2x^2$$
$$-3x=-7x-4$$
$$-3x+7x=-7x-4+7x$$
$$4x=-4$$
$$\frac{4x}{4}=\frac{-4}{4}$$
$$x=-1$$
The solution set is $\{-1\}$.

50. $$x(1+2x)=(2x-1)(x-2)$$
$$x+2x^2=2x^2-5x+2$$
$$x+2x^2-2x^2=2x^2-5x+2-2x^2$$
$$x=-5x+2$$
$$x+5x=-5x+2+5x$$
$$6x=2$$
$$\frac{6x}{6}=\frac{2}{6}\Rightarrow x=\frac{1}{3}$$
The solution set is $\left\{\frac{1}{3}\right\}$.

51. $$p\left(p^2+3\right)=12+p^3$$
$$p^3+3p=12+p^3$$
$$p^3-p^3+3p=12+p^3-p^3$$
$$3p=12$$
$$\frac{3p}{3}=\frac{12}{3}\Rightarrow p=4$$
The solution set is $\{4\}$.

52. $$w(4-w^2)=8-w^3$$
$$4w-w^3=8-w^3$$
$$4w-w^3+w^3=8-w^3+w^3$$
$$4w=8$$
$$\frac{4w}{4}=\frac{8}{4}$$
$$w=2$$
The solution set is $\{2\}$.

53. $$\frac{x}{x-2}+3=\frac{2}{x-2}$$
$$\left(\frac{x}{x-2}+3\right)(x-2)=\left(\frac{2}{x-2}\right)(x-2)$$
$$x+3(x-2)=2$$
$$x+3x-6=2$$
$$4x-6=2$$
$$4x-6+6=2+6$$
$$4x=8$$
$$\frac{4x}{4}=\frac{8}{4}$$
$$x=2$$
Since $x=2$ causes a denominator to equal zero, we must discard it. Therefore the original equation has no solution.

54.

$$\frac{2x}{x+3}=\frac{-6}{x+3}-2$$
$$\left(\frac{2x}{x+3}\right)(x+3)=\left(\frac{-6}{x+3}-2\right)(x+3)$$
$$2x=-6-(2)(x+3)$$
$$2x=-6-2x-6$$
$$2x=-12-2x$$
$$2x+2x=-12-2x+2x$$
$$4x=-12$$
$$\frac{4x}{4}=\frac{-12}{4}$$
$$x=-3$$

Since $x = –3$ causes a denominator to equal zero, we must discard it. Therefore the original equation has no solution.

55.

$$\frac{2x}{x^2-4}=\frac{4}{x^2-4}-\frac{3}{x+2}$$
$$\frac{2x}{(x+2)(x-2)}=\frac{4}{(x+2)(x-2)}-\frac{3}{x+2}$$
$$\left(\frac{2x}{(x+2)(x-2)}\right)(x+2)(x-2)=\left(\frac{4}{(x+2)(x-2)}-\frac{3}{x+2}\right)(x+2)(x-2)$$
$$2x=4-3(x-2)$$
$$2x=4-3x+6$$
$$2x=10-3x$$
$$2x+3x=10-3x+3x$$
$$5x=10$$
$$\frac{5x}{5}=\frac{10}{5}$$
$$x=2$$

Since $x = 2$ causes a denominator to equal zero, we must discard it. Therefore the original equation has no solution.

56.
$$\begin{aligned}
\frac{x}{x^2-9}+\frac{4}{x+3}&=\frac{3}{x^2-9}\\
\frac{x}{(x+3)(x-3)}+\frac{4}{x+3}&=\frac{3}{(x+3)(x-3)}\\
\left(\frac{x}{(x+3)(x-3)}+\frac{4}{x+3}\right)(x+3)(x-3)&=\left(\frac{3}{(x+3)(x-3)}\right)(x+3)(x-3)\\
x+4(x-3)&=3\\
x+4x-12&=3\\
5x-12&=3\\
5x-12+12&=3+12\\
5x&=15\\
\frac{5x}{5}&=\frac{15}{5}\\
x&=3
\end{aligned}$$
Since $x = 3$ causes a denominator to equal zero, we must discard it. Therefore the original equation has no solution.

57.
$$\begin{aligned}
\frac{x}{x+2}&=\frac{3}{2}\\
2(x+2)\left(\frac{x}{x+2}\right)&=2(x+2)\left(\frac{3}{2}\right)\\
2x&=3(x+2)\\
2x&=3x+6\\
2x-3x&=3x+6-3x\\
-x&=6\\
\frac{-x}{-1}&=\frac{6}{-1}\\
x&=-6
\end{aligned}$$
Since $x = -6$ does not cause any denominator to equal zero, the solution set is $\{-6\}$.

58.
$$\begin{aligned}
\frac{3x}{x-1}&=2\\
\left(\frac{3x}{x-1}\right)(x-1)&=2(x-1)\\
3x&=2x-2\\
3x-2x&=2x-2-2x\\
x&=-2
\end{aligned}$$
Since $x = -2$ does not cause any denominator to equal zero, the solution set is $\{-2\}$.

59.
$$\begin{aligned}
\frac{7}{3x+10}&=\frac{2}{x-3}\\
\left(\frac{7}{3x+10}\right)(3x+10)(x-3)&=\left(\frac{2}{x-3}\right)(3x+10)(x-3)\\
7(x-3)&=2(3x+10)\\
7x-21&=6x+20\\
7x-21-6x&=6x+20-6x\\
-21+x&=20\\
-21+x+21-25&=20+21\\
x&=41
\end{aligned}$$
Since $x = 41$ does not cause any denominator to equal zero, the solution is $\{41\}$.

60.
$$\begin{aligned}
\frac{-4}{x+4}&=\frac{-3}{x+6}\\
\left(\frac{-4}{x+4}\right)(x+6)(x+4)&=\left(\frac{-3}{x+6}\right)(x+6)(x+4)\\
-4(x+6)&=-3(x+4)\\
-4x-24&=-3x-12\\
-4x-24+4x&=-3x-12+4x\\
-24&=-12+x\\
-24+12&=-12+x+12\\
-12&=x
\end{aligned}$$
Since $x = -12$ does not cause any denominator to equal zero, the solution set is $\{-12\}$.

61.
$$\begin{aligned}
\frac{6t+7}{4t-1} &= \frac{3t+8}{2t-4}\\
\left(\frac{6t+7}{4t-1}\right)(4t-1)(2t-4) &= \left(\frac{3t+8}{2t-4}\right)(4t-1)(2t-4)\\
(6t+7)(2t-4) &= (3t+8)(4t-1)\\
12t^2-24t+14t-28 &= 12t^2-3t+32t-8\\
12t^2-10t-28 &= 12t^2+29t-8\\
12t^2-10t-28-12t^2 &= 12t^2+29t-8-12t^2\\
-10t-28 &= 29t-8\\
-10t-28-29t &= 29t-8-29t\\
-28-39t &= -8\\
-28-39t+28 &= -8+28\\
-39t &= 20\\
\frac{-39t}{-39} &= \frac{20}{-39}\\
t &= -\frac{20}{39}
\end{aligned}$$

Since $t=-\frac{20}{39}$ does not cause any denominator to equal zero, the solution set is $\left\{-\frac{20}{39}\right\}$.

62.
$$\begin{aligned}
\frac{8w+5}{10w-7} &= \frac{4w-3}{5w+7}\\
\left(\frac{8w+5}{10w-7}\right)(10w-7)(5w+7) &= \left(\frac{4w-3}{5w+7}\right)(10w-7)(5w+7)\\
(8w+5)(5w+7) &= (4w-3)(10w-7)\\
40w^2+56w+25w+35 &= 40w^2-28w-30w+21\\
40w^2+81w+35 &= 40w^2-58w+21\\
40w^2+81w+35-40w^2 &= 40w^2-58w+21-40w^2\\
81w+35 &= -58w+21\\
81w+35+58w &= -58w+21+58w\\
139w+35 &= 21\\
139w+35-35 &= 21-35\\
139w &= -14\\
\frac{139w}{139} &= \frac{-14}{139}\\
w &= -\frac{14}{139}
\end{aligned}$$

Since $w=-\frac{14}{139}$ does not cause any denominator to equal zero, the solution set is $\left\{-\frac{14}{139}\right\}$.

63. $$\frac{4}{x-2}=\frac{-3}{x+5}+\frac{7}{(x+5)(x-2)}$$
$$\left(\frac{4}{x-2}\right)(x+5)(x-2)=\left(\frac{-3}{x+5}+\frac{7}{(x+5)(x-2)}\right)(x+5)(x-2)$$
$$4(x+5)=-3(x-2)+7$$
$$4x+20=-3x+6+7$$
$$4x+20=-3x+13$$
$$4x+20+3x=-3x+13+3x$$
$$7x+20=13$$
$$7x+20-20=13-20$$
$$7x=-7$$
$$\frac{7x}{7}=\frac{-7}{7}$$
$$x=-1$$
Since $x=-1$ does not cause any denominator to equal zero, the solution set is $\{-1\}$.

64. $$\frac{-4}{2x+3}+\frac{1}{x-1}=\frac{1}{(2x+3)(x-1)}$$
$$\left(\frac{-4}{2x+3}+\frac{1}{x-1}\right)(2x+3)(x-1)=\left(\frac{1}{(2x+3)(x-1)}\right)(2x+3)(x-1)$$
$$-4(x-1)+1(2x+3)=1$$
$$-4x+4+2x+3=1$$
$$-2x+7=1$$
$$-2x+7-7=1-7$$
$$-2x=-6$$
$$\frac{-2x}{-2}=\frac{-6}{-2}$$
$$x=3$$
Since $x=3$ does not cause any denominator to equal zero, the solution set is $\{3\}$.

65.
$$\frac{2}{y+3}+\frac{3}{y-4}=\frac{5}{y+6}$$
$$\left(\frac{2}{y+3}+\frac{3}{y-4}\right)(y+3)(y-4)(y+6)=\left(\frac{5}{y+6}\right)(y+3)(y-4)(y+6)$$
$$2(y-4)(y+6)+3(y+3)(y+6)=5(y+3)(y-4)$$
$$2\left(y^2+6y-4y-24\right)+3\left(y^2+6y+3y+18\right)=5\left(y^2-4y+3y-12\right)$$
$$2\left(y^2+2y-24\right)+3\left(y^2+9y+18\right)=5\left(y^2-y-12\right)$$
$$2y^2+4y-48+3y^2+27y+54=5y^2-5y-60$$
$$5y^2+31y+6=5y^2-5y-60$$
$$5y^2+31y+6-5y^2=5y^2-5y-60-5y^2$$
$$31y+6=-5y-60$$
$$31y+6+5y=-5y-60+5y$$
$$36y+6=-60$$
$$36y+6-6=-60-6$$
$$36y=-66$$
$$\frac{36y}{36}=\frac{-66}{36}$$
$$y=-\frac{11}{6}$$
Since $y=-\frac{11}{6}$ does not cause any denominator to equal zero, the solution set is $\left\{-\frac{11}{6}\right\}$.

66.
$$\frac{5}{5z-11}+\frac{4}{2z-3}=\frac{-3}{5-z}$$
$$\left(\frac{5}{5z-11}+\frac{4}{2z-3}\right)(5z-11)(2z-3)(5-z)=\left(\frac{-3}{5-z}\right)(5z-11)(2z-3)(5-z)$$
$$5(2z-3)(5-z)+4(5z-11)(5-z)=-3(5z-11)(2z-3)$$
$$5\left(10z-2z^2-15+3z\right)+4\left(25z-5z^2-55+11z\right)=-3\left(10z^2-15z-22z+33\right)$$
$$5\left(-2z^2+13z-15\right)+4\left(-5z^2+36z-55\right)=-3\left(10z^2-37z+33\right)$$
$$-10z^2+65z-75-20z^2+144z-220=-30z^2+111z-99$$
$$-30z^2+209z-295=-30z^2+111z-99$$
$$-30z^2+209z-295+30z^2=-30z^2+111z-99+30z^2$$
$$209z-295=111z-99$$
$$209z-295-209z=111z-99-209z$$
$$-295=-98z-99$$
$$-295+99=-98z-99+99$$
$$-196=-98z$$
$$\frac{-196}{-98}=\frac{-118z}{-98}$$
$$2=z$$
Since $z=2$ does not cause any denominator to equal zero, the solution set is $\{2\}$.

67.

$$\frac{x}{x^2-9}-\frac{x-4}{x^2+3x}=\frac{10}{x^2-3x}$$
$$\frac{x}{(x+3)(x-3)}-\frac{x-4}{x(x+3)}=\frac{10}{x(x-3)}$$
$$\left(\frac{x}{(x+3)(x-3)}-\frac{x-4}{x(x+3)}\right)x(x+3)(x-3)=\left(\frac{10}{x(x-3)}\right)x(x+3)(x-3)$$
$$(x)(x)-(x-4)(x-3)=10(x+3)$$
$$x^2-\left(x^2-3x-4x+12\right)=10x+30$$
$$x^2-\left(x^2-7x+12\right)=10x+30$$
$$x^2-x^2+7x-12=10x+30$$
$$7x-12=10x+30$$
$$7x-12+12=10x+30+12$$
$$7x=10x+42$$
$$7x-10x=10x-10x+42$$
$$-3x=42$$
$$\frac{-3x}{-3}=\frac{42}{-3}$$
$$x=-14$$

Since $x=-6$ does not cause any denominator to equal zero, the solution set is $\{-14\}$.

68.

$$\frac{x+1}{x^2+2x}-\frac{x+4}{x^2+x}=\frac{-3}{x^2+3x+2}$$
$$\frac{x+1}{x(x+2)}-\frac{x+4}{x(x+1)}=\frac{-3}{(x+2)(x+1)}$$
$$\left(\frac{x+1}{x(x+2)}-\frac{x+4}{x(x+1)}\right)x(x+2)(x+1)=\left(\frac{-3}{(x+2)(x+1)}\right)x(x+2)(x+1)$$
$$(x+1)(x+1)-(x+4)(x+2)=-3x$$
$$\left(x^2+x+x+1\right)-\left(x^2+2x+4x+8\right)=-3x$$
$$x^2+2x+1-\left(x^2+6x+8\right)=-3x$$
$$x^2+2x+1-x^2-6x-8=-3x$$
$$2x+1-6x-8=-3x$$
$$-4x-7=-3x$$
$$-4x-7+4x=-3x+4x$$
$$-7=x$$

Since $x=-7$ does not cause any denominator to equal zero, the solution set is $\{-7\}$.

69.
$$3.2x + \frac{21.3}{65.871} = 19.23$$
$$3.2x + \frac{21.3}{65.871} - \frac{21.3}{65.871} = 19.23 - \frac{21.3}{65.871}$$
$$3.2x = 19.23 - \frac{21.3}{65.871}$$
$$\left(\frac{1}{3.2}\right)(3.2x) = \left(19.23 - \frac{21.3}{65.871}\right)\left(\frac{1}{3.2}\right)$$
$$x = \left(19.23 - \frac{21.3}{65.871}\right)\left(\frac{1}{3.2}\right) \approx 5.91$$
The solution set is approximately $\{5.91\}$.

70.
$$6.2x - \frac{19.1}{83.72} = 0.195$$
$$6.2x - \frac{19.1}{83.72} + \frac{19.1}{83.72} = 0.195 + \frac{19.1}{83.72}$$
$$6.2x = 0.195 + \frac{19.1}{83.72}$$
$$\left(\frac{1}{6.2}\right)(6.2x) = \left(0.195 + \frac{19.1}{83.72}\right)\left(\frac{1}{6.2}\right)$$
$$x = \left(0.195 + \frac{19.1}{83.72}\right)\left(\frac{1}{6.2}\right) \approx 0.07$$
The solution set is approximately $\{0.07\}$.

71.
$$14.72 - 21.58x = \frac{18}{2.11}x + 2.4$$
$$14.72 - 21.58x - \frac{18}{2.11}x = \frac{18}{2.11}x + 2.4 - \frac{18}{2.11}x$$
$$14.72 - 21.58x - \frac{18}{2.11}x = 2.4$$
$$14.72 - 21.58x - \frac{18}{2.11}x - 14.72 = 2.4 - 14.72$$
$$-21.58x - \frac{18}{2.11}x = -12.32$$
$$\left(-21.58 - \frac{18}{2.11}\right)x = -12.32$$
$$\left(\frac{1}{-21.58 - \frac{18}{2.11}}\right)\left(-21.58 - \frac{18}{2.11}\right)x = -12.32\left(\frac{1}{-21.58 - \frac{18}{2.11}}\right)$$
$$x = -12.32\left(\frac{1}{-21.58 - \frac{18}{2.11}}\right) \approx 0.41$$
The solution set is approximately $\{0.41\}$.

72.
$$\begin{aligned}
18.63x-\frac{21.2}{2.6}&=\frac{14}{2.32}x-20\\
18.63x-\frac{21.2}{2.6}-\frac{14}{2.32}x&=\frac{14}{2.32}x-20-\frac{14}{2.32}x\\
18.63x-\frac{21.2}{2.6}-\frac{14}{2.32}x&=-20\\
18.63x-\frac{21.2}{2.6}-\frac{14}{2.32}x+\frac{21.2}{2.6}&=-20+\frac{21.2}{2.6}\\
18.63x-\frac{14}{2.32}x&=-20+\frac{21.2}{2.6}\\
\left(18.63-\frac{14}{2.32}\right)x&=-20+\frac{21.2}{2.6}\\
\left(\frac{1}{18.63-\frac{14}{2.32}}\right)\left(18.63-\frac{14}{2.32}\right)x&=\left(-20+\frac{21.2}{2.6}\right)\left(\frac{1}{18.63-\frac{14}{2.32}}\right)\\
x&=\left(-20+\frac{21.2}{2.6}\right)\left(\frac{1}{18.63-\frac{14}{2.32}}\right)\approx-0.94
\end{aligned}$$

The solution set is approximately $\{-0.94\}$.

73.
$$\begin{aligned}
ax-b&=c,\quad a\neq 0\\
ax-b+b&=c+b\\
ax&=b+c\\
\frac{ax}{a}&=\frac{b+c}{a}\\
x&=\frac{b+c}{a}
\end{aligned}$$

74.
$$\begin{aligned}
1-ax&=b,\ a\neq 0\\
1-ax-1&=b-1\\
-ax&=b-1\\
\frac{-ax}{-a}&=\frac{b-1}{-a}\\
x&=\frac{b-1}{-a}=\frac{1-b}{a}
\end{aligned}$$

75.
$$\begin{aligned}
\frac{x}{a}+\frac{x}{b}&=c,\quad a\neq 0, b\neq 0, a\neq -b\\
ab\left(\frac{x}{a}+\frac{x}{b}\right)&=ab\cdot c\\
bx+ax&=abc\\
(a+b)x&=abc\\
\frac{(a+b)x}{a+b}&=\frac{abc}{a+b}\\
x&=\frac{abc}{a+b}
\end{aligned}$$

76.
$$\begin{aligned}
\frac{a}{x}+\frac{b}{x}&=c,\quad c\neq 0\\
x\left(\frac{a}{x}+\frac{b}{x}\right)&=x\cdot c\\
a+b&=cx\\
\frac{a+b}{c}&=\frac{cx}{c}\\
x&=\frac{a+b}{c}
\end{aligned}$$

77. $x+2a=16+ax-6a$, if $x=4$

$$\begin{aligned}
4+2a&=16+a(4)-6a\\
4+2a&=16+4a-6a\\
4+2a&=16-2a\\
4a&=12\\
\frac{4a}{4}&=\frac{12}{4}\\
a&=3
\end{aligned}$$

78. $x+2b=x-4+2bx$, for $x=2$

$$\begin{aligned}
2+2b&=2-4+2b(2)\\
2+2b&=2-4+4b\\
2+2b&=-2+4b\\
4&=2b\\
\frac{4}{2}&=b\\
b&=2
\end{aligned}$$

79.

$$\begin{aligned}
\frac{1}{R}&=\frac{1}{R_1}+\frac{1}{R_2}\\
RR_1R_2\left(\frac{1}{R}\right)&=RR_1R_2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\\
R_1R_2&=RR_2+RR_1\\
R_1R_2&=R(R_2+R_1)\\
\frac{R_1R_2}{R_2+R_1}&=\frac{R(R_2+R_1)}{R_2+R_1}\\
\frac{R_1R_2}{R_2+R_1}&=R
\end{aligned}$$

80.

$$\begin{aligned}
A&=P(1+rt)\\
A&=P+Prt\\
A-P&=Prt\\
\frac{A-P}{Pt}&=\frac{Prt}{Pt}\\
\frac{A-P}{Pt}&=r
\end{aligned}$$

81.

$$\begin{aligned}
F&=\frac{mv^2}{R}\\
RF&=R\left(\frac{mv^2}{R}\right)\\
RF&=mv^2\\
\frac{RF}{F}&=\frac{mv^2}{F}\\
R&=\frac{mv^2}{F}
\end{aligned}$$

82.

$$\begin{aligned}
PV&=nRT\\
\frac{PV}{nR}&=\frac{nRT}{nR}\\
\frac{PV}{nR}&=T
\end{aligned}$$

83.

$$\begin{aligned}
S&=\frac{a}{1-r}\\
S(1-r)&=\left(\frac{a}{1-r}\right)(1-r)\\
S-Sr&=a\\
S-Sr-S&=a-S\\
-Sr&=a-S\\
\frac{-Sr}{-S}&=\frac{a-S}{-S}\\
r&=\frac{S-a}{S}
\end{aligned}$$

84.

$$\begin{aligned}
v&=-gt+v_0\\
v-v_0&=-gt\\
\frac{v-v_0}{-g}&=\frac{-gt}{-g}\\
t&=\frac{v-v_0}{-g}=\frac{v_0-v}{g}
\end{aligned}$$

85.

Amount in bonds	Amount in CDs	Total
x	$x-3000$	20,000

$$\begin{aligned}
x+(x-3000)&=20{,}000\\
2x-3000&=20{,}000\\
2x&=23{,}000\\
x&=11{,}500
\end{aligned}$$

\$11,500 will be invested in bonds and \$8500 will be invested in CD's.

86.

Sean's Amount	George's Amount	Total
x	$x-3000$	$10,000$

$$\begin{aligned} x+(x-3000)&=10,000\\ 2x-3000&=10,000\\ 2x&=13,000\\ x&=6500 \end{aligned}$$

Sean will receive \$6500 and Jorge will receive \$3500.

87.

	Dollars per hour	Hours worked	Money earned
Regular wage	x	40	$40x$
Overtime wage	$1.5x$	8	$8(1.5x)$

$$\begin{aligned} 40x+8(1.5x)&=910\\ 40x+12x&=910\\ 52x&=910\\ x&=\frac{910}{52}=17.50 \end{aligned}$$

Sandra's regular hourly wage is \$17.50.

88.

	Dollars per hour	Hours worked	Money earned
Regular wage	x	40	$40x$
Overtime wage	$1.5x$	6	$6(1.5x)$
Sunday wage	$2x$	4	$4(2x)$

$$\begin{aligned} 40x+6(1.5x)+4(2x)&=1083\\ 40x+9x+8x&=1083\\ 57x&=1083\\ x&=\frac{1083}{57}=19 \end{aligned}$$

Leigh's regular hourly wage is \$19.00.

89. Let x represent the score on the final exam.

$$\begin{aligned} \frac{80+83+71+61+95+x+x}{7}&=80\\ \frac{390+2x}{7}&=80\\ 390+2x&=560\\ 2x&=170\\ x&=85 \end{aligned}$$

Camila needs a score of 85 on the final exam.

90. Let x represent the score on the final exam.
Note: since the final exam counts for two-thirds of the overall grade, the average of the four test scores count for one-third of the overall grade.
For a B, the average score must be 80.

$$\begin{aligned} \frac{1}{3}\left(\frac{86+80+84+90}{4}\right)+\frac{2}{3}x&=80\\ \frac{1}{3}\left(\frac{340}{4}\right)+\frac{2}{3}x&=80\\ \frac{85}{3}+\frac{2}{3}x&=80\\ 3\left(\frac{85}{3}+\frac{2}{3}x\right)&=3(80)\\ 85+2x&=240\\ 2x&=155\\ x&=77.5 \end{aligned}$$

Ali needs a score of 78 to earn a B.

For an A, the average score must be 90.

$$\begin{aligned} \frac{1}{3}\left(\frac{86+80+84+90}{4}\right)+\frac{2}{3}x&=90\\ \frac{1}{3}\left(\frac{340}{4}\right)+\frac{2}{3}x&=90\\ \frac{85}{3}+\frac{2}{3}x&=90\\ 3\left(\frac{85}{3}+\frac{2}{3}x\right)&=3(90)\\ 85+2x&=270\\ 2x&=185\\ x&=92.5 \end{aligned}$$

Ali needs a score of 93 to earn an A.

91. Let x represent the original price of the phone.
Then $0.12x$ represents the reduction in the price of the phone.
The new price of the phone is \$572.
original price − reduction = new price

$$x - 0.12x = 572$$
$$0.88x = 572$$
$$x = 650$$

The original price of the phone was \$650.
The amount of the reduction (i.e., the savings) is 0.12(\$650) = \$78.

92. Let x represent the original price of the car.
Then $0.15x$ represents the reduction in the price of the car.
The new price of the car is \$8000.
list price − reduction = new price

$$x - 0.15x = 18000$$
$$0.85x = 18000$$
$$x \approx 21176.47$$

The list price of the car was \$21,176.47.
The amount of the reduction (i.e., the savings) is $0.15(\$21176.47) \approx \$3176.47$.

93. Let x represent the price the theater pays for the candy.
Then $2.75x$ represents the markup on the candy.
The selling price of the candy is \$4.50.
suppier price + markup = selling price

$$x + 2.75x = 4.50$$
$$3.75x = 4.50$$
$$x = 1.20$$

The theater paid \$1.20 for the candy.

94. Let x represent selling price for the new car.
The dealer's cost is $0.85(\$24,000) = \$20,400$.
The markup is \$300.
selling price = dealer's cost + markup
$x = 20,400 + 300 = \$20,700$
At \$300 over the dealer's cost, the price of the care is \$20,700.

95.

	Tickets sold	Price per ticket	Money earned
Adults	x	7.50	$7.50x$
Children	$5200 - x$	4.50	$4.50(5200 - x)$

$$7.50x + 4.50(5200 - x) = 29,961$$
$$7.50x + 23,400 - 4.50x = 29,961$$
$$3.00x + 23,400 = 29,961$$
$$3.00x = 6561$$
$$x = 2187$$

There were 2187 adult patrons.

96. Let p represent the original price for the boots.
Then, $0.30p$ represents the discounted amount.
original price − discount = clearance price

$$p - 0.30p = 399$$
$$0.70p = 399$$
$$p = 570$$

The boots originally cost \$570.

97. Let w represent the width of the rectangle.
Then $w + 8$ is the length.
Perimeter is given by the formula $P = 2l + 2w$.

$$2(w + 8) + 2w = 60$$
$$2w + 16 + 2w = 60$$
$$4w + 16 = 60$$
$$4w = 44$$
$$w = 11$$

Now, 11 + 8 = 19.
The width of the rectangle is 11 feet and the length is 19 feet.

98. Let w represent the width of the rectangle.
Then $2w$ is the length.
Perimeter is given by the formula $P = 2l + 2w$.

$$2(2w) + 2w = 42$$
$$4w + 2w = 42$$
$$6w = 42$$
$$w = 7$$

Now, 2(7) = 14.
The width of the rectangle is 7 meters and the length is 14 meters.

99. We will let B be the calories from breakfast, L the calories from lunch and D the calories from dinner. So we have the following equations:

$$B = L + 125$$
$$D = 2L - 300$$
$$2025 = B + L + D$$

Now we substitute the first two into the last one and solve for L.

$$2025 = (L+125) + L + (2L-300)$$
$$2025 = 4L - 175$$
$$2200 = 4L$$
$$L = 550$$

Now we substitute L into the first two equations to get B and D.

$$B = 550 + 125 = 675$$
$$D = 2(550) - 300 = 800$$

So Herschel took in 675 calories from breakfast, 550 calories from lunch and 800 calories from dinner.

100. We will let B be the calories from breakfast, L the calories from lunch, D the calories from dinner and S the calories from snacks. So we have the following equations:

$$L = 0.5B \qquad D = B + 200$$
$$S = B - 120 \quad E = 700$$
$$1480 = B + L + D - E$$

Now we substitute the first four into the last one and solve for B.

$$1480 = B + 0.5B + (B+200) + (B-200) - 700$$
$$1480 = 3.5B - 620$$
$$2100 = 3.5B$$
$$B = 600$$

Now we substitute B to get S.

$$S = B - 120 = 600 - 120 = 480$$

So Tyshira took in 480 calories from snacks.

101.

Judy's Amount	Tom's Amount	Total
x	$\frac{2}{3}x$	18

$$x + \frac{2}{3}x = 18$$
$$\frac{5}{3}x = 18$$
$$x = \frac{3}{5}(18)$$
$$x = 10.80$$

Judy pays \$10.80 and Tom pays \$7.20.

102. An isosceles triangle has three equal sides. Therefore: $4x+10 = 2x+40 = 3x+18$. Solve each set separately:

$$4x + 10 = 2x + 40$$
$$2x = 30$$
$$x = 15$$

$$4x + 10 = 3x + 18$$
$$x = 8$$

$$2x + 40 = 3x + 18$$
$$-x = -22$$
$$x = 22$$

Since 22 is the largest of the numbers then the largest perimeter is:

$$4(22) + 10 + 2(22) + 40 + 3(22) + 18 = 266$$

103. $\frac{3}{4}x - \frac{1}{5}\left(\frac{1}{2} - 3x\right) + 1 = \frac{1}{4}\left(\frac{1}{20}x + 6\right) - \frac{4}{5}$

$$\frac{3}{4}x - \frac{1}{10} + \frac{3}{5}x + 1 = \frac{1}{80}x + \frac{3}{2} - \frac{4}{5}$$

Multiply both sides by the LCD 80 to clear fractions.

$$60x - 8 + 48x + 80 = x + 120 - 64$$
$$108x + 72 = x + 58$$
$$107x = -16$$
$$x = -\frac{16}{107}$$

104. If a hexagon is inscribed in a circle then the sides of the hexagon are equal to the radius of the circle. Let the P = 6r be the perimeter of the hexagon. Let r be the radius of the circle.

$$6r = r + 10$$
$$5r = 10$$
$$r = 2$$

Thus r = 2 inches is the radius of the circle where the perimeter of the hexagon is 10 inches more than the radius.

105. To move from step (6) to step (7), we divided both sides of the equation by the expression $x-2$. From step (1), however, we know $x = 2$, so this means we divided both sides of the equation by zero.

106– 107. Answers will vary.

Section 1.2

1. $x^2-5x-6=(x-6)(x+1)$

2. $2x^2-x-3=(2x-3)(x+1)$

3. $\left\{-\frac{5}{3},3\right\}$

4. True

5. $\frac{1}{2}\bullet 5=\frac{5}{2};\left(\frac{5}{2}\right)^2=\frac{25}{4};x^2+5x+\frac{25}{4}$

   $x^2+5x+\frac{25}{4}=\left(x+\frac{5}{2}\right)^2$

6. discriminant; negative

7. False; a quadratic equation may have no real solutions.

8. False; If $x^2=p$ then x could also be negative.

9. b

10. d

11. $x^2-9x=0$
    $x(x-9)=0$
    $x=0$ or $x-9=0$
    $x=0$ or $x=9$
    The solution set is $\{0, 9\}$.

12. $x^2+4x=0$
    $x(x+4)=0$
    $x=0$ or $x+4=0$
    $x=0$ or $x=-4$
    The solution set is $\{-4, 0\}$.

13. $x^2-25=0$
    $(x+5)(x-5)=0$
    $x+5=0$ or $x-5=0$
    $x=-5$ or $x=5$
    The solution set is $\{-5, 5\}$.

14. $x^2-9=0$
    $(x+3)(x-3)=0$
    $x+3=0$ or $x-3=0$
    $x=-3$ or $x=3$
    The solution set is $\{-3, 3\}$.

15. $z^2+z-6=0$
    $(z+3)(z-2)=0$
    $z+3=0$ or $z-2=0$
    $z=-3$ or $z=2$
    The solution set is $\{-3, 2\}$.

16. $v^2+7v+6=0$
    $(v+6)(v+1)=0$
    $v+6=0$ or $v+1=0$
    $v=-6$ or $v=-1$
    The solution set is $\{-6, -1\}$

17. $2x^2-5x-3=0$
    $(2x+1)(x-3)=0$
    $2x+1=0$ or $x-3=0$
    $x=-\frac{1}{2}$ or $x=3$
    The solution set is $\left\{-\frac{1}{2}, 3\right\}$

18. $3x^2+5x+2=0$
    $(3x+2)(x+1)=0$
    $3x+2=0$ or $x+1=0$
    $x=-\frac{2}{3}$ or $x=-1$
    The solution set is $\left\{-1,-\frac{2}{3}\right\}$.

19. $-5w^2+180=0$
    $-5(w^2-36)=0$
    $-5(w+6)(w-6)=0$
    $w+6=0$ or $w-6=0$
    $w=-6$ or $w=6$
    The solution set is $\{-6, 6\}$.

20.
$$2y^2-50=0$$
$$2(y^2-25)=0$$
$$2(y+5)(y-5)=0$$
$$y+5=0 \quad \text{or} \quad y-5=0$$
$$y=-5 \quad \text{or} \quad y=5$$
The solution set is $\{-5, 5\}$.

21.
$$x(x+3)-10=0$$
$$x^2+3x-10=0$$
$$(x-2)(x+5)=0$$
$$x-2=0 \quad \text{or} \quad x+5=0$$
$$x=2 \quad \text{or} \quad x=-5$$
The solution set is $\{-5, 2\}$.

22.
$$x(x+4)=12$$
$$x^2+4x-12=0$$
$$(x+6)(x-2)=0$$
$$x+6=0 \quad \text{or} \quad x-2=0$$
$$x=-6 \quad \text{or} \quad x=2$$
The solution set is $\{-6,\ 2\}$.

23.
$$4x^2+9=12x$$
$$4x^2-12x+9=0$$
$$(2x-3)^2=0$$
$$2x-3=0$$
$$x=\frac{3}{2}$$
The solution set is $\left\{\frac{3}{2}\right\}$.

24.
$$25x^2+16=40x$$
$$25x^2-40x+16=0$$
$$(5x-4)^2=0$$
$$5x-4=0$$
$$x=\frac{4}{5}$$
The solution set is $\left\{\frac{4}{5}\right\}$.

25.
$$6(p^2-1)=5p$$
$$6p^2-6=5p$$
$$6p^2-5p-6=0$$
$$(3p+2)(2p-3)=0$$
$$3p+2=0 \quad \text{or} \quad 2p-3=0$$
$$p=-\frac{2}{3} \quad \text{or} \quad p=\frac{3}{2}$$
The solution set is $\left\{-\frac{2}{3}, \frac{3}{2}\right\}$.

26.
$$2(2u^2-4u)+3=0$$
$$4u^2-8u+3=0$$
$$(2u-1)(2u-3)=0$$
$$2u-1=0 \quad \text{or} \quad 2u-3=0$$
$$u=\frac{1}{2} \quad \text{or} \quad u=\frac{3}{2}$$
The solution set is $\left\{\frac{1}{2}, \frac{3}{2}\right\}$.

27.
$$6x-5=\frac{6}{x}$$
$$(6x-5)x=\left(\frac{6}{x}\right)x$$
$$6x^2-5x=6$$
$$6x^2-5x-6=0$$
$$(3x+2)(2x-3)=0$$
$$3x+2=0 \quad \text{or} \quad 2x-3=0$$
$$x=-\frac{2}{3} \quad \text{or} \quad x=\frac{3}{2}$$
Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{2}{3}, \frac{3}{2}\right\}$.

28.
$$x+\frac{12}{x}=7$$
$$\left(x+\frac{12}{x}\right)x=7x$$
$$x^2+12=7x$$
$$x^2-7x+12=0$$
$$(x-3)(x-4)=0$$
$$x-3=0 \quad \text{or} \quad x-4=0$$
$$x=3 \quad \text{or} \quad x=4$$

Neither of these values causes a denominator to equal zero, so the solution set is $\{3, 4\}$.

29.
$$\frac{4(x-2)}{x-3}+\frac{3}{x}=\frac{-3}{x(x-3)}$$
$$\left(\frac{4(x-2)}{x-3}+\frac{3}{x}\right)x(x-3)=\left(\frac{-3}{x(x-3)}\right)x(x-3)$$
$$4x(x-2)+3(x-3)=-3$$
$$4x^2-8x+3x-9=-3$$
$$4x^2-5x-6=0$$
$$(4x+3)(x-2)=0$$
$$4x+3=0 \quad \text{or} \quad x-2=0$$
$$x=-\frac{3}{4} \quad \text{or} \quad x=2$$
Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{3}{4}, 2\right\}$.

30.
$$\frac{5}{x+4}=4+\frac{3}{x-2}$$
$$\left(\frac{5}{x+4}\right)(x+4)(x-2)=\left(4+\frac{3}{x-2}\right)(x+4)(x-2)$$
$$5(x-2)=4(x+4)(x-2)+3(x+4)$$
$$5x-10=4\left(x^2+2x-8\right)+3x+12$$
$$5x-10=4x^2+8x-32+3x+12$$
$$0=4x^2+6x-10$$
$$0=2\left(2x^2+3x-5\right)$$
$$0=2(2x+5)(x-1)$$
$$2x+5=0 \quad \text{or} \quad x-1=0$$
$$x=-\frac{5}{2} \quad \text{or} \quad x=1$$
Neither of these values causes a denominator to equal zero, so the solution set is $\left\{-\frac{5}{2}, 1\right\}$.

31. $x^2=25$
$$x=\pm\sqrt{25}$$
$$x=\pm 5$$
The solution set is $\{-5, 5\}$.

32. $x^2=36$
$$x=\pm\sqrt{36}$$
$$x=\pm 6$$
The solution set is $\{-6, 6\}$.

33. $(x-1)^2=4$
$$x-1=\pm\sqrt{4}$$
$$x-1=\pm 2$$
$$x-1=2 \quad \text{or} \quad x-1=-2$$
$$x=3 \quad \text{or} \quad x=-1$$
The solution set is $\{-1, 3\}$.

34. $(x+2)^2=1$
$$x+2=\pm\sqrt{1}$$
$$x+2=\pm 1$$
$$x+2=1 \quad \text{or} \quad x+2=-1$$
$$x=-1 \quad \text{or} \quad x=-3$$
The solution set is $\{-3, -1\}$.

35. $\left(\frac{1}{3}h+4\right)^2=16$
$$\frac{1}{3}h+4=\pm\sqrt{16}$$
$$\frac{1}{3}h+4=\pm 4$$
$$\frac{1}{3}h+4=4 \quad \text{or} \quad \frac{1}{3}h+4=-4$$
$$\frac{1}{3}h=0 \quad \text{or} \quad \frac{1}{3}h=-8$$
$$h=0 \quad \text{or} \quad h=-24$$
The solution set is $\{-24, 0\}$.

36. $(3z-2)^2=4$
$$3z-2=\pm\sqrt{4}$$
$$3z-2=\pm 2$$
$$3z-2=2 \quad \text{or} \quad 3z-2=-2$$
$$3z=4 \quad \text{or} \quad 3z=0$$
$$z=\frac{4}{3} \quad \text{or} \quad z=0$$
The solution set is $\left\{0, \frac{4}{3}\right\}$.

37.
$$\begin{aligned} x^2+4x&=21\\ x^2+4x+4&=21+4\\ (x+2)^2&=25\\ x+2&=\pm\sqrt{25}\\ x+2&=\pm 5\\ x&=-2\pm 5\\ x&=3 \text{ or } x=-7 \end{aligned}$$
The solution set is $\{-7,3\}$.

38.
$$\begin{aligned} x^2-6x&=13\\ x^2-6x+9&=13+9\\ (x-3)^2&=22\\ x-3&=\pm\sqrt{22}\\ x&=3\pm\sqrt{22} \end{aligned}$$
The solution set is $\{3-\sqrt{22},3+\sqrt{22}\}$.

39.
$$\begin{aligned} x^2-\frac{1}{2}x-\frac{3}{16}&=0\\ x^2-\frac{1}{2}x&=\frac{3}{16}\\ x^2-\frac{1}{2}x+\frac{1}{16}&=\frac{3}{16}+\frac{1}{16}\\ \left(x-\frac{1}{4}\right)^2&=\frac{1}{4}\\ x-\frac{1}{4}&=\pm\sqrt{\frac{1}{4}}=\pm\frac{1}{2}\\ x&=\frac{1}{4}\pm\frac{1}{2}\\ x&=\frac{3}{4} \text{ or } x=-\frac{1}{4} \end{aligned}$$
The solution set is $\left\{-\frac{1}{4},\frac{3}{4}\right\}$.

40.
$$\begin{aligned} x^2+\frac{2}{3}x-\frac{1}{3}&=0\\ x^2+\frac{2}{3}x&=\frac{1}{3}\\ x^2+\frac{2}{3}x+\frac{1}{9}&=\frac{1}{3}+\frac{1}{9}\\ \left(x+\frac{1}{3}\right)^2&=\frac{4}{9}\\ x+\frac{1}{3}&=\pm\sqrt{\frac{4}{9}}=\pm\frac{2}{3}\\ x&=-\frac{1}{3}\pm\frac{2}{3}\\ x&=\frac{1}{3} \text{ or } x=-1 \end{aligned}$$
The solution set is $\left\{-1,\frac{1}{3}\right\}$.

41.
$$\begin{aligned} 3x^2+x-\frac{1}{2}&=0\\ x^2+\frac{1}{3}x-\frac{1}{6}&=0\\ x^2+\frac{1}{3}x&=\frac{1}{6}\\ x^2+\frac{1}{3}x+\frac{1}{36}&=\frac{1}{6}+\frac{1}{36}\\ \left(x+\frac{1}{6}\right)^2&=\frac{7}{36}\\ x+\frac{1}{6}&=\pm\sqrt{\frac{7}{36}}\\ x+\frac{1}{6}&=\pm\frac{\sqrt{7}}{6}\\ x&=\frac{-1\pm\sqrt{7}}{6} \end{aligned}$$
The solution set is $\left\{\frac{-1-\sqrt{7}}{6},\frac{-1+\sqrt{7}}{6}\right\}$.

42.
$$\begin{aligned} 2x^2-3x-1&=0\\ x^2-\frac{3}{2}x-\frac{1}{2}&=0\\ x^2-\frac{3}{2}x&=\frac{1}{2}\\ x^2-\frac{3}{2}x+\frac{9}{16}&=\frac{1}{2}+\frac{9}{16} \end{aligned}$$

$$\left(x-\frac{3}{4}\right)^2=\frac{17}{16}$$
$$x-\frac{3}{4}=\pm\sqrt{\frac{17}{16}}$$
$$x-\frac{3}{4}=\pm\frac{\sqrt{17}}{4}$$
$$x=\frac{3\pm\sqrt{17}}{4}$$

The solution set is $\left\{\frac{3-\sqrt{17}}{4},\frac{3+\sqrt{17}}{4}\right\}$.

43. $x^2-4x+2=0$

$a=1,\quad b=2,\quad c=-13$

$$x=\frac{-(2)\pm\sqrt{(2)^2-4(1)(-13)}}{2(1)}=\frac{-2\pm\sqrt{4+52}}{2}$$
$$=\frac{-2\pm\sqrt{56}}{2}=\frac{-2\pm2\sqrt{14}}{2}=-1\pm\sqrt{14}$$

The solution set is $\left\{-1-\sqrt{14},-1+\sqrt{14}\right\}$.

44. $x^2+4x+2=0$

$a=1,\quad b=4,\quad c=2$

$$x=\frac{-4\pm\sqrt{4^2-4(1)(2)}}{2(1)}=\frac{-4\pm\sqrt{16-8}}{2}$$
$$=\frac{-4\pm\sqrt{8}}{2}=\frac{-4\pm2\sqrt{2}}{2}=-2\pm\sqrt{2}$$

The solution set is $\left\{-2-\sqrt{2},-2+\sqrt{2}\right\}$.

45. $x^2-4x-1=0$

$a=1,\quad b=-4,\quad c=-1$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-1)}}{2(1)}=\frac{4\pm\sqrt{16+4}}{2}$$
$$=\frac{4\pm\sqrt{20}}{2}=\frac{4\pm2\sqrt{5}}{2}=2\pm\sqrt{5}$$

The solution set is $\left\{2-\sqrt{5},2+\sqrt{5}\right\}$.

46. $x^2+6x+1=0$

$a=1,\quad b=6,\quad c=1$

$$x=\frac{-6\pm\sqrt{6^2-4(1)(1)}}{2(1)}=\frac{-6\pm\sqrt{36-4}}{2}$$
$$=\frac{-6\pm\sqrt{32}}{2}=\frac{-6\pm4\sqrt{2}}{2}=-3\pm2\sqrt{2}$$

The solution set is $\left\{-3-2\sqrt{2},-3+2\sqrt{2}\right\}$.

47. $2x^2-5x+3=0$

$a=2,\quad b=-5,\quad c=3$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(2)(3)}}{2(2)}$$
$$=\frac{5\pm\sqrt{25-24}}{4}=\frac{5\pm\sqrt{1}}{4}=\frac{5\pm1}{4}$$
$$x=\frac{5+1}{4}\ \text{ or }\ x=\frac{5-1}{4}$$
$$x=\frac{6}{4}\ \text{ or }\ x=\frac{4}{4}$$
$$x=\frac{3}{2}\ \text{ or }\ x=1$$

The solution set is $\left\{1,\frac{3}{2}\right\}$.

48. $2x^2+5x+3=0$

$a=2,\quad b=5,\quad c=3$

$$x=\frac{-5\pm\sqrt{5^2-4(2)(3)}}{2(2)}$$
$$=\frac{-5\pm\sqrt{25-24}}{4}=\frac{-5\pm\sqrt{1}}{4}=\frac{-5\pm1}{4}$$
$$x=\frac{-5+1}{4}\ \text{ or }\ x=\frac{-5-1}{4}$$
$$x=\frac{-4}{4}\ \text{ or }\ x=\frac{-6}{4}$$
$$x=-1\ \text{ or }\ x=-\frac{3}{2}$$

The solution set is $\left\{-\frac{3}{2},-1\right\}$.

49. $4y^2 - y + 2 = 0$

$a = 4, \quad b = -1, \quad c = 2$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(2)}}{2(4)}$$

$$= \frac{1 \pm \sqrt{1-32}}{8} = \frac{1 \pm \sqrt{-31}}{8}$$

No real solution.

50. $4t^2 + t + 1 = 0$

$a = 4, \quad b = 1, \quad c = 1$

$$t = \frac{-1 \pm \sqrt{1^2 - 4(4)(1)}}{2(4)}$$

$$= \frac{-1 \pm \sqrt{1-16}}{8} = \frac{-1 \pm \sqrt{-15}}{8}$$

No real solution.

51. $9x^2 + 8x = 5$

$9x^2 + 8x - 5 = 0$

$a = 9, \quad b = 8, \quad c = -5$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(9)(-5)}}{2(9)}$$

$$= \frac{-8 \pm \sqrt{64+180}}{18} = \frac{-8 \pm \sqrt{244}}{18}$$

$$= \frac{-8 \pm 2\sqrt{61}}{18} = \frac{-4 \pm \sqrt{61}}{9}$$

The solution set is $\left\{\frac{-4-\sqrt{61}}{9}, \frac{-4+\sqrt{61}}{9}\right\}$.

52. $2x^2 = 1 - 2x$

$2x^2 + 2x - 1 = 0$

$a = 2, \quad b = 2, \quad c = -1$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)} = \frac{-2 \pm \sqrt{4+8}}{4}$$

$$= \frac{-2 \pm \sqrt{12}}{4} = \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

The solution set is $\left\{\frac{-1-\sqrt{3}}{2}, \frac{-1+\sqrt{3}}{2}\right\}$.

53. $4x^2 = 9x$

$4x^2 - 9x = 0$

$x(4x - 9) = 0$

$x = 0 \quad \text{or} \quad 4x - 9 = 0$

$x = 0 \quad \text{or} \quad x = \frac{9}{4}$

The solution set is $\left\{0, \frac{9}{4}\right\}$.

54. $5x = 4x^2$

$0 = 4x^2 - 5x$

$0 = x(4x - 5)$

$x = 0 \quad \text{or} \quad 4x - 5 = 0$

$x = 0 \quad \text{or} \quad x = \frac{5}{4}$

The solution set is $\left\{0, \frac{5}{4}\right\}$.

55. $9t^2 - 6t + 1 = 0$

$a = 9, \quad b = -6, \quad c = 1$

$$t = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)}$$

$$= \frac{6 \pm \sqrt{36-36}}{18} = \frac{6 \pm 0}{18} = \frac{1}{3}$$

The solution set is $\left\{\frac{1}{3}\right\}$.

56. $4u^2 - 6u + 9 = 0$

$a = 4, \quad b = -6, \quad c = 9$

$$u = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(9)}}{2(4)}$$

$$= \frac{6 \pm \sqrt{36-144}}{8} = \frac{6 \pm \sqrt{-108}}{8}$$

No real solution.

57. $\frac{3}{4}x^2 - \frac{1}{4}x - \frac{1}{2} = 0$

$$4\left(\frac{3}{4}x^2 - \frac{1}{4}x - \frac{1}{2}\right) = 4(0)$$

$3x^2 - x - 2 = 0$

$a = 3, \quad b = -1, \quad c = -2$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-2)}}{2(3)}$$
$$= \frac{1 \pm \sqrt{1+24}}{6} = \frac{1 \pm \sqrt{25}}{6} = \frac{1 \pm 5}{6}$$
$$x = \frac{1+5}{6} \quad \text{or} \quad x = \frac{1-5}{6}$$
$$x = \frac{6}{6} \quad \text{or} \quad x = \frac{-4}{6}$$
$$x = 1 \quad \text{or} \quad x = -\frac{2}{3}$$

The solution set is $\left\{-\frac{2}{3}, 1\right\}$.

58. $\frac{2}{3}x^2 - x - 3 = 0$
$$3\left(\frac{2}{3}x^2 - x - 3\right) = 3(0)$$
$$2x^2 - 3x - 9 = 0$$
$a = 2, \quad b = -3, \quad c = -9$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-9)}}{2(2)}$$
$$= \frac{3 \pm \sqrt{9+72}}{4} = \frac{3 \pm \sqrt{81}}{4} = \frac{3 \pm 9}{4}$$
$$x = \frac{3+9}{4} \quad \text{or} \quad x = \frac{3-9}{4}$$
$$x = \frac{12}{4} \quad \text{or} \quad x = \frac{-6}{4}$$
$$x = 3 \quad \text{or} \quad x = -\frac{3}{2}$$

The solution set is $\left\{-\frac{3}{2}, 3\right\}$.

59. $\frac{5}{3}x^2 - x = \frac{1}{3}$
$$3\left(\frac{5}{3}x^2 - x\right) = 3\left(\frac{1}{3}\right)$$
$$5x^2 - 3x = 1$$
$$5x^2 - 3x - 1 = 0$$
$a = 5, \quad b = -3, \quad c = -1$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-1)}}{2(5)}$$
$$= \frac{3 \pm \sqrt{9+20}}{10} = \frac{3 \pm \sqrt{29}}{10}$$

The solution set is $\left\{\frac{3-\sqrt{29}}{10}, \frac{3+\sqrt{29}}{10}\right\}$.

60. $\frac{3}{5}x^2 - x = \frac{1}{5}$
$$5\left(\frac{3}{5}x^2 - x\right) = 5\left(\frac{1}{5}\right)$$
$$3x^2 - 5x = 1$$
$$3x^2 - 5x - 1 = 0$$
$a = 3, \quad b = -5, \quad c = -1$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-1)}}{2(3)}$$
$$= \frac{5 \pm \sqrt{25+12}}{6} = \frac{5 \pm \sqrt{37}}{6}$$

The solution set is $\left\{\frac{5-\sqrt{37}}{6}, \frac{5+\sqrt{37}}{6}\right\}$.

61. $2x(x+2) = 3$
$$2x^2 + 4x - 3 = 0$$
$a = 2, \quad b = 4, \quad c = -3$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2(2)} = \frac{-4 \pm \sqrt{16+24}}{4}$$
$$= \frac{-4 \pm \sqrt{40}}{4} = \frac{-4 \pm 2\sqrt{10}}{4} = \frac{-2 \pm \sqrt{10}}{2}$$

The solution set is $\left\{\frac{-2-\sqrt{10}}{2}, \frac{-2+\sqrt{10}}{2}\right\}$.

62. $3x(x+2)=1$

$$3x^2+6x-1=0$$
$$a=3,\quad b=6,\quad c=-1$$
$$x=\frac{-6\pm\sqrt{6^2-4(3)(-1)}}{2(3)}=\frac{-6\pm\sqrt{36+12}}{6}$$
$$=\frac{-6\pm\sqrt{48}}{6}=\frac{-6\pm4\sqrt{3}}{6}=\frac{-3\pm2\sqrt{3}}{3}$$

The solution set is $\left\{\frac{-3-2\sqrt{3}}{3},\frac{-3+2\sqrt{3}}{3}\right\}$.

63. $4+\frac{1}{x}-\frac{1}{x^2}=0$

$$x^2\left(4+\frac{1}{x}-\frac{1}{x^2}\right)=x^2(0)$$
$$4x^2+x-1=0$$
$$a=4,\quad b=1,\quad c=-1$$
$$x=\frac{-1\pm\sqrt{1^2-4(4)(-1)}}{2(4)}$$
$$=\frac{-1\pm\sqrt{1+16}}{8}=\frac{-1\pm\sqrt{17}}{8}$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{\frac{-1-\sqrt{17}}{8},\frac{-1+\sqrt{17}}{8}\right\}$.

64. $2+\frac{8}{x}+\frac{3}{x^2}=0$

$$x^2\left(2+\frac{8}{x}+\frac{3}{x^2}\right)=x^2(0)$$
$$2x^2+8x+3=0$$
$$a=2,\quad b=8,\quad c=3$$
$$x=\frac{-(8)\pm\sqrt{(8)^2-4(2)(3)}}{2(2)}$$
$$=\frac{-8\pm\sqrt{64-24}}{4}=\frac{-8\pm\sqrt{40}}{4}$$
$$=\frac{-8\pm2\sqrt{10}}{4}=\frac{-4\pm\sqrt{10}}{2}$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{\frac{-4-\sqrt{10}}{4},\frac{-4+\sqrt{10}}{4}\right\}$.

65. $\frac{3x}{x-2}+\frac{1}{x}=4$

$$\left(\frac{3x}{x-2}+\frac{1}{x}\right)x(x-2)=4x(x-2)$$
$$3x(x)+(x-2)=4x^2-8x$$
$$3x^2+x-2=4x^2-8x$$
$$0=x^2-9x+2$$
$$a=1,\quad b=-9,\quad c=2$$
$$x=\frac{-(-9)\pm\sqrt{(-9)^2-4(1)(2)}}{2(1)}$$
$$=\frac{9\pm\sqrt{81-8}}{2}=\frac{9\pm\sqrt{73}}{2}$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{\frac{9-\sqrt{73}}{2},\frac{9+\sqrt{73}}{2}\right\}$.

66. $\frac{2x}{x-3}+\frac{1}{x}=4$

$$\left(\frac{2x}{x-3}+\frac{1}{x}\right)x(x-3)=4x(x-3)$$
$$2x(x)+(x-3)=4x^2-12x$$
$$2x^2+x-3=4x^2-12x$$
$$0=2x^2-13x+3$$
$$a=2,\quad b=-13,\quad c=3$$
$$x=\frac{-(-13)\pm\sqrt{(-13)^2-4(2)(3)}}{2(2)}$$
$$=\frac{13\pm\sqrt{169-24}}{4}=\frac{13\pm\sqrt{145}}{4}$$

Neither of these values causes a denominator to equal zero, so the solution set is $\left\{\frac{13-\sqrt{145}}{4},\frac{13+\sqrt{145}}{4}\right\}$.

67. $x^2-4.1x+2.2=0$

$$a=1,\quad b=-4.1,\quad c=2.2$$
$$x=\frac{-(-4.1)\pm\sqrt{(-4.1)^2-4(1)(2.2)}}{2(1)}$$
$$=\frac{4.1\pm\sqrt{16.81-8.8}}{2}=\frac{4.1\pm\sqrt{8.01}}{2}$$
$$x\approx3.47\quad\text{or}\quad x\approx0.63$$

The solution set is $\{0.63,\ 3.47\}$.

68. $x^2+3.9x+1.8=0$
$a=1, \quad b=3.9, \quad c=1.8$
$$x=\frac{-3.9\pm\sqrt{(3.9)^2-4(1)(1.8)}}{2(1)}$$
$$=\frac{-3.9\pm\sqrt{15.21-7.2}}{2}=\frac{-3.9\pm\sqrt{8.01}}{2}$$
$x\approx-0.53$ or $x\approx-3.37$
The solution set is $\{-3.37,-0.53\}$.

69. $x^2+\sqrt{3}x-3=0$
$a=1, \quad b=\sqrt{3}, \quad c=-3$
$$x=\frac{-\sqrt{3}\pm\sqrt{\left(\sqrt{3}\right)^2-4(1)(-3)}}{2(1)}$$
$$=\frac{-\sqrt{3}\pm\sqrt{3+12}}{2}=\frac{-\sqrt{3}\pm\sqrt{15}}{2}$$
$x\approx1.07$ or $x\approx-2.80$
The solution set is $\{-2.80,\ 1.07\}$.

70. $x^2+\sqrt{2}x-2=0$
$a=1, \quad b=\sqrt{2}, \quad c=-2$
$$x=\frac{-\sqrt{2}\pm\sqrt{\left(\sqrt{2}\right)^2-4(1)(-2)}}{2(1)}$$
$$=\frac{-\sqrt{2}\pm\sqrt{2+8}}{2}=\frac{-\sqrt{2}\pm\sqrt{10}}{2}$$
$x\approx0.87$ or $x\approx-2.29$
The solution set is $\{-2.29,\ 0.87\}$.

71. $\pi x^2-x-\pi=0$
$a=\pi, \quad b=-1, \quad c=-\pi$
$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(\pi)(-\pi)}}{2(\pi)}$$
$$=\frac{1\pm\sqrt{1+4\pi^2}}{2\pi}$$
$x\approx1.17$ or $x\approx-0.85$
The solution set is $\{-0.85,\ 1.17\}$.

72. $\pi x^2+\pi x-2=0$
$a=\pi, \quad b=\pi, \quad c=-2$
$$x=\frac{-\pi\pm\sqrt{(\pi)^2-4(\pi)(-2)}}{2(\pi)}$$
$$=\frac{-\pi\pm\sqrt{\pi^2+8\pi}}{2\pi}$$
$x\approx0.44$ or $x\approx-1.44$
The solution set is $\{-1.44,\ 0.44\}$.

73. $2x^2-6x+7=0$
$a=2, \quad b=-6, \quad c=7$
$b^2-4ac=(-6)^2-4(2)(7)=36-56=-20$
Since the $b^2-4ac<0,$ the equation has no real solution.

74. $x^2+4x+7=0$
$a=1, \quad b=4, \quad c=7$
$b^2-4ac=(4)^2-4(1)(7)=16-28=-12$
Since the $b^2-4ac<0,$ the equation has no real solution.

75. $9x^2-30x+25=0$
$a=9, \quad b=-30, \quad c=25$
$b^2-4ac=(-30)^2-4(9)(25)=900-900=0$
Since $b^2-4ac=0,$ the equation has one repeated real solution.

76. $25x^2-20x+4=0$
$a=25, \quad b=-20, \quad c=4$
$b^2-4ac=(-20)^2-4(25)(4)=400-400=0$
Since $b^2-4ac=0,$ the equation has one repeated real solution.

77. $3x^2+5x-8=0$
$a=3, \quad b=5, \quad c=-8$
$b^2-4ac=(5)^2-4(3)(-8)=25+96=121$
Since $b^2-4ac>0,$ the equation has two unequal real solutions.

78. $2x^2-3x-7=0$
$a=2, \quad b=-3, \quad c=-7$
$b^2-4ac=(-3)^2-4(2)(-7)=9+56=65$
Since $b^2-4ac>0,$ the equation has two unequal real solutions.

79. $x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$

The solution set is $\left\{-\sqrt{5}, \sqrt{5}\right\}$.

80. $x^2 - 6 = 0$

$x^2 = 6$

$x = \pm\sqrt{6}$

The solution set is $\left\{-\sqrt{6}, \sqrt{6}\right\}$.

81. $16x^2 - 8x + 1 = 0$

$(4x-1)(4x-1) = 0$

$4x - 1 = 0$

$x = \frac{1}{4}$

The solution set is $\left\{\frac{1}{4}\right\}$.

82. $9x^2 - 12x + 4 = 0$

$(3x-2)(3x-2) = 0$

$3x - 2 = 0$

$x = \frac{2}{3}$

The solution set is $\left\{\frac{2}{3}\right\}$.

83. $10x^2 - 19x - 15 = 0$

$(5x+3)(2x-5) = 0$

$5x + 3 = 0$ or $2x - 5 = 0$

$x = -\frac{3}{5}$ or $x = \frac{5}{2}$

The solution set is $\left\{-\frac{3}{5}, \frac{5}{2}\right\}$.

84. $6x^2 + 7x - 20 = 0$

$(3x-4)(2x+5) = 0$

$3x - 4 = 0$ or $2x + 5 = 0$

$x = \frac{4}{3}$ or $x = -\frac{5}{2}$

The solution set is $\left\{-\frac{5}{2}, \frac{4}{3}\right\}$.

85. $2 + z = 6z^2$

$0 = 6z^2 - z - 2$

$0 = (3z-2)(2z+1)$

$3z - 2 = 0$ or $2z + 1 = 0$

$z = \frac{2}{3}$ or $z = -\frac{1}{2}$

The solution set is $\left\{-\frac{1}{2}, \frac{2}{3}\right\}$.

86. $2 = y + 6y^2$

$0 = 6y^2 + y - 2$

$0 = (3y+2)(2y-1)$

$3y + 2 = 0$ or $2y - 1 = 0$

$y = -\frac{2}{3}$ or $y = \frac{1}{2}$

The solution set is $\left\{-\frac{2}{3}, \frac{1}{2}\right\}$.

87. $x^2 + \sqrt{2}x = \frac{1}{2}$

$x^2 + \sqrt{2}x - \frac{1}{2} = 0$

$2\left(x^2 + \sqrt{2}x - \frac{1}{2}\right) = 2(0)$

$2x^2 + 2\sqrt{2}x - 1 = 0$

$a = 2, \quad b = 2\sqrt{2}, \quad c = -1$

$$x = \frac{-(2\sqrt{2}) \pm \sqrt{(2\sqrt{2})^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-2\sqrt{2} \pm \sqrt{8+8}}{4} = \frac{-2\sqrt{2} \pm \sqrt{16}}{4}$$

$$= \frac{-2\sqrt{2} \pm 4}{4} = \frac{-\sqrt{2} \pm 2}{2}$$

The solution set is $\left\{\frac{-\sqrt{2}-2}{2}, \frac{-\sqrt{2}+2}{2}\right\}$

88. $\frac{1}{2}x^2 = \sqrt{2}x + 1$

$\frac{1}{2}x^2 - \sqrt{2}x - 1 = 0$

$2\left(\frac{1}{2}x^2 - \sqrt{2}x - 1\right) = 2(0)$

$x^2 - 2\sqrt{2}x - 2 = 0$

$a = 1, \quad b = -2\sqrt{2}, \quad c = -2$

$$x=\frac{-(-2\sqrt{2})\pm\sqrt{(-2\sqrt{2})^2-4(1)(-2)}}{2(1)}$$
$$=\frac{2\sqrt{2}\pm\sqrt{8+8}}{2}=\frac{2\sqrt{2}\pm\sqrt{16}}{2}$$
$$=\frac{2\sqrt{2}\pm 4}{2}=\frac{\sqrt{2}\pm 2}{1}$$

The solution set is $\left\{\sqrt{2}-2, \sqrt{2}+2\right\}$.

89. $x^2+x=4$

$$x^2+x-4=0$$
$$a=1,\quad b=1,\quad c=-4$$
$$x=\frac{-(1)\pm\sqrt{(1)^2-4(1)(-4)}}{2(1)}$$
$$=\frac{-1\pm\sqrt{1+16}}{2}=\frac{-1\pm\sqrt{17}}{2}$$

The solution set is $\left\{\frac{-1-\sqrt{17}}{2}, \frac{-1+\sqrt{17}}{2}\right\}$.

90. $x^2+x=1$

$$x^2+x-1=0$$
$$a=1,\quad b=1,\quad c=-1$$
$$x=\frac{-(1)\pm\sqrt{(1)^2-4(1)(-1)}}{2(1)}$$
$$=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1\pm\sqrt{5}}{2}$$

The solution set is $\left\{\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right\}$.

91.

$$\frac{x}{x-2}+\frac{2}{x+1}=\frac{7x+1}{x^2-x-2}$$
$$\frac{x}{x-2}+\frac{2}{x+1}=\frac{7x+1}{(x-2)(x+1)}$$
$$\left(\frac{x}{x-2}+\frac{2}{x+1}\right)(x-2)(x+1)=\left(\frac{7x+1}{(x-2)(x+1)}\right)(x-2)(x+1)$$
$$x(x+1)+2(x-2)=7x+1$$
$$x^2+x+2x-4=7x+1$$
$$x^2+3x-4=7x+1$$
$$x^2-4x-5=0$$
$$(x+1)(x-5)=0$$
$$x+1=0 \quad \text{or} \quad x-5=0$$
$$x=-1 \quad \text{or} \quad x=5$$

The value $x=-1$ causes a denominator to equal zero, so we disregard it. Thus, the solution set is $\{5\}$.

92.
$$\frac{3x}{x+2}+\frac{1}{x-1}=\frac{4-7x}{x^2+x-2}$$
$$\frac{3x}{x+2}+\frac{1}{x-1}=\frac{4-7x}{(x+2)(x-1)}$$
$$\left(\frac{3x}{x+2}+\frac{1}{x-1}\right)(x+2)(x-1)=\left(\frac{4-7x}{(x+2)(x-1)}\right)(x+2)(x-1)$$
$$3x(x-1)+(x+2)=4-7x$$
$$3x^2-3x+x+2=4-7x$$
$$3x^2-2x+2=4-7x$$
$$3x^2+5x-2=0$$
$$(3x-1)(x+2)=0$$
$$3x-1=0 \quad \text{or} \quad x+2=0$$
$$x=\frac{1}{3} \quad \text{or} \quad x=-2$$

The value $x=-2$ causes a denominator to equal zero, so we disregard it. Thus, the solution set is $\left\{\frac{1}{3}\right\}$.

93. Since this is a right triangle then we can use the Pythagorean Theorem. So
$$(2x+3)^2=(2x-5)^2+(x+7)^2$$
$$4x^2+12x+9=4x^2-20x+25+x^2+14x+49$$
$$12x+9=x^2-6x+74$$
$$0=x^2-18x+65$$
$$0=(x-5)(x-13)$$
$$x-5=0 \quad \text{or} \quad x-13=0$$
$$x=5 \quad \text{or} \quad x=13$$

This means there are 2 possible that meet these requirements. Substituting x into the given sides gives:
When x = 5: 5m, 12m, 13m
When x = 13: 20m, 21m, 29m
Thus there are 2 solutions.

94. Since this is a right triangle then we can use the Pythagorean Theorem. So
$$(4x+5)^2=(3x+13)^2+x^2$$
$$16x^2+40x+25=9x^2+78x+169+x^2$$
$$6x^2-38x-144=0$$
$$2(3x^2-19x-72)=0$$
$$2(3x+8)(x-9)=0$$
$$3x+8=0 \quad \text{or} \quad x-9=0$$
$$x=-\frac{8}{3} \quad \text{or} \quad x=9$$

This means there are 2 possible solutions that meet these requirements. Substituting x into the given sides gives:
When x = 9: 41m, 40m, 9m
When x = $-\frac{8}{3}$ at least one side of the triangle has a negative measurement which is impossible.
Thus there is only 1 triangle possible

95. Let w represent the width of window.
Then $l=w+2$ represents the length of the window.
Since the area is 143 square feet, we have:
$$w(w+2)=143$$
$$w^2+2w-143=0$$
$$(w+13)(w-11)=0$$
$$\cancel{w=-13} \quad \text{or} \quad w=11$$
Discard the negative solution since width cannot be negative. The width of the rectangular window is 11 feet and the length is 13 feet.

96. Let w represent the width of window.
Then $l=w+1$ represents the length of the window.
Since the area is 306 square centimeters, we have: $w(w+1)=306$
$$w^2+w-306=0$$
$$(w+18)(w-17)=0$$
$$\cancel{w=-18} \quad \text{or} \quad w=17$$
Discard the negative solution since width cannot

be negative. The width of the rectangular window is 17 centimeters and the length is 18 centimeters.

97. Let l represent the length of the rectangle.
Let w represent the width of the rectangle.
The perimeter is 26 meters and the area is 40 square meters.

$$2l + 2w = 26$$
$$l + w = 13 \quad \text{so} \quad w = 13 - l$$
$$l\,w = 40$$
$$l(13 - l) = 40$$
$$13l - l^2 = 40$$
$$l^2 - 13l + 40 = 0$$
$$(l-8)(l-5) = 0$$
$$l = 8 \ \text{ or } \ l = 5$$
$$w = 5 \qquad w = 8$$

The dimensions are 5 meters by 8 meters.

98. Let r represent the radius of the circle.
Since the field is a square with area 1250 square feet, the length of a side of the square is $\sqrt{1250} = 25\sqrt{2}$ feet. The length of the diagonal is $2r$.
Use the Pythagorean Theorem to solve for r:

$$(2r)^2 = \left(25\sqrt{2}\right)^2 + \left(25\sqrt{2}\right)^2$$
$$4r^2 = 1250 + 1250$$
$$4r^2 = 2500$$
$$r^2 = 625$$
$$r = 25$$

The shortest radius setting for the sprinkler is 25 feet.

99. Let x = length of side of original sheet in feet.
Length of box: $x - 2$ feet
Width of box: $x - 2$ feet
Height of box: 1 foot

$$V = l \cdot w \cdot h$$
$$4 = (x-2)(x-2)(1)$$
$$4 = x^2 - 4x + 4$$
$$0 = x^2 - 4x$$
$$0 = x(x-4)$$
$$x = 0 \quad \text{or} \quad x = 4$$

Discard $x = 0$ since that is not a feasible length for the original sheet. Therefore, the original sheet should measure 4 feet on each side.

100. Let x = width of original sheet in feet.
Length of sheet: $2x$
Length of box: $2x - 2$ feet
Width of box: $x - 2$ feet
Height of box: 1 foot

$$V = l \cdot w \cdot h$$
$$4 = (2x-2)(x-2)(1)$$
$$4 = 2x^2 - 6x + 4$$
$$0 = 2x^2 - 6x$$
$$0 = x^2 - 3x$$
$$0 = x(x-3)$$
$$x = 0 \quad \text{or} \quad x = 3$$

Discard $x = 0$ since that is not a feasible length for the original sheet. Therefore, the original sheet is 3 feet wide and 6 feet long.

101. **a.** When the ball strikes the ground, the distance from the ground will be 0. Therefore, we solve

$$96 + 80t - 16t^2 = 0$$
$$-16t^2 + 80t + 96 = 0$$
$$t^2 - 5t - 6 = 0$$
$$(t-6)(t+1) = 0$$
$$t = 6 \quad \text{or} \quad t = -1$$

Discard the negative solution since the time of flight must be positive. The ball will strike the ground after 6 seconds.

b. When the ball passes the top of the building, it will be 96 feet from the ground. Therefore, we solve

$$96 + 80t - 16t^2 = 96$$
$$-16t^2 + 80t = 0$$
$$t^2 - 5t = 0$$
$$t(t-5) = 0$$
$$t = 0 \quad \text{or} \quad t = 5$$

The ball is at the top of the building at time $t = 0$ when it is thrown. It will pass the top of the building on the way down after 5 seconds.

102. a. To find when the object will be 15 meters above the ground, we solve

$-4.9t^2 + 20t = 15$

$-4.9t^2 + 20t - 15 = 0$

$a = -4.9,\ b = 20,\ c = -15$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-15)}}{2(-4.9)}$$

$$= \frac{-20 \pm \sqrt{106}}{-9.8} = \frac{20 \pm \sqrt{106}}{9.8}$$

$t \approx 0.99$ or $t \approx 3.09$

The object will be 15 meters above the ground after about 0.99 seconds (on the way up) and about 3.09 seconds (on the way down).

b. The object will strike the ground when the distance from the ground is 0. Therefore, we solve

$-4.9t^2 + 20t = 0$

$t(-4.9t + 20) = 0$

$t = 0$ or $-4.9t + 20 = 0$

$-4.9t = -20$

$t \approx 4.08$

The object will strike the ground after about 4.08 seconds.

c. $-4.9t^2 + 20t = 100$

$-4.9t^2 + 20t - 100 = 0$

$a = -4.9,\ b = 20,\ c = -100$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-100)}}{2(-4.9)}$$

$$= \frac{-20 \pm \sqrt{-1560}}{-9.8}$$

There is no real solution. The object never reaches a height of 100 meters.

103. Let x represent the number of centimeters the length and width should be reduced.

$12 - x$ = the new length, $7 - x$ = the new width.

The new volume is 90% of the old volume.

$(12 - x)(7 - x)(3) = 0.9(12)(7)(3)$

$3x^2 - 57x + 252 = 226.8$

$3x^2 - 57x + 25.2 = 0$

$x^2 - 19x + 8.4 = 0$

$$x = \frac{-(-19) \pm \sqrt{(-19)^2 - 4(1)(8.4)}}{2(1)} = \frac{19 \pm \sqrt{327.4}}{2}$$

$x \approx 0.45$ or $x \approx 18.55$

Since 18.55 exceeds the dimensions, it is discarded. The dimensions of the new chocolate bar are: 11.55 cm by 6.55 cm by 3 cm.

104. Let x represent the number of centimeters the length and width should be reduced.

$12 - x$ = the new length, $7 - x$ = the new width.

The new volume is 80% of the old volume.

$(12 - x)(7 - x)(3) = 0.8(12)(7)(3)$

$3x^2 - 57x + 252 = 201.6$

$3x^2 - 57x + 50.4 = 0$

$x^2 - 19x + 16.8 = 0$

$$x = \frac{-(-19) \pm \sqrt{(-19)^2 - 4(1)(16.8)}}{2(1)} = \frac{19 \pm \sqrt{293.8}}{2}$$

$x \approx 0.93$ or $x \approx 18.07$

Since 18.07 exceeds the dimensions, it is discarded. The dimensions of the new chocolate bar are: 11.07 cm by 6.07 cm by 3 cm.

105. Let x represent the width of the border measured in feet. The radius of the pool is 5 feet. Then $x + 5$ represents the radius of the circle, including both the pool and the border. The total area of the pool and border is

$A_T = \pi(x+5)^2$.

The area of the pool is $A_P = \pi(5)^2 = 25\pi$.

The area of the border is

$A_B = A_T - A_P = \pi(x+5)^2 - 25\pi$.

Since the concrete is 3 inches or 0.25 feet thick, the volume of the concrete in the border is

$0.25A_B = 0.25\left(\pi(x+5)^2 - 25\pi\right)$

Solving the volume equation:

$0.25\left(\pi(x+5)^2 - 25\pi\right) = 27$

$\pi\left(x^2 + 10x + 25 - 25\right) = 108$

$\pi x^2 + 10\pi x - 108 = 0$

$$x = \frac{-10\pi \pm \sqrt{(10\pi)^2 - 4(\pi)(-108)}}{2(\pi)}$$

$$= \frac{-31.42 \pm \sqrt{100\pi^2 + 432\pi}}{6.28}$$

$x \approx 2.71$ or $x \approx -12.71$

Discard the negative solution. The width of the border is roughly 2.71 feet.

106. Let x represent the width of the border measured in feet. The radius of the pool is 5 feet. Then $x+5$ represents the radius of the circle, including both the pool and the border. The total area of the pool and border is $A_T = \pi(x+5)^2$.

The area of the pool is $A_P = \pi(5)^2 = 25\pi$.

The area of the border is $A_B = A_T - A_P = \pi(x+5)^2 - 25\pi$.

Since the concrete is 4 inches = $\frac{1}{3}$ foot thick, the volume of the concrete in the border is

$$\frac{1}{3}A_B = \frac{1}{3}\left(\pi(x+5)^2 - 25\pi\right)$$

Solving the volume equation:

$$\frac{1}{3}\left(\pi(x+5)^2 - 25\pi\right) = 27$$

$$\pi\left(x^2 + 10x + 25 - 25\right) = 81$$

$$\pi x^2 + 10\pi x - 81 = 0$$

$$x = \frac{-10\pi \pm \sqrt{(10\pi)^2 - 4(\pi)(-81)}}{2(\pi)}$$

$$= \frac{-31.42 \pm \sqrt{100\pi^2 + 324\pi}}{6.28}$$

$x \approx 2.13$ or $x \approx -12.13$

Discard the negative solution. The width of the border is approximately 2.13 feet.

107. Let x represent the width of the border measured in feet.

The total area is $A_T = (6+2x)(10+2x)$.

The area of the garden is $A_G = 6 \cdot 10 = 60$.

The area of the border is $A_B = A_T - A_G = (6+2x)(10+2x) - 60$.

Since the concrete is 3 inches or 0.25 feet thick, the volume of the concrete in the border is

$$0.25A_B = 0.25\left((6+2x)(10+2x) - 60\right)$$

Solving the volume equation:

$$0.25\left((6+2x)(10+2x) - 60\right) = 27$$

$$60 + 32x + 4x^2 - 60 = 108$$

$$4x^2 + 32x - 108 = 0$$

$$x^2 + 8x - 27 = 0$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-27)}}{2(1)} = \frac{-8 \pm \sqrt{172}}{2}$$

$x \approx 2.56$ or $x \approx -10.56$

Discard the negative solution. The width of the border is approximately 2.56 feet.

108. Let x = the width and $2x$ = the length of the patio. The height is $\frac{1}{3}$ foot and the concrete available is $8(27) = 216$ cubic feet..

$$V = lwh = x(2x)\cdot\frac{1}{3} = 216$$

$$\frac{2}{3}x^2 = 216$$

$$x^2 = 324 \Rightarrow x = \pm 18$$

The dimensions of the patio are 18 feet by 36 feet.

109. Let x = the length of a 12.9-inch iPad Pro in a 16:9 format.

Then $\frac{9}{16}x$ = the width of the iPad. The diagonal of the 12.9-inch iPad is 9.7 inches, so by the Pythagorean theorem we have:

$$x^2 + \left(\frac{9}{16}x\right)^2 = 12.9^2$$

$$x^2 + \frac{81}{256}x^2 = 166.41$$

$$256\left(x^2 + \frac{81}{256}x^2\right) = 256(166.41)$$

$$256x^2 + 81x^2 = 42600.96$$

$$337x^2 = 42600.96 \Rightarrow x^2 = \frac{42600.96}{337}$$

$$x = \pm\sqrt{\frac{42600.96}{337}} \approx \pm 11.24$$

Since the length cannot be negative, the length of the iPad is $\sqrt{\frac{42600.96}{337}}$ inches and the width is $\frac{9}{16}\sqrt{\frac{42600.96}{337}} \approx 6.32$ inches. Thus, the area of the iPad is $\sqrt{\frac{42600.96}{337}}\cdot\frac{9}{16}\sqrt{\frac{42600.96}{337}} = 71.11$ square inches.

Let y = the length of a 14.4-inch 3:2 format Microsoft Surface Pro. Then $\frac{2}{3}y$ = the width of the Surface Pro. The diagonal of a 14.4-inch Surface Pro is 14.4 inches, so by the Pythagorean theorem we have:

$$y^2 + \left(\frac{2}{3}y\right)^2 = 14.4^2$$

$$y^2 + \frac{4}{9}y^2 = 207.36$$

$$9\left(y^2 + \frac{4}{9}y^2\right) = 9(207.36)$$

$$9y^2+4y^2=1866.24$$
$$13y^2=1866.24$$
$$y^2=\frac{1866.24}{13}$$
$$y=\pm\sqrt{\frac{1866.24}{13}}\approx\pm 11.98$$

Since the length cannot be negative, the length of the Surface Pro is 11.98 inches and the width is $\frac{2}{3}\sqrt{\frac{1866.24}{13}}\approx 7.99$ inches. Thus, the area of the 14.4-inch 3:2 format Surface Pro is $\sqrt{\frac{1866.24}{13}}\cdot\frac{2}{3}\sqrt{\frac{1866.24}{13}}$ ≈ 95.7 square inches.

The Surface Pro format has the larger screen since its area is larger.

110. Let x = the length of a 8.3-inch iPad Mini in a 4:3 format.

Then $\frac{3}{4}x$ = the width of the iPad. The diagonal of the 8.3-inch iPad is 8.3 inches, so by the Pythagorean theorem we have:

$$x^2+\left(\frac{3}{4}x\right)^2=8.3^2$$
$$x^2+\frac{9}{16}x^2=68.89$$
$$16\left(x^2+\frac{9}{16}x^2\right)=16(68.89)$$
$$16x^2+9x^2=1102.24$$
$$25x^2=1102.24$$
$$x^2=44.0896$$
$$x=\pm\sqrt{44.0896}=\pm 6.64$$

Since the length cannot be negative, the length of the iPad is 6.64 inches and the width is $\frac{3}{4}(6.32)=4.98$ inches. Thus, the area of the iPad is $(6.64)(4.98)=33.1$ square inches.

Let y = the length of a 8-inch 16:10 format Amazon Fire HD 8™. Then $\frac{10}{16}y$ = the width of the Fire. The diagonal of a 8-inch Fire is 8 inches, so by the Pythagorean theorem we have:

$$y^2+\left(\frac{10}{16}y\right)^2=8^2$$
$$y^2+\frac{100}{256}y^2=64$$
$$256\left(y^2+\frac{100}{256}y^2\right)=256(64)$$
$$256y^2+100y^2=16384$$
$$356y^2=16384$$
$$y^2=\frac{16384}{356}$$
$$y=\pm\sqrt{\frac{16384}{356}}\approx\pm 6.78399$$

Since the length cannot be negative, the length of the Fire is $\sqrt{\frac{16384}{356}}\approx 6.78399$ inches and the width is $\frac{10}{16}\sqrt{\frac{16384}{356}}\approx 4.240$ inches. Thus, the area of the Amazon Fire is $(6.78399)(4.240)\approx 28.8$ square inches.

The iPad Mini™ 4:3 format has the larger screen since its area is larger.

111. Let h be 1.1. Then

$$1.1=-0.00025x^2+0.04x$$
$$0=-0.00025x^2+0.04x-1.1$$
$$x=\frac{-0.04\pm\sqrt{(0.04)^2-4(-0.00025)(-1.1)}}{2(-0.00025)}$$
$$=35.3\text{ ft or }124.7\text{ ft}$$

124.7 ft does not make sense in the context of the problem, so the answer is 35.3 ft.

112. Since d is expressed in 1000's we will set d = 12 and solve for x using the Quadratic Formula.

$$d=0.33x^2+0.26x+2.29$$
$$12=0.33x^2+0.26x+2.29$$
$$0=0.33x^2+0.26x-9.71$$
$$x=\frac{-0.26\pm\sqrt{(0.26)^2-4(0.33)(-9.71)}}{2(0.33)}$$
$$=\frac{-0.26\pm\sqrt{13.0772}}{0.66}$$
$$x\approx 5.085\quad\text{or}\quad x\approx -5.873$$

So the nearest year when the earnings were 12 occurred about 5 years after 2018 or 2023. The negative value -5.873 has no meaning.

113. We will set g = 2.97 and solve for h using the Quadratic Formula.

$$g = -0.0006x^2 + 0.015x + 3.04$$
$$2.97 = -0.0006x^2 + 0.015x + 3.04$$
$$0 = -0.0006x^2 + 0.015x + 0.07$$
$$x = \frac{-0.015 \pm \sqrt{(0.015)^2 - 4(-0.0006)(0.07)}}{2(-0.0006)}$$
$$= \frac{-0.015 \pm \sqrt{0.000393}}{-0.0012}$$
$$x \approx 29 \text{ or } x \approx -4.02$$

So the estimated numbers of hours worked by a student with a GPA of 2.97 is 29 hours. The value -4.02 has no meaning since it is negative.

114. Let x be the numbers of members in the fraternity and s be the share paid by each member. Then $s = \frac{1470}{x}$. If there are 7 members who cannot contribute then the share goes up by $5. So we have the following equation:

$$s + 5 = \frac{1470}{x-7} \text{ or } (s+5)(x-7) = 1470$$

Solving these two equations together:

$$(s+5)(x-7) = 1470 \text{ and } s = \frac{1470}{x}$$
$$\left(\frac{1470}{x} + 5\right)(x-7) = 1470$$
$$1470 - \frac{10290}{x} + 5x - 35 = 1470$$
$$5x - \frac{10290}{x} - 35 = 0$$
$$5x^2 - 35x - 10290 = 0$$

$$5x^2 - 35x - 10290 = 0$$
$$x^2 - 7x - 2058 = 0$$
$$(x+42)(x-49) = 0$$
$$x = -42 \text{ or } x = 49$$

Since x is the number of members, it must be positive so the number of members is 49.

115. Let a be the age the individual is able to start saving money. Then we need to find where the models are equal. Solving these two equations together:

$$-25a^2 + 2400a - 30700 = 160a + 7840$$
$$25a^2 - 2240a + 38540 = 0$$
$$a = \frac{2240 \pm \sqrt{(2240)^2 - 4(25)(38540)}}{2(25)}$$
$$a = \frac{2240 \pm \sqrt{1163600}}{50}$$
$$a = \frac{2240 \pm 1078.7}{50}$$
$$a = \frac{2240 + 1078.7}{50} \quad \text{or} \quad a = \frac{2240 - 1078.7}{50}$$
$$a = 66.4 \qquad \text{or} \quad a = 23.2$$

Since x is the age to start saving, it makes sense that the answer is approximate at age 23.

116.
$$\frac{1}{2}n(n-3) = 65$$
$$n(n-3) = 130$$
$$n^2 - 3n - 130 = 0$$
$$(n-13)(n+10) = 0$$
$$n = 13 \quad \text{or} \quad n = -10$$

Since the number of sides cannot be negative, we discard the negative value. A polygon with 65 diagonals will have 13 sides.

$$\frac{1}{2}n(n-3) = 80$$
$$n(n-3) = 160$$
$$n^2 - 3n - 160 = 0$$
$$a = 1, b = -3, c = -160$$
$$n = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-160)}}{2(1)} = \frac{3 \pm \sqrt{646}}{2}$$

Neither solution is an integer, so there is no polygon that has 80 diagonals.

117. The roots of a quadratic equation are

$$x_1 = \frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$\begin{aligned} x_1 + x_2 &= \frac{-b-\sqrt{b^2-4ac}}{2a} + \frac{-b+\sqrt{b^2-4ac}}{2a} \\ &= \frac{-b-\sqrt{b^2-4ac}-b+\sqrt{b^2-4ac}}{2a} \\ &= \frac{-2b}{2a} \\ &= -\frac{b}{a} \end{aligned}$$

118. The roots of a quadratic equation are

$$x_1 = \frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$\begin{aligned} x_1 \cdot x_2 &= \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right) \\ &= \frac{(-b)^2-\left(\sqrt{b^2-4ac}\right)^2}{(2a)^2} = \frac{b^2-b^2+4ac}{4a^2} \\ &= \frac{4ac}{4a^2} \\ &= \frac{c}{a} \end{aligned}$$

119. In order to have one repeated solution, we need the discriminant to be 0.

$$\begin{aligned} b^2-4ac &= 0 \\ 1^2-4(k)(k) &= 0 \\ 1-4k^2 &= 0 \\ 4k^2 &= 1 \\ k^2 &= \frac{1}{4} \\ k &= \pm\sqrt{\frac{1}{4}} \\ k = \frac{1}{2} \quad &\text{or} \quad k = -\frac{1}{2} \end{aligned}$$

120. In order to have one repeated solution, we need the discriminant to be 0.

$$\begin{aligned} b^2-4ac &= 0 \\ (-k)^2-4(1)(4) &= 0 \\ k^2-16 &= 0 \\ (k-4)(k+4) &= 0 \\ k = 4 \quad &\text{or} \quad k = -4 \end{aligned}$$

121. For $ax^2+bx+c=0$:

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

For $ax^2-bx+c=0$:

$$\begin{aligned} x &= \frac{b\pm\sqrt{(-b)^2-4ac}}{2a} \\ &= -\left(\frac{-b\pm\sqrt{b^2-4ac}}{2a}\right) \end{aligned}$$

122. For $ax^2+bx+c=0$:

$$x_1 = \frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

For $cx^2+bx+a=0$:

$$\begin{aligned} x_1^* &= \frac{-b-\sqrt{b^2-4(c)(a)}}{2c} = \frac{-b-\sqrt{b^2-4ac}}{2c} \\ &= \frac{-b-\sqrt{b^2-4ac}}{2c} \cdot \frac{-b+\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}} \\ &= \frac{b^2-\left(b^2-4ac\right)}{2c\left(-b+\sqrt{b^2-4ac}\right)} = \frac{4ac}{2c\left(-b+\sqrt{b^2-4ac}\right)} \\ &= \frac{2a}{-b+\sqrt{b^2-4ac}} \\ &= \frac{1}{x_2} \end{aligned}$$

and

$$x_2^* = \frac{-b+\sqrt{b^2-4(c)(a)}}{2c} = \frac{-b+\sqrt{b^2-4ac}}{2c}$$

$$= \frac{-b+\sqrt{b^2-4ac}}{2c} \cdot \frac{-b-\sqrt{b^2-4ac}}{-b-\sqrt{b^2-4ac}}$$

$$= \frac{b^2-\left(b^2-4ac\right)}{2c\left(-b-\sqrt{b^2-4ac}\right)} = \frac{4ac}{2c\left(-b-\sqrt{b^2-4ac}\right)}$$

$$= \frac{2a}{-b-\sqrt{b^2-4ac}}$$

$$= \frac{1}{x_1}$$

123. If x = original width and y = original length, then $xy = 1$ or $x = \frac{1}{y}$. The ratio of side lengths is $\frac{x}{y} = \frac{1}{y^2}$. Folding along the longest side results in sides of length $x = \frac{1}{y}$ and $\frac{y}{2}$ whose ratio is

$$\frac{\frac{y}{2}}{\frac{1}{y}} = \frac{y^2}{2}$$

Equating the ratios gives

$$\frac{1}{y^2} = \frac{y^2}{2}$$

$$y^4 = 2$$

$$y = \sqrt[4]{2} \text{ m}$$

So

$$x = \frac{1}{\sqrt[4]{2}} = \frac{\sqrt[4]{8}}{2} \text{ m.}$$

124. **a.** $x^2 = 9$ and $x = 3$ are not equivalent because they do not have the same solution set. In the first equation we can also have $x = -3$.

b. $x = \sqrt{9}$ and $x = 3$ are equivalent because $\sqrt{9} = 3$.

c. $(x-1)(x-2) = (x-1)^2$ and $x-2 = x-1$ are not equivalent because they do not have the same solution set. The first equation has the solution set $\{1\}$ while the second equation has no solutions.

125. Answers will vary. Methods may include the quadratic formula, completing the square, graphing, etc.

126. Answers will vary. Knowing the discriminant allows us to know how many real solutions the equation will have.

127. Answers will vary. One possibility:
Two distinct: $x^2-3x-18=0$
One repeated: $x^2-14x+49=0$
No real: $x^2+x+4=0$

128. Answers will vary.

Section 1.3

1. Integers: $\{-3, 0\}$

Rationals: $\left\{-3, 0, \frac{6}{5}\right\}$

2. True; the set of real numbers consists of all rational and irrational numbers.

3.
$$\frac{3}{2+\sqrt{3}} = \frac{3}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{3\left(2-\sqrt{3}\right)}{2^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{3\left(2-\sqrt{3}\right)}{4-3}$$

$$= 3\left(2-\sqrt{3}\right)$$

4. real; imaginary; imaginary unit

5. False; the conjugate of $2+5i$ is $2-5i$.

6. True; the set of real numbers is a subset of the set of complex numbers.

7. False; if $2-3i$ is a solution of a quadratic equation with real coefficients, then its conjugate, $2+3i$, is also a solution.

8. b

9. a

10. c

11. $(2-3i)+(6+8i)=(2+6)+(-3+8)i=8+5i$

12. $(4+5i)+(-8+2i)=(4+(-8))+(5+2)i$
$=-4+7i$

13. $(-3+2i)-(4-4i)=(-3-4)+(2-(-4))i$
$=-7+6i$

14. $(3-4i)-(-3-4i)=(3-(-3))+(-4-(-4))i$
$=6+0i=6$

15. $(2-5i)-(8+6i)=(2-8)+(-5-6)i$
$=-6-11i$

16. $(-8+4i)-(2-2i)=(-8-2)+(4-(-2))i$
$=-10+6i$

17. $3(2-6i)=6-18i$

18. $-4(2+8i)=-8-32i$

19. $-3i(7+6i)=-21i-18i^2=-21i-18(-1)$
$=18-21i$

20. $3i(-3+4i)=-9i+12i^2=-9i+12(-1)=-12-9i$

21. $(3-4i)(2+i)=6+3i-8i-4i^2$
$=6-5i-4(-1)$
$=10-5i$

22. $(5+3i)(2-i)=10-5i+6i-3i^2$
$=10+i-3(-1)$
$=13+i$

23. $(-5-i)(-5+i)=25-5i+5i-i^2$
$=25-(-1)$
$=26$

24. $(-3+i)(3+i)=-9-3i+3i+i^2$
$=-9+(-1)$
$=-10$

25. $\dfrac{10}{3-4i}=\dfrac{10}{3-4i}\cdot\dfrac{3+4i}{3+4i}=\dfrac{30+40i}{9+12i-12i-16i^2}$
$=\dfrac{30+40i}{9-16(-1)}=\dfrac{30+40i}{25}$
$=\dfrac{30}{25}+\dfrac{40}{25}i$
$=\dfrac{6}{5}+\dfrac{8}{5}i$

26. $\dfrac{13}{5-12i}=\dfrac{13}{5-12i}\cdot\dfrac{5+12i}{5+12i}$
$=\dfrac{65+156i}{25+60i-60i-144i^2}$
$=\dfrac{65+156i}{25-144(-1)}=\dfrac{65+156i}{169}$
$=\dfrac{65}{169}+\dfrac{156}{169}i$
$=\dfrac{5}{13}+\dfrac{12}{13}i$

27. $\dfrac{2+i}{i}=\dfrac{2+i}{i}\cdot\dfrac{-i}{-i}=\dfrac{-2i-i^2}{-i^2}$
$=\dfrac{-2i-(-1)}{-(-1)}=\dfrac{1-2i}{1}=1-2i$

28. $\dfrac{2-i}{-2i}=\dfrac{2-i}{-2i}\cdot\dfrac{i}{i}=\dfrac{2i-i^2}{-2i^2}$
$=\dfrac{2i-(-1)}{-2(-1)}=\dfrac{1+2i}{2}=\dfrac{1}{2}+i$

29. $\dfrac{6-i}{1+i}=\dfrac{6-i}{1+i}\cdot\dfrac{1-i}{1-i}=\dfrac{6-6i-i+i^2}{1-i+i-i^2}$
$=\dfrac{6-7i+(-1)}{1-(-1)}=\dfrac{5-7i}{2}=\dfrac{5}{2}-\dfrac{7}{2}i$

30. $\dfrac{2+3i}{1-i}=\dfrac{2+3i}{1-i}\cdot\dfrac{1+i}{1+i}=\dfrac{2+2i+3i+3i^2}{1+i-i-i^2}$
$=\dfrac{2+5i+3(-1)}{1-(-1)}=\dfrac{-1+5i}{2}=-\dfrac{1}{2}+\dfrac{5}{2}i$

31. $\left(\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)^2=\dfrac{1}{4}+2\left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}i\right)+\dfrac{3}{4}i^2$
$=\dfrac{1}{4}+\dfrac{\sqrt{3}}{2}i+\dfrac{3}{4}(-1)=-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

32. $\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^2=\frac{3}{4}-2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}i\right)+\frac{1}{4}i^2$
$=\frac{3}{4}-\frac{\sqrt{3}}{2}i+\frac{1}{4}(-1)=\frac{1}{2}-\frac{\sqrt{3}}{2}i$

33. $(1+i)^2=1+2i+i^2=1+2i+(-1)=2i$

34. $(1-i)^2=1-2i+i^2=1-2i+(-1)=-2i$

35. $i^{23}=i^{22+1}=i^{22}\cdot i=\left(i^2\right)^{11}\cdot i=(-1)^{11}i=-i$

36. $i^{14}=\left(i^2\right)^7=(-1)^7=-1$

37. $i^{-20}=\frac{1}{i^{20}}=\frac{1}{i^{20}}=\frac{1}{(i^2)^{10}}$
$=\frac{1}{(-1)^{10}}=\frac{1}{1}=1$

38. $i^{-23}=\frac{1}{i^{23}}=\frac{1}{i^{22+1}}=\frac{1}{i^{22}\cdot i}=\frac{1}{(i^2)^{11}\cdot i}$
$=\frac{1}{(-1)^{11}i}=\frac{1}{-i}=\frac{1}{-i}\cdot\frac{i}{i}=\frac{i}{-i^2}=\frac{i}{-(-1)}=i$

39. $i^6-5=\left(i^2\right)^3-5=(-1)^3-5=-1-5=-6$

40. $4+i^3=4+i^2\cdot i=4+(-1)i=4-i$

41. $6i^3-4i^5=i^3(6-4i^2)$
$=i^2\cdot i(6-4(-1))=-1\cdot i(10)=-10i$

42. $4i^3-2i^2+1=4i^2\cdot i-2i^2+1$
$=4(-1)i-2(-1)+1$
$=-4i+2+1$
$=3-4i$

43. $(1+i)^3=(1+i)(1+i)(1+i)=(1+2i+i^2)(1+i)$
$=(1+2i-1)(1+i)=2i(1+i)$
$=2i+2i^2=2i+2(-1)$
$=-2+2i$

44. $(3i)^4+1=81i^4+1=81(1)+1=82$

45. $i^7(1+i^2)=i^7(1+(-1))=i^7(0)=0$

46. $2i^4(1+i^2)=2(1)(1+(-1))=2(0)=0$

47. $i^8+i^6-i^4-i^2=\left(i^2\right)^4+\left(i^2\right)^3+\left(i^2\right)^2+i^2$
$=(-1)^4+(-1)^3+(-1)^2-1$
$=1-1+1-1$
$=0$

48. $i^7+i^5+i^3+i=\left(i^2\right)^3\cdot i+\left(i^2\right)^2\cdot i+i^2\cdot i+i$
$=(-1)^3\cdot i+(-1)^2\cdot i+(-1)\cdot i+i$
$=-i+i-i+i$
$=0$

49. $\sqrt{-4}=2i$

50. $\sqrt{-9}=3i$

51. $\sqrt{-25}=5i$

52. $\sqrt{-64}=8i$

53. $\sqrt{-12}=i\sqrt{4\cdot 3}=2\sqrt{3}i$

54. $\sqrt{-18}=i\sqrt{9\cdot 2}=3\sqrt{2}i$

55. $\sqrt{-200}=i\sqrt{100\cdot 2}=10\sqrt{2}i$

56. $\sqrt{-45}=i\sqrt{9\cdot 5}=3\sqrt{5}i$

57. $\sqrt{(3+4i)(4i-3)}=\sqrt{12i-9+16i^2-12i}$
$=\sqrt{-9+16(-1)}$
$=\sqrt{-25}$
$=5i$

58. $\sqrt{(4+3i)(3i-4)}=\sqrt{12i-16+9i^2-12i}$
$=\sqrt{-16+9(-1)}$
$=\sqrt{-25}$
$=5i$

59. $x^2+4=0$

$x^2=-4$

$x=\pm\sqrt{-4}$

$x=\pm 2i$

The solution set is $\{-2i, 2i\}$.

60. $x^2-4=0$

$(x+2)(x-2)=0$

$x=-2$ or $x=2$

The solution set is $\{-2, 2\}$.

61. $x^2-16=0$

$(x+4)(x-4)=0$

$x=-4$ or $x=4$

The solution set is $\{-4, 4\}$.

62. $x^2+25=0$

$x^2=-25$

$x=\pm\sqrt{-25}=\pm 5i$

The solution set is $\{-5i, 5i\}$.

63. $x^2-6x+13=0$

$a=1, b=-6, c=13,$

$b^2-4ac=(-6)^2-4(1)(13)=36-52=-16$

$$x=\frac{-(-6)\pm\sqrt{-16}}{2(1)}=\frac{6\pm 4i}{2}=3\pm 2i$$

The solution set is $\{3-2i, 3+2i\}$.

64. $x^2+4x+8=0$

$a=1, b=4, c=8$

$b^2-4ac=4^2-4(1)(8)=16-32=-16$

$$x=\frac{-4\pm\sqrt{-16}}{2(1)}=\frac{-4\pm 4i}{2}=-2\pm 2i$$

The solution set is $\{-2-2i, -2+2i\}$.

65. $x^2-6x+10=0$

$a=1, b=-6, c=10$

$b^2-4ac=(-6)^2-4(1)(10)=36-40=-4$

$$x=\frac{-(-6)\pm\sqrt{-4}}{2(1)}=\frac{6\pm 2i}{2}=3\pm i$$

The solution set is $\{3-i, 3+i\}$.

66. $x^2-2x+5=0$

$a=1, b=-2, c=5$

$b^2-4ac=(-2)^2-4(1)(5)=4-20=-16$

$$x=\frac{-(-2)\pm\sqrt{-16}}{2(1)}=\frac{2\pm 4i}{2}=1\pm 2i$$

The solution set is $\{1-2i, 1+2i\}$.

67. $25x^2-10x+2=0$

$a=25, b=-10, c=2$

$b^2-4ac=(-10)^2-4(25)(2)=100-200=-100$

$$x=\frac{-(-10)\pm\sqrt{-100}}{50}=\frac{10\pm 10i}{50}=\frac{1}{5}\pm\frac{1}{5}i$$

The solution set is $\left\{\frac{1}{5}-\frac{1}{5}i, \frac{1}{5}+\frac{1}{5}i\right\}$.

68. $10x^2+6x+1=0$

$a=10, b=6, c=1$

$b^2-4ac=6^2-4(10)(1)=36-40=-4$

$$x=\frac{-6\pm\sqrt{-4}}{2(10)}=\frac{-6\pm 2i}{20}=-\frac{3}{10}\pm\frac{1}{10}i$$

The solution set is $\left\{-\frac{3}{10}-\frac{1}{10}i, -\frac{3}{10}+\frac{1}{10}i\right\}$.

69. $5x^2+1=2x$

$5x^2-2x+1=0$

$a=5, b=-2, c=1$

$b^2-4ac=(-2)^2-4(5)(1)=4-20=-16$

$$x=\frac{-(-2)\pm\sqrt{-16}}{2(5)}=\frac{2\pm 4i}{10}=\frac{1}{5}\pm\frac{2}{5}i$$

The solution set is $\left\{\frac{1}{5}-\frac{2}{5}i, \frac{1}{5}+\frac{2}{5}i\right\}$.

70. $13x^2+1=6x$

$13x^2-6x+1=0$

$a=13, b=-6, c=1$

$b^2-4ac=(-6)^2-4(13)(1)=36-52=-16$

$$x=\frac{-(-6)\pm\sqrt{-16}}{2(13)}=\frac{6\pm 4i}{26}=\frac{3}{13}\pm\frac{2}{13}i$$

The solution set is $\left\{\frac{3}{13}-\frac{2}{13}i, \frac{3}{13}+\frac{2}{13}i\right\}$.

71. $x^2+x+1=0$
$a=1, b=1, c=1,$
$b^2-4ac=1^2-4(1)(1)=1-4=-3$
$x=\dfrac{-1\pm\sqrt{-3}}{2(1)}=\dfrac{-1\pm\sqrt{3}\,i}{2}=-\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i$
The solution set is $\left\{-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i,\ -\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right\}$.

72. $x^2-x+1=0$
$a=1, b=-1, c=1$
$b^2-4ac=(-1)^2-4(1)(1)=1-4=-3$
$x=\dfrac{-(-1)\pm\sqrt{-3}}{2(1)}=\dfrac{1\pm\sqrt{3}\,i}{2}=\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i$
The solution set is $\left\{\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i,\ \dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right\}$.

73. $x^3-64=0$
$(x-4)\left(x^2+4x+16\right)=0$
$x-4=0 \Rightarrow x=4$
or $x^2+4x+16=0$
$a=1, b=4, c=16$
$b^2-4ac=4^2-4(1)(16)=16-64=-48$
$x=\dfrac{-4\pm\sqrt{-48}}{2(1)}=\dfrac{-4\pm4\sqrt{3}\,i}{2}=-2\pm2\sqrt{3}i$
The solution set is $\left\{4,-2-2\sqrt{3}i,-2+2\sqrt{3}i\right\}$.

74. $x^3+27=0$
$(x+3)\left(x^2-3x+9\right)=0$
$x+3=0 \Rightarrow x=-3$
or $x^2-3x+9=0$
$a=1, b=-3, c=9$
$b^2-4ac=(-3)^2-4(1)(9)=9-36=-27$
$x=\dfrac{-(-3)\pm\sqrt{-27}}{2(1)}=\dfrac{3\pm3\sqrt{3}\,i}{2}=\dfrac{3}{2}\pm\dfrac{3\sqrt{3}}{2}i$
The solution set is $\left\{-3, \dfrac{3}{2}-\dfrac{3\sqrt{3}}{2}i, \dfrac{3}{2}+\dfrac{3\sqrt{3}}{2}i\right\}$.

75. $x^4=16$
$x^4-16=0$
$\left(x^2-4\right)\left(x^2+4\right)=0$
$(x-2)(x+2)\left(x^2+4\right)=0$
$x-2=0$ or $x+2=0$ or $x^2+4=0$
$x=2$ or $x=-2$ or $x^2=-4$
$x=2$ or $x=-2$ or $x=\pm\sqrt{-4}=\pm2i$
The solution set is $\{-2, 2, -2i, 2i\}$.

76. $x^4=1$
$x^4-1=0$
$\left(x^2-1\right)\left(x^2+1\right)=0$
$(x-1)(x+1)\left(x^2+1\right)=0$
$x-1=0$ or $x+1=0$ or $x^2+1=0$
$x=1$ or $x=-1$ or $x^2=-1$
$x=1$ or $x=-1$ or $x=\pm\sqrt{-1}=\pm i$
The solution set is $\{-1, 1, -i, i\}$.

77. $x^4+13x^2+36=0$
$\left(x^2+9\right)\left(x^2+4\right)=0$
$x^2+9=0$ or $x^2+4=0$
$x^2=-9$ or $x^2=-4$
$x=\pm\sqrt{-9}$ or $x=\pm\sqrt{-4}$
$x=\pm3i$ or $x=\pm2i$
The solution set is $\{-3i, 3i, -2i, 2i\}$.

78. $x^4+3x^2-4=0$
$\left(x^2-1\right)\left(x^2+4\right)=0$
$(x-1)(x+1)\left(x^2+4\right)=0$
$x-1=0$ or $x+1=0$ or $x^2+4=0$
$x=1$ or $x=-1$ or $x^2=-4$
$x=1$ or $x=-1$ or $x=\pm\sqrt{-4}=\pm2i$
The solution set is $\{-1, 1, -2i, 2i\}$.

79. $3x^2-3x+4=0$
$a=3, b=-3, c=4$
$b^2-4ac=(-3)^2-4(3)(4)=9-48=-39$
The equation has two complex solutions that are conjugates of each other.

80. $2x^2 - 4x + 1 = 0$
$a = 2, b = -4, c = 1$
$b^2 - 4ac = (-4)^2 - 4(2)(1) = 16 - 8 = 8$
The equation has two unequal real number solutions.

81. $2x^2 + 3x = 4$
$2x^2 + 3x - 4 = 0$
$a = 2, b = 3, c = -4$
$b^2 - 4ac = 3^2 - 4(2)(-4) = 9 + 32 = 41$
The equation has two unequal real solutions.

82. $x^2 + 6 = 2x$
$x^2 - 2x + 6 = 0$
$a = 1, b = -2, c = 6$
$b^2 - 4ac = (-2)^2 - 4(1)(6) = 4 - 24 = -20$
The equation has two complex solutions that are conjugates of each other.

83. $9x^2 - 12x + 4 = 0$
$a = 9, b = -12, c = 4$
$b^2 - 4ac = (-12)^2 - 4(9)(4) = 144 - 144 = 0$
The equation has a repeated real solution.

84. $4x^2 + 12x + 9 = 0$
$a = 4, b = 12, c = 9$
$b^2 - 4ac = 12^2 - 4(4)(9) = 144 - 144 = 0$
The equation has a repeated real solution.

85. The other solution is $\overline{2+3i} = 2 - 3i$.

86. The other solution is $\overline{4-i} = 4 + i$.

87. $z + \overline{z} = 3 - 4i + \overline{3-4i} = 3 - 4i + 3 + 4i = 6$

88.
$$\begin{aligned} w - \overline{w} &= 8 + 3i - \left(\overline{8+3i}\right) \\ &= 8 + 3i - (8 - 3i) \\ &= 8 + 3i - 8 + 3i \\ &= 0 + 6i = 6i \end{aligned}$$

89.
$$\begin{aligned} z \cdot \overline{z} &= (3-4i)(\overline{3-4i}) \\ &= (3-4i)(3+4i) \\ &= 9 + 12i - 12i - 16i^2 \\ &= 9 - 16(-1) = 25 \end{aligned}$$

90.
$$\begin{aligned} \overline{z - w} &= \overline{3 - 4i - (8 + 3i)} \\ &= \overline{3 - 4i - 8 - 3i} \\ &= \overline{-5 - 7i} \\ &= -5 + 7i \end{aligned}$$

91.
$$\begin{aligned} Z &= \frac{V}{I} = \frac{18+i}{3-4i} = \frac{18+i}{3-4i} \cdot \frac{3+4i}{3+4i} \\ &= \frac{54 + 72i + 3i + 4i^2}{9 + 12i - 12i - 16i^2} = \frac{54 + 75i - 4}{9 + 16} \\ &= \frac{50 + 75i}{25} = 2 + 3i \end{aligned}$$
The impedance is $2 + 3i$ ohms.

92.
$$\begin{aligned} \frac{1}{Z} &= \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{2+i} + \frac{1}{4-3i} = \frac{(4-3i) + (2+i)}{(2+i)(4-3i)} \\ &= \frac{6-2i}{8 - 6i + 4i - 3i^2} = \frac{6-2i}{8 - 2i + 3} = \frac{6-2i}{11-2i} \end{aligned}$$

So,
$$\begin{aligned} Z &= \frac{11-2i}{6-2i} = \frac{11-2i}{6-2i} \cdot \frac{6+2i}{6+2i} \\ &= \frac{66 + 22i - 12i - 4i^2}{36 + 12i - 12i - 4i^2} = \frac{66 + 10i + 4}{36 + 4} \\ &= \frac{70 + 10i}{40} = \frac{7}{4} + \frac{1}{4}i \end{aligned}$$

The total impedance is $\frac{7}{4} + \frac{1}{4}i$ ohms.

93.
$$\begin{aligned} z + \overline{z} &= (a + bi) + (\overline{a + bi}) \\ &= a + bi + a - bi \\ &= 2a \end{aligned}$$

$$\begin{aligned} z - \overline{z} &= a + bi - (\overline{a + bi}) \\ &= a + bi - (a - bi) \\ &= a + bi - a + bi \\ &= 2bi \end{aligned}$$

94. $\overline{\overline{z}} = \overline{\overline{a + bi}} = \overline{a - bi} = a + bi = z$

95.
$$\begin{aligned} \overline{z + w} &= \overline{(a + bi) + (c + di)} \\ &= \overline{(a + c) + (b + d)i} \\ &= (a + c) - (b + d)i \\ &= (a - bi) + (c - di) \\ &= \overline{a + bi} + \overline{c + di} \\ &= \overline{z} + \overline{w} \end{aligned}$$

96. $\overline{z \cdot w} = \overline{(a+bi)\cdot(c+di)}$
$= \overline{ac + adi + bci + bdi^2}$
$= \overline{(ac-bd)+(ad+bc)i}$
$= (ac-bd)-(ad+bc)i$

$\overline{z}\cdot\overline{w} = \overline{a+bi}\cdot\overline{c+di}$
$= (a-bi)(c-di)$
$= ac - adi - bci + bdi^2$
$= (ac-bd)-(ad+bc)i$

97.
$$(a+bi)^2 = -(a-bi)^2$$
$$a^2 + 2abi + (bi)^2 = -\left(a^2 - 2abi + (bi)^2\right)$$
$$a^2 + 2abi + b^2i^2 = -\left(a^2 - 2abi + b^2i^2\right)$$
$$a^2 + 2abi - b^2 = -a^2 + 2abi + b^2$$
$$a^2 - b^2 = -(a^2 - b^2)$$
$$2(a^2 - b^2) = 0$$
$$a^2 = b^2 \to b = \pm a$$
Any complex number of the form $a+ai$ or $a-ai$ will work.

98. Let $u = \sqrt[3]{2}$ in $x^3 + 2 = 0$ so that $x^3 + u^3 = 0$. Then, $(x+u)(x^2 - ux + u^2) = 0$. From the first factor we find $x = -u = -\sqrt[3]{2}$. From the second factor, use the quadratic formula to get
$$x = \frac{-(-u) \pm \sqrt{(-u)^2 - 4\cdot 1\cdot u^2}}{2\cdot 1}$$
$$= \frac{u \pm \sqrt{-3u^2}}{2} = \frac{u}{2} \pm \frac{u\sqrt{3}}{2}i = \frac{\sqrt[3]{2}}{2} \pm \frac{\sqrt[3]{2}\sqrt{3}}{2}i$$
The solution set is:
$$\left\{-\sqrt[3]{2}, \frac{\sqrt[3]{2}}{2} \pm \frac{\sqrt[3]{2}\sqrt{3}}{2}i\right\}$$

99. $(x+5)(y-5) = (x+y)^2$; let $u = x+5$ (so $x = u-5$ and $v = y-5$ so $y = v+5$.
Substituting gives $uv = (u+v)^2$ or $u^2 + uv + v^2 = 0$ which is quadratic in u. Using the quadratic formula gives
$$x = \frac{-v \pm \sqrt{v^2 - 4\cdot 1\cdot v^2}}{2\cdot 1} = \frac{-v \pm |v|\sqrt{-3}}{2}.$$ Since x is a real number, u must also be a real number. This is only possible if v = 0 which then makes u = 0. Therefore, $x = 0-5 = -5$ and $y = 0+5 = 5$, so $x - y = -5 - 5 = -10$

100 – 102. Answers will vary.

103. Answers will vary. A complex number is the sum or difference of two numbers (real and imaginary parts of the complex number) just as a binomial is the sum or difference of two monomial terms. We multiply two binomials by using the FOIL method, an approach we can also use to multiply two complex numbers.

104. Although the set of real numbers is a subset of the set of complex numbers, not all rules that work in the real number system can be used in the larger complex number system. The rule that allows us to write the product of two square roots as the square root of the product only works in the real number system. That is, $\sqrt{a}\cdot\sqrt{b} = \sqrt{ab}$ only when $\sqrt{a}$ and $\sqrt{b}$ are real numbers. In the complex number system we must first convert the radicals to complex form. In this case this means we need to write $\sqrt{-9}$ as $\sqrt{-1\cdot 9} = \sqrt{9}\cdot\sqrt{-1} = 3i$. Then we can multiply to get $\sqrt{-9}\cdot\sqrt{-9} = 3i\cdot 3i = 9i^2 = 9(-1) = -9$.

Section 1.4

1. True
2. $\left(\sqrt[3]{x}\right)^3 = x$
3. $6x^3 - 2x^2 = 2x^2(3x-1)$
4. False; you can also use the Quadratic Formula or completing the square.
5. quadratic in form
6. True
7. a
8. c

9. $\sqrt{2t-1}=1$

$$\left(\sqrt{2t-1}\right)^2=1^2$$
$$2t-1=1$$
$$2t=2$$
$$t=1$$

Check: $\sqrt{2(1)-1}=\sqrt{1}=1$

The solution set is $\{1\}$.

10. $\sqrt{3t+4}=2$

$$\left(\sqrt{3t+4}\right)^2=2^2$$
$$3t+4=4$$
$$3t=0$$
$$t=0$$

Check: $\sqrt{3(0)+4}=\sqrt{4}=2$

The solution set is $\{0\}$.

11. $\sqrt{3t+4}=-6$

Since the principal square root is never negative, the equation has no real solution.

12. $\sqrt{5t+3}=-2$

Since the principal square root is never negative, the equation has no real solution.

13. $\sqrt[3]{1-2x}-3=0$

$$\sqrt[3]{1-2x}=3$$
$$\left(\sqrt[3]{1-2x}\right)^3=3^3$$
$$1-2x=27$$
$$-2x=26$$
$$x=-13$$

Check: $\sqrt[3]{1-2(-13)}-3=\sqrt[3]{27}-3=0$

The solution set is $\{-13\}$.

14. $\sqrt[3]{1-2x}-1=0$

$$\sqrt[3]{1-2x}=1$$
$$\left(\sqrt[3]{1-2x}\right)^3=1^3$$
$$1-2x=1$$
$$-2x=0$$
$$x=0$$

Check: $\sqrt[3]{1-2(0)}-1=\sqrt[3]{1}-1=0$

The solution set is $\{0\}$.

15. $\sqrt[5]{x^2+2x}=-1$

$$\left(\sqrt[5]{x^2+2x}\right)^5=(-1)^5$$
$$x^2+2x=-1$$
$$x^2+2x+1=0$$
$$(x+1)^2=0$$
$$x+1=0$$
$$x=-1$$

Check: $\sqrt[5]{(-1)^2+2(-1)}=\sqrt[5]{1-2}=\sqrt[5]{-1}=-1$

The solution set is $\{-1\}$.

16. $\sqrt[4]{x^2+16}=\sqrt{5}$

$$\left(\sqrt[4]{x^2+16}\right)^4=\left(\sqrt{5}\right)^4$$
$$x^2+16=25$$
$$x^2=9$$
$$x=\pm 3$$

Check -3: $\sqrt[4]{(-3)^2+16}=\sqrt[4]{9+16}=\sqrt[4]{25}=\sqrt{5}$

Check 3: $\sqrt[4]{(3)^2+16}=\sqrt[4]{9+16}=\sqrt[4]{25}=\sqrt{5}$

The solution set is $\{-3,3\}$.

17. $x=8\sqrt{x}$

$$(x)^2=\left(8\sqrt{x}\right)^2$$
$$x^2=64x$$
$$x^2-64x=0$$
$$x(x-64)=0$$
$$x=0 \quad\text{or}\quad x=64$$

Check 0: $0=8\sqrt{0}$, $0=0$

Check 64: $64=8\sqrt{64}$, $64=64$

The solution set is $\{0,64\}$.

18. $x=3\sqrt{x}$

$$(x)^2=\left(3\sqrt{x}\right)^2$$
$$x^2=9x$$
$$x^2-9x=0$$
$$x(x-9)=0$$
$$x=0 \quad\text{or}\quad x=9$$

Check 0: $0 = 3\sqrt{0}$ $0 = 0$ Check 9: $9 = 3\sqrt{9}$ $9 = 9$

The solution set is $\{0, 9\}$.

19. $\sqrt{15-2x} = x$

$$\left(\sqrt{15-2x}\right)^2 = x^2$$
$$15 - 2x = x^2$$
$$x^2 + 2x - 15 = 0$$
$$(x+5)(x-3) = 0$$
$$x = -5 \text{ or } x = 3$$

Check –5: $\sqrt{15-2(-5)} = \sqrt{25} = 5 \neq -5$

Check 3: $\sqrt{15-2(3)} = \sqrt{9} = 3 = 3$

Disregard $x = -5$ as extraneous.

The solution set is $\{3\}$.

20. $\sqrt{12-x} = x$

$$\left(\sqrt{12-x}\right)^2 = x^2$$
$$12 - x = x^2$$
$$x^2 + x - 12 = 0$$
$$(x+4)(x-3) = 0$$
$$x = -4 \text{ or } x = 3$$

Check –4: $\sqrt{12-(-4)} = \sqrt{16} = 4 \neq -4$

Check 3: $\sqrt{12-3} = \sqrt{9} = 3 = 3$

Disregard $x = -4$ as extraneous.

The solution set is $\{3\}$.

21. $x = 2\sqrt{x-1}$

$$x^2 = \left(2\sqrt{x-1}\right)^2$$
$$x^2 = 4(x-1)$$
$$x^2 = 4x - 4$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)^2 = 0$$
$$x = 2$$

Check: $2 = 2\sqrt{2-1}$

$2 = 2$

The solution set is $\{2\}$.

22. $x = 2\sqrt{-x-1}$

$$x^2 = \left(2\sqrt{-x-1}\right)^2$$
$$x^2 = 4(-x-1)$$
$$x^2 = -4x - 4$$
$$x^2 + 4x + 4 = 0$$
$$(x+2)^2 = 0$$
$$x = -2$$

Check: $-2 = 2\sqrt{-(-2)-1}$

$-2 \neq 2$

The equation has no real solution.

23. $\sqrt{x^2 - x - 4} = x + 2$

$$\left(\sqrt{x^2-x-4}\right)^2 = (x+2)^2$$
$$x^2 - x - 4 = x^2 + 4x + 4$$
$$-8 = 5x$$
$$-\frac{8}{5} = x$$

Check: $\sqrt{\left(-\frac{8}{5}\right)^2 - \left(-\frac{8}{5}\right) - 4} = \left(-\frac{8}{5}\right) + 2$

$$\sqrt{\frac{64}{25} + \frac{8}{5} - 4} = \frac{2}{5}$$
$$\sqrt{\frac{4}{25}} = \frac{2}{5}$$
$$\frac{2}{5} = \frac{2}{5}$$

The solution set is $\left\{-\frac{8}{5}\right\}$.

24. $\sqrt{3 - x + x^2} = x - 2$

$$\left(\sqrt{3-x+x^2}\right)^2 = (x-2)^2$$
$$3 - x + x^2 = x^2 - 4x + 4$$
$$3x = 1$$
$$x = \frac{1}{3}$$

Check: $\sqrt{3 - \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2} = \left(\frac{1}{3}\right) - 2$

$$\sqrt{3 - \frac{1}{3} + \frac{1}{9}} = -\frac{5}{3}$$

Since the principal square root is always a non-negative number; $x = \frac{1}{3}$ does not check.

Therefore this equation has no real solution.

25. $3+\sqrt{3x+1}=x$

$$\sqrt{3x+1}=x-3$$
$$\left(\sqrt{3x+1}\right)^2=(x-3)^2$$
$$3x+1=x^2-6x+9$$
$$0=x^2-9x+8$$
$$0=(x-1)(x-8)$$
$$x=1 \text{ or } x=8$$

Check 1: $3+\sqrt{3(1)+1}=3+\sqrt{4}=5\neq 1$

Check 8: $3+\sqrt{3(8)+1}=3+\sqrt{25}=8=8$

Discard $x=1$ as extraneous.
The solution set is $\{8\}$.

26. $2+\sqrt{12-2x}=x$

$$\sqrt{12-2x}=x-2$$
$$\left(\sqrt{12-2x}\right)^2=(x-2)^2$$
$$12-2x=x^2-4x+4$$
$$0=x^2-2x-8$$
$$(x+2)(x-4)=0$$
$$x=-2 \text{ or } x=4$$

Check -2: $2+\sqrt{12-2(-2)}=2+\sqrt{16}=6\neq -2$

Check 4: $2+\sqrt{12-2(4)}=2+\sqrt{4}=4=4$

Discard $x=-2$ as extraneous.
The solution set is $\{4\}$.

27. $\sqrt{3(x+10)}-4=x$

$$\sqrt{3(x+10)}=x+4$$
$$\left(\sqrt{3(x+10)}\right)^2=(x+4)^2$$
$$3x+30=x^2+8x+16$$
$$0=x^2+5x-14$$
$$0=(x+7)(x-2)$$
$$x=-7 \text{ or } x=2$$

Check -7: $\sqrt{3(-7+10)}-4=\sqrt{9}-4=-1\neq -7$

Check 2: $\sqrt{3(2+10)}-4=\sqrt{36}-4=2=2$

Discard $x=-7$ as extraneous.
The solution set is $\{2\}$.

28. $\sqrt{1-x}-3=x+2$

$$\sqrt{1-x}=x+5$$
$$\left(\sqrt{1-x}\right)^2=(x+5)^2$$
$$1-x=x^2+10x+25$$
$$0=x^2+11x+24$$
$$0=(x+3)(x+8)$$
$$x=-3 \text{ or } x=-8$$

Check -3: $\sqrt{1-(-3)}-3=-3+2\to -1=-1$

Check -8: $\sqrt{1-(-8)}-3=-8+2\to 0=-6$

Discard $x=-8$ as extraneous.
The solution set is {-3}.

29. $\sqrt{3x-5}-\sqrt{x+7}=2$

$$\sqrt{3x-5}=2+\sqrt{x+7}$$
$$\left(\sqrt{3x-5}\right)^2=\left(2+\sqrt{x+7}\right)^2$$
$$3x-5=4+4\sqrt{x+7}+x+7$$
$$2x-16=4\sqrt{x+7}$$
$$(2x-16)^2=\left(4\sqrt{x+7}\right)^2$$
$$4x^2-64x+256=16(x+7)$$
$$4x^2-64x+256=16x+112$$
$$4x^2-80x+144=0$$
$$x^2-20x+36=0$$
$$(x-2)(x-18)=0$$
$$x=2 \text{ or } x=18$$

Check 2: $\sqrt{3(2)-5}-\sqrt{2+7}$
$=\sqrt{1}-\sqrt{9}=1-3=-2\neq 2$

Check 18: $\sqrt{3(18)-5}-\sqrt{18+7}$
$=\sqrt{49}-\sqrt{25}=7-5=2=2$

Discard $x=2$ as extraneous.
The solution set is $\{18\}$.

30. $\sqrt{3x+7}+\sqrt{x+2}=1$

$$\sqrt{3x+7}=1-\sqrt{x+2}$$
$$\left(\sqrt{3x+7}\right)^2=\left(1-\sqrt{x+2}\right)^2$$
$$3x+7=1-2\sqrt{x+2}+x+2$$
$$2x+4=-2\sqrt{x+2}$$
$$-x-2=\sqrt{x+2}$$
$$(-x-2)^2=\left(\sqrt{x+2}\right)^2$$
$$x^2+4x+4=x+2$$
$$x^2+3x+2=0$$
$$(x+1)(x+2)=0$$
$$x=-1 \text{ or } x=-2$$

Check -1: $\sqrt{3(-1)+7}+\sqrt{-1+2}$
$=\sqrt{4}+\sqrt{1}=2+1=3\neq 1$

Check -2: $\sqrt{3(-2)+7}+\sqrt{-2+2}$
$=\sqrt{1}+\sqrt{0}=1+0=1=1$

Discard $x=-1$ as extraneous.
The solution set is $\{-2\}$.

31. $\sqrt{3x+1}-\sqrt{x-1}=2$

$$\sqrt{3x+1}=2+\sqrt{x-1}$$
$$\left(\sqrt{3x+1}\right)^2=\left(2+\sqrt{x-1}\right)^2$$
$$3x+1=4+4\sqrt{x-1}+x-1$$
$$2x-2=4\sqrt{x-1}$$
$$(2x-2)^2=\left(4\sqrt{x-1}\right)^2$$
$$4x^2-8x+4=16(x-1)$$
$$x^2-2x+1=4x-4$$
$$x^2-6x+5=0$$
$$(x-1)(x-5)=0$$
$$x=1 \text{ or } x=5$$

Check 1: $\sqrt{3(1)+1}-\sqrt{1-1}$
$=\sqrt{4}-\sqrt{0}=2-0=2=2$

Check 5: $\sqrt{3(5)+1}-\sqrt{5-1}$
$=\sqrt{16}-\sqrt{4}=4-2=2=2$

The solution set is $\{1,5\}$.

32. $\sqrt{2x+3}-\sqrt{x+1}=1$

$$\sqrt{2x+3}=1+\sqrt{x+1}$$
$$\left(\sqrt{2x+3}\right)^2=\left(1+\sqrt{x+1}\right)^2$$
$$2x+3=1+2\sqrt{x+1}+x+1$$
$$x+1=2\sqrt{x+1}$$
$$(x+1)^2=\left(2\sqrt{x+1}\right)^2$$
$$x^2+2x+1=4(x+1)$$
$$x^2+2x+1=4x+4$$
$$x^2-2x-3=0$$
$$(x+1)(x-3)=0$$
$$x=-1 \text{ or } x=3$$

Check -1: $\sqrt{2(-1)+3}-\sqrt{-1+1}$
$=\sqrt{1}-\sqrt{0}=1-0=1=1$

Check 3: $\sqrt{2(3)+3}-\sqrt{3+1}$
$=\sqrt{9}-\sqrt{4}=3-2=1=1$

The solution set is $\{-1,3\}$.

33. $\sqrt{3-2\sqrt{x}}=\sqrt{x}$

$$\left(\sqrt{3-2\sqrt{x}}\right)^2=\left(\sqrt{x}\right)^2$$
$$3-2\sqrt{x}=x$$
$$-2\sqrt{x}=x-3$$
$$\left(-2\sqrt{x}\right)^2=(x-3)^2$$
$$4x=x^2-6x+9$$
$$0=x^2-10x+9$$
$$0=(x-1)(x-9)$$
$$x=1 \quad\text{or}\quad x=9$$

Check 1:	Check 9:
$\sqrt{3-2\sqrt{1}}=\sqrt{1}$	$\sqrt{3-2\sqrt{9}}=\sqrt{9}$
$\sqrt{3-2}=1$	$\sqrt{3-2\cdot 3}=3$
$\sqrt{1}=1$	$\sqrt{-3}\neq 3$
$1=1$	

Discard $x=9$ as extraneous.
The solution set is $\{1\}$.

34. $\sqrt{10+3\sqrt{x}}=\sqrt{x}$

$$\left(\sqrt{10+3\sqrt{x}}\right)^2=\left(\sqrt{x}\right)^2$$
$$10+3\sqrt{x}=x$$
$$3\sqrt{x}=x-10$$
$$\left(3\sqrt{x}\right)^2=(x-10)^2$$
$$9x=x^2-20x+100$$
$$0=x^2-29x+100$$
$$0=(x-4)(x-25)$$
$$x=4 \quad\text{or}\quad x=25$$

Check 4:
$$\sqrt{10+3\sqrt{4}}=\sqrt{4}$$
$$\sqrt{10+3\cdot 2}=2$$
$$\sqrt{16}=2$$
$$4\neq 2$$

Check 25:
$$\sqrt{10+3\sqrt{25}}=\sqrt{25}$$
$$\sqrt{10+3\cdot 5}=5$$
$$\sqrt{25}=5$$
$$5=5$$

Discard $x=4$ as extraneous.
The solution set is $\{25\}$.

35. $(3x+1)^{1/2}=4$

$$\left((3x+1)^{1/2}\right)^2=(4)^2$$
$$3x+1=16$$
$$3x=15$$
$$x=5$$

Check: $(3(5)+1)^{1/2}=16^{1/2}=4$

The solution set is $\{5\}$.

36. $(3x-5)^{1/2}=2$

$$\left((3x-5)^{1/2}\right)^2=(2)^2$$
$$3x-5=4$$
$$3x=9$$
$$x=3$$

Check: $(3(3)-5)^{1/2}=4^{1/2}=2$

The solution set is $\{3\}$.

37. $(5x-2)^{1/3}=2$

$$\left((5x-2)^{1/3}\right)^3=(2)^3$$
$$5x-2=8$$
$$5x=10$$
$$x=2$$

Check: $(5(2)-2)^{1/3}=8^{1/3}=2$

The solution set is $\{2\}$.

38. $(2x+1)^{1/3}=-1$

$$\left((2x+1)^{1/3}\right)^3=(-1)^3$$
$$2x+1=-1$$
$$2x=-2$$
$$x=-1$$

Check: $(2(-1)+1)^{1/3}=(-1)^{1/3}=-1$

The solution set is $\{-1\}$.

39. $\left(x^2+9\right)^{1/2}=5$

$$\left(\left(x^2+9\right)^{1/2}\right)^2=(5)^2$$
$$x^2+9=25$$
$$x^2=16$$
$$x=\pm\sqrt{16}=\pm 4$$

Check -4: $\left((-4)^2+9\right)^{1/2}=25^{1/2}=5$

Check 4: $\left((4)^2+9\right)^{1/2}=25^{1/2}=5$

The solution set is $\{-4,4\}$.

40. $\left(x^2-16\right)^{1/2}=9$

$$\left(\left(x^2-16\right)^{1/2}\right)^2=(9)^2$$
$$x^2-16=81$$
$$x^2=97$$
$$x=\pm\sqrt{97}$$

Check $-\sqrt{97}$: $\left(\left(-\sqrt{97}\right)^2-16\right)^{1/2}=81^{1/2}=9$

Check $\sqrt{97}$: $\left(\left(\sqrt{97}\right)^2-16\right)^{1/2}=81^{1/2}=9$

The solution set is $\left\{-\sqrt{97},\sqrt{97}\right\}$.

41. $x^{3/2} - 3x^{1/2} = 0$

$x^{1/2}(x-3) = 0$

$x^{1/2} = 0$ or $x - 3 = 0$

$x = 0$ or $x = 3$

Check 0: $0^{3/2} - 3 \cdot 0^{1/2} = 0 - 0 = 0$

Check 3: $3^{3/2} - 3 \cdot 3^{1/2} = 3\sqrt{3} - 3\sqrt{3} = 0$

The solution set is $\{0, 3\}$.

42. $x^{3/4} - 9x^{1/4} = 0$

$x^{1/4}(x^{1/2} - 9) = 0$

$x^{1/4} = 0$ or $x^{1/2} = 9$

$x = 0$ $\quad x = 81$

Check 0: $0^{3/4} - 9 \cdot 0^{1/4} = 0 - 0 = 0$

Check 81: $81^{3/4} - 9 \cdot 81^{1/4} = 27 - 27 = 0$

The solution set is $\{0, 81\}$.

43. $x^4 - 5x^2 + 4 = 0$

$(x^2 - 4)(x^2 - 1) = 0$

$x^2 - 4 = 0$ or $x^2 - 1 = 0$

$x = \pm 2$ or $x = \pm 1$

The solution set is $\{-2, -1, 1, 2\}$.

44. $x^4 - 10x^2 + 25 = 0$

$(x^2 - 5)(x^2 - 5) = 0$

$x^2 - 5 = 0$

$x = \pm\sqrt{5}$

The solution set is $\{-\sqrt{5}, \sqrt{5}\}$.

45. $6x^4 - 5x^2 - 1 = 0$

$(6x^2 + 1)(x^2 - 1) = 0$

$6x^2 + 1 = 0$ or $x^2 - 1 = 0$

$6x^2 = -1$ or $x^2 = 1$

Not real or $x = \pm 1$

The solution set is $\{-1, 1\}$.

46. $2x^4 - 5x^2 - 12 = 0$

$(2x^2 + 3)(x^2 - 4) = 0$

$2x^2 + 3 = 0$ or $x^2 - 4 = 0$

$2x^2 = -3$ or $x^2 = 4$

Not real or $x = \pm 2$

The solution set is $\{-2, 2\}$.

47. $x^6 + 7x^3 - 8 = 0$

$(x^3 + 8)(x^3 - 1) = 0$

$x^3 + 8 = 0$ or $x^3 - 1 = 0$

$x^3 = -8$ or $x^3 = 1$

$x = -2$ or $x = 1$

The solution set is $\{-2, 1\}$.

48. $x^6 - 7x^3 - 8 = 0$

$(x^3 - 8)(x^3 + 1) = 0$

$x^3 - 8 = 0$ or $x^3 + 1 = 0$

$x^3 = 8$ or $x^3 = -1$

$x = 2$ or $x = -1$

The solution set is $\{-1, 2\}$.

49. $(x+2)^2 + 7(x+2) + 12 = 0$

Let $u = x + 2$, so that $u^2 = (x+2)^2$.

$u^2 + 7u + 12 = 0$

$(u+3)(u+4) = 0$

$u + 3 = 0$ or $u + 4 = 0$

$u = -3$ or $u = -4$

$x + 2 = -3$ or $x + 2 = -4$

$x = -5$ or $x = -6$

The solution set is $\{-6, -5\}$.

50. $(2x+5)^2 - (2x+5) - 6 = 0$

Let $u = 2x + 5$ so that $u^2 = (2x+5)^2$.

$u^2 - u - 6 = 0$

$(u-3)(u+2) = 0$

$u - 3 = 0$ or $u + 2 = 0$

$u = 3$ or $u = -2$

$2x + 5 = 3$ or $2x + 5 = -2$

$x = -1$ or $x = -\frac{7}{2}$

The solution set is $\left\{-\frac{7}{2}, -1\right\}$.

51. $(4x-9)^2 - 10(4x-9) + 25 = 0$

Let $u = 4x - 9$ so that $u^2 = (4x-9)^2$.

$$u^2-10u+25=0$$
$$(u-5)^2=0$$
$$u-5=0$$
$$u=5$$
$$4x-9=5$$
$$4x=14$$
$$x=\frac{7}{2}$$

The solution set is $\left\{\frac{7}{2}\right\}$.

52. $(2-x)^2+(2-x)-20=0$

Let $u=2-x$ so that $u^2=(2-x)^2$.

$$u^2+u-20=0$$
$$(u+5)(u-4)=0$$
$$u+5=0 \quad \text{or} \quad u-4=0$$
$$u=-5 \quad \text{or} \quad u=4$$
$$2-x=-5 \quad \text{or} \quad 2-x=4$$
$$x=7 \quad \text{or} \quad x=-2$$

The solution set is $\{-2,7\}$.

53. $2(s+1)^2-5(s+1)=3$

Let $u=s+1$ so that $u^2=(s+1)^2$.

$$2u^2-5u=3$$
$$2u^2-5u-3=0$$
$$(2u+1)(u-3)=0$$
$$2u+1=0 \quad \text{or} \quad u-3=0$$
$$u=-\frac{1}{2} \quad \text{or} \quad u=3$$
$$s+1=-\frac{1}{2} \quad \text{or} \quad s+1=3$$
$$s=-\frac{3}{2} \quad \text{or} \quad s=2$$

The solution set is $\left\{-\frac{3}{2},2\right\}$.

54. $3(1-y)^2+5(1-y)+2=0$

Let $u=1-y$ so that $u^2=(1-y)^2$.

$$3u^2+5u+2=0$$
$$(3u+2)(u+1)=0$$
$$3u+2=0 \quad \text{or} \quad u+1=0$$
$$u=-\frac{2}{3} \quad \text{or} \quad u=-1$$
$$1-y=-\frac{2}{3} \quad \text{or} \quad 1-y=-1$$
$$y=\frac{5}{3} \quad \text{or} \quad y=2$$

The solution set is $\left\{\frac{5}{3},2\right\}$.

55. $x-4x\sqrt{x}=0$

$$x\left(1-4\sqrt{x}\right)=0$$
$$x=0 \quad \text{or} \quad 1-4\sqrt{x}=0$$
$$1=4\sqrt{x}$$
$$\tfrac{1}{4}=\sqrt{x}$$
$$\left(\tfrac{1}{4}\right)^2=\left(\sqrt{x}\right)^2$$
$$\tfrac{1}{16}=x$$

Check:

$x=0$: $0-4(0)\sqrt{0}=0$

$0=0$

$x=\frac{1}{16}$: $\left(\frac{1}{16}\right)-4\left(\frac{1}{16}\right)\sqrt{\frac{1}{16}}=0$

$\frac{1}{16}-4\left(\frac{1}{16}\right)\left(\frac{1}{4}\right)=0$

$\frac{1}{16}-\frac{1}{16}=0$

$0=0$

The solution set is $\left\{0,\frac{1}{16}\right\}$.

56. $x+8\sqrt{x}=0$

$$8\sqrt{x}=-x$$
$$\left(8\sqrt{x}\right)^2=(-x)^2$$
$$64x=x^2$$
$$0=x^2-64x$$
$$0=x(x-64)$$
$$x=0 \quad \text{or} \quad x=64$$

Check: $x=0$: $0+8\sqrt{0}=0$

$0=0$

$x=64$: $64+8\sqrt{64}=0$

$64+64\neq 0$

The solution set is $\{0\}$.

57. $x+\sqrt{x}=20$

Let $u=\sqrt{x}$ so that $u^2=x$.

$$u^2+u=20$$
$$u^2+u-20=0$$
$$(u+5)(u-4)=0$$
$$u+5=0 \quad \text{or} \quad u-4=0$$
$$u=-5 \quad \text{or} \quad u=4$$
$$\sqrt{x}=-5 \quad \text{or} \quad \sqrt{x}=4$$
$$\text{not possible} \quad \text{or} \quad x=16$$

Check: $16+\sqrt{16}=20$

$16+4=20$

The solution set is $\{16\}$.

58. $x+\sqrt{x}=6$

Let $u=\sqrt{x}$ so that $u^2=x$.

$$u^2+u=6$$
$$u^2+u-6=0$$
$$(u+3)(u-2)=0$$
$$u+3=0 \quad \text{or} \quad u-2=0$$
$$u=-3 \quad \text{or} \quad u=2$$
$$\sqrt{x}=-3 \quad \text{or} \quad \sqrt{x}=2$$
$$\text{not possible} \quad \text{or} \quad x=4$$

Check: $4+\sqrt{4}=6$

$4+2=6$

The solution set is $\{4\}$.

59. $t^{1/2}-2t^{1/4}+1=0$

Let $u=t^{1/4}$ so that $u^2=t^{1/2}$.

$$u^2-2u+1=0$$
$$(u-1)^2=0$$
$$u-1=0$$
$$u=1$$
$$t^{1/4}=1$$
$$t=1$$

Check: $1^{1/2}-2(1)^{1/4}+1=0$

$1-2+1=0$

$0=0$

The solution set is $\{1\}$.

60. $z^{1/2}-4t^{1/4}+4=0$

Let $u=z^{1/4}$ so that $u^2=z^{1/2}$.

$$u^2-4u+4=0$$
$$(u-2)^2=0$$
$$u-2=0$$
$$u=2$$
$$z^{1/4}=2$$
$$z=16$$

Check: $16^{1/2}-4(16)^{1/4}+4=0$

$4-8+4=0$

$0=0$

The solution set is $\{16\}$.

61. $x^{1/2}-3x^{1/4}+2=0$

Let $u=x^{1/4}$ so that $u^2=x^{1/2}$.

$$u^2-3u+2=0$$
$$(u-2)(u-1)=0$$
$$u=2 \quad \text{or} \quad u=1$$
$$x^{1/4}=2 \quad \text{or} \quad x^{1/4}=1$$
$$x=16 \quad \text{or} \quad x=1$$

Check:

$x=16:\ 16^{1/2}-3(16)^{1/4}+2=0$

$4-6+2=0$

$0=0$

$x=1:\ 1^{1/2}-3(1)^{1/4}+2=0$

$1-3+2=0$

$0=0$

The solution set is $\{1, 16\}$.

62. $4x^{1/2}-9x^{1/4}+4=0$

Let $u=x^{1/4}$ so that $u^2=x^{1/2}$.

$$4u^2-9u+4=0$$
$$u=\frac{-(-9)\pm\sqrt{(-9)^2-4(4)(4)}}{2(4)}=\frac{9\pm\sqrt{17}}{8}$$
$$x^{1/4}=\frac{9\pm\sqrt{17}}{8}$$
$$x=\left(\frac{9\pm\sqrt{17}}{8}\right)^4$$

Check $x = \left(\frac{9+\sqrt{17}}{8}\right)^4$:

$$4\left(\left(\frac{9+\sqrt{17}}{8}\right)^4\right)^{1/2} - 9\left(\left(\frac{9+\sqrt{17}}{8}\right)^4\right)^{1/4} + 4 = 0$$

$$4\left(\frac{9+\sqrt{17}}{8}\right)^2 - 9\left(\frac{9+\sqrt{17}}{8}\right) + 4 = 0$$

$$4\frac{\left(9+\sqrt{17}\right)^2}{64} - 9\left(\frac{9+\sqrt{17}}{8}\right) + 4 = 0$$

$$64\left(4\frac{\left(9+\sqrt{17}\right)^2}{64} - 9\left(\frac{9+\sqrt{17}}{8}\right) + 4\right) = (0)(64)$$

$$4\left(9+\sqrt{17}\right)^2 - 72\left(9+\sqrt{17}\right) + 256 = 0$$

$$4\left(81+18\sqrt{17}+17\right) - 72\left(9+\sqrt{17}\right) + 256 = 0$$

$$324 + 72\sqrt{17} + 68 - 648 - 72\sqrt{17} + 256 = 0$$

$$0 = 0$$

Check $x = \left(\frac{9-\sqrt{17}}{8}\right)^4$:

$$4\left(\left(\frac{9-\sqrt{17}}{8}\right)^4\right)^{1/2} - 9\left(\left(\frac{9-\sqrt{17}}{8}\right)^4\right)^{1/4} + 4 = 0$$

$$4\left(\frac{9-\sqrt{17}}{8}\right)^2 - 9\left(\frac{9-\sqrt{17}}{8}\right) + 4 = 0$$

$$4\left(81-18\sqrt{17}+17\right) - 72\left(9-\sqrt{17}\right) + 256 = 0$$

$$324 - 72\sqrt{17} + 68 - 648 + 72\sqrt{17} + 256 = 0$$

$$0 = 0$$

The solution set is $\left\{\left(\frac{9-\sqrt{17}}{8}\right)^4, \left(\frac{9+\sqrt{17}}{8}\right)^4\right\}$.

63. $\sqrt[4]{5x^2-6} = x$

$$\left(\sqrt[4]{5x^2-6}\right)^4 = x^4$$

$$5x^2 - 6 = x^4$$

$$0 = x^4 - 5x^2 + 6$$

Let $u = x^2$ so that $u^2 = x^4$.

$$0 = u^2 - 5u + 6$$

$$0 = (u-3)(u-2)$$

$u = 3$ or $u = 2$

$x^2 = 3$ or $x^2 = 2$

$x = \pm\sqrt{3}$ or $x = \pm\sqrt{2}$

Check:

$x = -\sqrt{3}$: $\sqrt[4]{5\left(-\sqrt{3}\right)^2 - 6} = -\sqrt{3}$

$$\sqrt[4]{15-6} = -\sqrt{3}$$

$$\sqrt[4]{9} \neq -\sqrt{3}$$

$x = \sqrt{3}$: $\sqrt[4]{5\left(\sqrt{3}\right)^2 - 6} = \sqrt{3}$

$$\sqrt[4]{15-6} = \sqrt{3}$$

$$\sqrt[4]{9} = \sqrt{3}$$

$$\sqrt{3} = \sqrt{3}$$

$x = -\sqrt{2}$: $\sqrt[4]{5\left(-\sqrt{2}\right)^2 - 6} = -\sqrt{2}$

$$\sqrt[4]{10-6} = -\sqrt{2}$$

$$\sqrt[4]{4} \neq -\sqrt{2}$$

$x = \sqrt{2}$: $\sqrt[4]{5\left(\sqrt{2}\right)^2 - 6} = \sqrt{2}$

$$\sqrt[4]{10-6} = \sqrt{2}$$

$$\sqrt[4]{4} = \sqrt{2}$$

$$\sqrt{2} = \sqrt{2}$$

The solution set is $\left\{\sqrt{2}, \sqrt{3}\right\}$.

64. $\sqrt[4]{4-5x^2} = x$

$$\left(\sqrt[4]{4-5x^2}\right)^4 = x^4$$

$$4 - 5x^2 = x^4$$

$$0 = x^4 + 5x^2 - 4$$

Let $u = x^2$ so that $u^2 = x^4$.

$$0 = u^2 + 5u - 4$$

$$u = \frac{-5 \pm \sqrt{5^2 - 4(1)(-4)}}{2} = \frac{-5 \pm \sqrt{41}}{2}$$

$$x^2 = \frac{-5 \pm \sqrt{41}}{2}$$

$$x = \pm\sqrt{\frac{-5 \pm \sqrt{41}}{2}}$$

Since $-5-\sqrt{41}<0$, $x=\pm\sqrt{\dfrac{-5-\sqrt{41}}{2}}$ is not real.

Since x is a fourth root, $x=-\sqrt{\dfrac{-5+\sqrt{41}}{2}}$ is also not real. Therefore, we have only one possible solution to check: $x=\sqrt{\dfrac{-5+\sqrt{41}}{2}}$:

Check $x=\sqrt{\dfrac{-5+\sqrt{41}}{2}}$:

$$\sqrt[4]{4-5\left(\pm\sqrt{\dfrac{-5+\sqrt{41}}{2}}\right)^2}=\sqrt{\dfrac{-5+\sqrt{41}}{2}}$$
$$\sqrt[4]{4-5\left(\dfrac{-5+\sqrt{41}}{2}\right)}=\sqrt{\dfrac{-5+\sqrt{41}}{2}}$$
$$\sqrt[4]{\dfrac{8-5\left(-5+\sqrt{41}\right)}{2}}=\sqrt{\dfrac{-5+\sqrt{41}}{2}}$$
$$\sqrt[4]{\dfrac{33-5\sqrt{41}}{2}}=\sqrt{\dfrac{-5+\sqrt{41}}{2}}$$
$$\sqrt[4]{\dfrac{66-10\sqrt{41}}{4}}=\sqrt{\dfrac{-5+\sqrt{41}}{2}}$$
$$\sqrt[4]{\dfrac{25-10\sqrt{41}+41}{4}}=\sqrt{\dfrac{-5+\sqrt{41}}{2}}$$
$$\sqrt[4]{\dfrac{\left(-5+\sqrt{41}\right)^2}{4}}=\sqrt{\dfrac{-5+\sqrt{41}}{2}}$$
$$\sqrt{\dfrac{-5+\sqrt{41}}{2}}=\sqrt{\dfrac{-5+\sqrt{41}}{2}}$$

The solution set is $\left\{\sqrt{\dfrac{-5+\sqrt{41}}{2}}\right\}$.

65. $x^2+3x+\sqrt{x^2+3x}=6$

Let $u=\sqrt{x^2+3x}$ so that $u^2=x^2+3x$.

$$u^2+u=6$$
$$u^2+u-6=0$$
$$(u+3)(u-2)=0$$
$$u=-3 \quad \text{or} \quad u=2$$
$$\sqrt{x^2+3x}=-3 \quad \text{or} \quad \sqrt{x^2+3x}=2$$
$$\text{Not possible} \quad \text{or} \quad x^2+3x=4$$
$$x^2+3x-4=0$$
$$(x+4)(x-1)=0$$
$$x=-4 \text{ or } x=1$$

Check $x=-4$:

$$(-4)^2+3(-4)+\sqrt{(-4)^2+3(-4)}=6$$
$$16-12+\sqrt{16-12}=6$$
$$16-12+\sqrt{4}=6$$
$$6=6$$

Check $x=1$:

$$(1)^2+3(1)+\sqrt{(1)^2+3(1)}=6$$
$$1+3+\sqrt{1+3}=6$$
$$4+\sqrt{4}=6$$
$$6=6$$

The solution set is $\{-4,\ 1\}$.

66. $x^2-3x-\sqrt{x^2-3x}=2$

Let $u=\sqrt{x^2-3x}$ so that $u^2=x^2-3x$.

$$u^2-u=2$$
$$u^2-u-2=0$$
$$(u+1)(u-2)=0$$
$$u=-1 \quad \text{or} \quad u=2$$
$$\sqrt{x^2-3x}=-1 \quad \text{or} \quad \sqrt{x^2-3x}=2$$
$$\text{Not possible} \quad \text{or} \quad x^2-3x=4$$
$$x^2-3x-4=0$$
$$(x-4)(x+1)=0$$
$$x=4 \text{ or } x=-1$$

Check $x=4$:

$$(4)^2-3(4)-\sqrt{(4)^2-3(4)}=16-12-\sqrt{4}$$
$$=4-2=2$$

Check $x=-1$:

$$(-1)^2-3(-1)-\sqrt{(-1)^2-3(-1)}=1+3-\sqrt{4}$$
$$=4-2=2$$

The solution set is $\{-1,\ 4\}$.

67. $\dfrac{1}{(x+1)^2}=\dfrac{1}{x+1}+2$

Let $u=\dfrac{1}{x+1}$ so that $u^2=\left(\dfrac{1}{x+1}\right)^2$.

$$u^2=u+2$$
$$u^2-u-2=0$$
$$(u+1)(u-2)=0$$
$$u=-1 \quad\text{or}\quad u=2$$
$$\frac{1}{x+1}=-1 \quad\text{or}\quad \frac{1}{x+1}=2$$
$$1=-x-1 \quad\text{or}\quad 1=2x+2$$
$$x=-2 \quad\text{or}\quad -2x=1$$
$$x=-\frac{1}{2}$$

Check:

$x=-2$: $\dfrac{1}{(-2+1)^2}=\dfrac{1}{-2+1}+2$
$$1=-1+2$$
$$1=1$$

$x=-\dfrac{1}{2}$: $\dfrac{1}{\left(-\frac{1}{2}+1\right)^2}=\dfrac{1}{\left(-\frac{1}{2}+1\right)}+2$
$$4=2+2$$
$$4=4$$

The solution set is $\left\{-2,-\frac{1}{2}\right\}$.

68. $\dfrac{1}{(x-1)^2}+\dfrac{1}{x-1}=12$

Let $u=\dfrac{1}{x-1}$ so that $u^2=\left(\dfrac{1}{x-1}\right)^2$.

$$u^2+u=12$$
$$u^2+u-12=0$$
$$(u+4)(u-3)=0$$
$$u=-4 \quad\text{or}\quad u=3$$
$$\frac{1}{x-1}=-4 \quad\text{or}\quad \frac{1}{x-1}=3$$
$$1=-4x+4 \quad\text{or}\quad 1=3x-3$$
$$4x=3 \quad\text{or}\quad 4=3x$$
$$x=\frac{3}{4} \quad\text{or}\quad x=\frac{4}{3}$$

Check:

$x=\dfrac{3}{4}$: $\dfrac{1}{\left(\frac{3}{4}-1\right)^2}+\dfrac{1}{\left(\frac{3}{4}-1\right)}=12$
$$\frac{1}{\left(\frac{1}{16}\right)}+\frac{1}{\left(-\frac{1}{4}\right)}=12$$
$$16-4=12$$
$$12=12$$

$x=\dfrac{4}{3}$: $\dfrac{1}{\left(\frac{4}{3}-1\right)^2}+\dfrac{1}{\left(\frac{4}{3}-1\right)}=12$
$$\frac{1}{\left(\frac{1}{9}\right)}+\frac{1}{\left(\frac{1}{3}\right)}=12$$
$$9+3=12$$
$$12=12$$

The solution set is $\left\{\frac{3}{4},\frac{4}{3}\right\}$.

69. $3x^{-2}-7x^{-1}-6=0$

Let $u=x^{-1}$ so that $u^2=x^{-2}$.

$$3u^2-7u-6=0$$
$$(3u+2)(u-3)=0$$
$$u=-\frac{2}{3} \quad\text{or}\quad u=3$$
$$x^{-1}=-\frac{2}{3} \quad\text{or}\quad x^{-1}=3$$
$$\left(x^{-1}\right)^{-1}=\left(-\frac{2}{3}\right)^{-1} \quad\text{or}\quad \left(x^{-1}\right)^{-1}=(3)^{-1}$$
$$x=-\frac{3}{2} \quad\text{or}\quad x=\frac{1}{3}$$

Check:

$x=-\dfrac{3}{2}$: $3\left(-\frac{3}{2}\right)^{-2}-7\left(-\frac{3}{2}\right)^{-1}-6=0$
$$3\left(\frac{4}{9}\right)-7\left(-\frac{2}{3}\right)-6=0$$
$$\frac{4}{3}+\frac{14}{3}-6=0$$
$$0=0$$

$x=\dfrac{1}{3}$: $3\left(\dfrac{1}{3}\right)^{-2}-7\left(\dfrac{1}{3}\right)^{-1}-6=0$
$$3(9)-7(3)-6=0$$
$$27-21-6=0$$
$$0=0$$

The solution set is $\left\{-\frac{3}{2},\frac{1}{3}\right\}$.

70. $2x^{-2}-3x^{-1}-4=0$

Let $u=x^{-1}$ so that $u^2=x^{-2}$.

$2u^2-3u-4=0$

$$u=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-4)}}{2(2)}=\frac{3\pm\sqrt{41}}{4}$$

$$u=\frac{3+\sqrt{41}}{4} \quad \text{or} \quad u=\frac{3-\sqrt{41}}{4}$$

$$x^{-1}=\frac{3+\sqrt{41}}{4} \quad \text{or} \quad x^{-1}=\frac{3-\sqrt{41}}{4}$$

$$\left(x^{-1}\right)^{-1}=\left(\frac{3+\sqrt{41}}{4}\right)^{-1} \quad \text{or} \quad \left(x^{-1}\right)^{-1}=\left(\frac{3-\sqrt{41}}{4}\right)^{-1}$$

$$x=\frac{4}{3+\sqrt{41}}\left(\frac{3-\sqrt{41}}{3-\sqrt{41}}\right) \quad \text{or} \quad x=\frac{4}{3-\sqrt{41}}\left(\frac{3+\sqrt{41}}{3+\sqrt{41}}\right)$$

$$=\frac{12-4\sqrt{41}}{-32} \qquad =\frac{12+4\sqrt{41}}{-32}$$

$$=\frac{-3+\sqrt{41}}{8} \qquad =\frac{-3-\sqrt{41}}{8}$$

Check $x=\dfrac{-3+\sqrt{41}}{8}$:

$$2\left(\frac{-3+\sqrt{41}}{8}\right)^{-2}-3\left(\frac{-3+\sqrt{41}}{8}\right)^{-1}-4=0$$

$$2\left(\frac{64}{\left(-3+\sqrt{41}\right)^2}\right)-3\left(\frac{8}{-3+\sqrt{41}}\right)-4=0$$

$$2(64)-3(8)\left(-3+\sqrt{41}\right)-4\left(-3+\sqrt{41}\right)^2=0$$

$$128+72-24\sqrt{41}-4\left(9-6\sqrt{41}+41\right)=0$$

$$128+72-24\sqrt{41}-36+24\sqrt{41}-164=0$$

$$0=0$$

Check $x=\dfrac{-3-\sqrt{41}}{8}$:

$$2\left(\frac{-3-\sqrt{41}}{8}\right)^{-2}-3\left(\frac{-3-\sqrt{41}}{8}\right)^{-1}-4=0$$

$$2\left(\frac{64}{\left(-3-\sqrt{41}\right)^2}\right)-3\left(\frac{8}{-3-\sqrt{41}}\right)-4=0$$

$$2(64)-3(8)\left(-3-\sqrt{41}\right)-4\left(-3-\sqrt{41}\right)^2=0$$

$$128+72+24\sqrt{41}-4\left(9+6\sqrt{41}+41\right)=0$$

$$128+72+24\sqrt{41}-36-24\sqrt{41}-164=0$$

$$0=0$$

The solution set is $\left\{\dfrac{-3-\sqrt{41}}{8},\dfrac{-3+\sqrt{41}}{8}\right\}$.

71. $2x^{2/3}-5x^{1/3}-3=0$

Let $u=x^{1/3}$ so that $u^2=x^{2/3}$.

$2u^2-5u-3=0$

$(2u+1)(u-3)=0$

$$u=-\frac{1}{2} \quad \text{or} \quad u=3$$

$$x^{1/3}=-\frac{1}{2} \quad \text{or} \quad x^{1/3}=3$$

$$\left(x^{1/3}\right)^3=\left(-\frac{1}{2}\right)^3 \quad \text{or} \quad \left(x^{1/3}\right)^3=(3)^3$$

$$x=-\frac{1}{8} \quad \text{or} \quad x=27$$

Check $x=-\frac{1}{8}$: $2\left(-\frac{1}{8}\right)^{2/3}-5\left(-\frac{1}{8}\right)^{1/3}-3=0$

$$2\left(\frac{1}{4}\right)-5\left(-\frac{1}{2}\right)-3=0$$

$$\frac{1}{2}+\frac{5}{2}-3=0$$

$$3-3=0$$

$$0=0$$

Check $x=27$: $2(27)^{2/3}-5(27)^{1/3}-3=0$

$$2(9)-5(3)-3=0$$

$$18-15-3=0$$

$$3-3=0$$

$$0=0$$

The solution set is $\left\{-\frac{1}{8},27\right\}$.