TT TEXTBOOK TESTBANKS

Solutions Manual for Engineering Mechanics 6th Bedford

SOLUTIONS MANUAL SAMPLE PREVIEW

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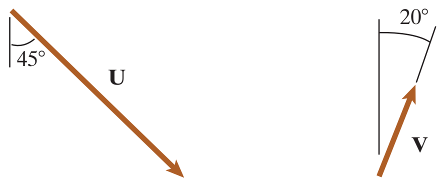
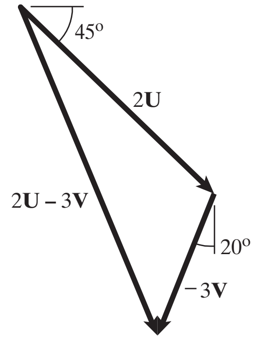
Problem 2.1 Consider vectors **U** and **V** oriented as shown. Their magnitudes are $|\mathbf{U}| = 8$ and $|\mathbf{V}| = 3$. Graphically determine the magnitude of the vector $2\mathbf{U} - 3\mathbf{V}$.



Solution: The vector $2\mathbf{U}$ has the same direction as **U** and magnitude 16. The vector $-3\mathbf{V}$ has the direction opposite to **V** and magnitude 9. We choose a scale and add these vectors to obtain the vector $2\mathbf{U} - 3\mathbf{V}$:

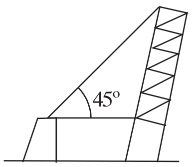


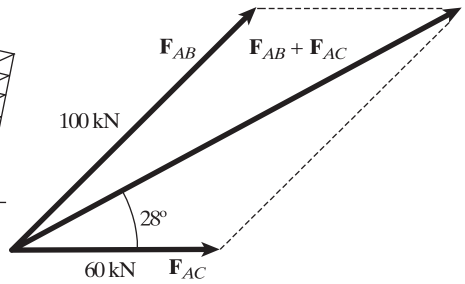
By measuring its magnitude, we estimate that $|2\mathbf{U} - 3\mathbf{V}| = 21.5$.

$\boxed{|2\mathbf{U} - 3\mathbf{V}| = 21.5.}$

Problem 2.2 Suppose that the pylon in Example 2.2 is moved closer to the stadium so that the angle between the forces $\mathbf{F}_{AB}$ and $\mathbf{F}_{AC}$ is 45°. Draw a sketch of the structure with the cables in their new orientation. The magnitudes of the forces are $|\mathbf{F}_{AB}| = 100$ kN and $|\mathbf{F}_{AC}| = 60$ kN. Graphically determine the magnitude and direction of the sum of the forces exerted on the pylon at *A* by the two cables.

Solution:

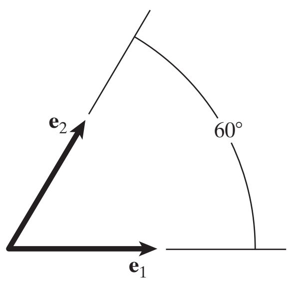




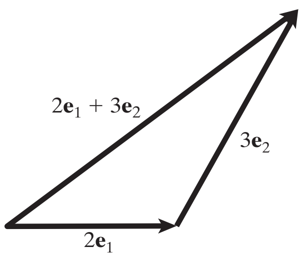
From the diagram we estimate that $|\mathbf{F}_{AB} + \mathbf{F}_{AC}| = 148$ kN at 48° relative to the horizontal.

$\boxed{\text{148 kN at 28° from horizontal.}}$

Problem 2.3 Two unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are oriented as shown. Graphically determine the magnitude of the vector $\mathbf{U} = 2\mathbf{e}_1 + 3\mathbf{e}_2$.



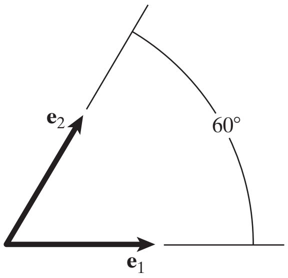
Solution: The vector $2\mathbf{e}_1$ has the same direction as $\mathbf{e}_1$ and magnitude 2. The vector $3\mathbf{e}_2$ has the same direction as $\mathbf{e}_2$ and magnitude 3. We choose a scale and add these vectors to obtain the vector $2\mathbf{e}_1 + 3\mathbf{e}_2$:



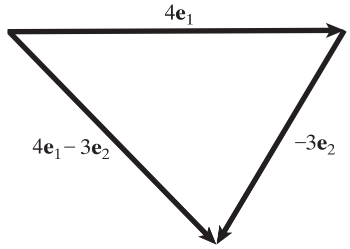
By measuring its magnitude, we estimate that $|2\mathbf{e}_1 + 3\mathbf{e}_2| = 4.4$.

$\boxed{|\mathbf{U}| = 4.4.}$

Problem 2.4 Two unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are oriented as shown. Graphically determine the magnitude of the vector $\mathbf{U} = 4\mathbf{e}_1 - 3\mathbf{e}_2$.



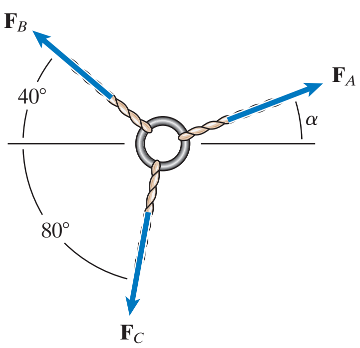
Solution: The vector $4\mathbf{e}_1$ has the same direction as $\mathbf{e}_1$ and magnitude 4. The vector $-3\mathbf{e}_2$ has the direction opposite to $\mathbf{e}_2$ and magnitude 3. We choose a scale and add these vectors to obtain the vector $4\mathbf{e}_1 - 3\mathbf{e}_2$:



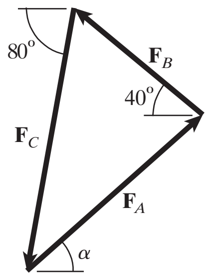
By measuring its magnitude, we estimate that $|4\mathbf{e}_1 - 3\mathbf{e}_2| = 3.6$.

$\boxed{|\mathbf{U}| = 3.6.}$

Problem 2.5 Three forces act on the ring. Their sum is zero: $\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{0}$. The magnitudes $|\mathbf{F}_B| = 100$ N and $|\mathbf{F}_C| = 160$ N. Graphically determine the magnitude of $\mathbf{F}_A$ and the angle α.



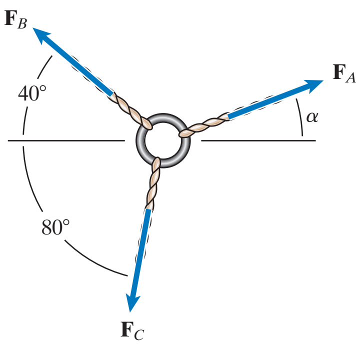
Solution: If the sum of the three forces equals zero, the three force vectors form a closed triangle when placed head-to-tail. We choose a scale and draw the force triangle:



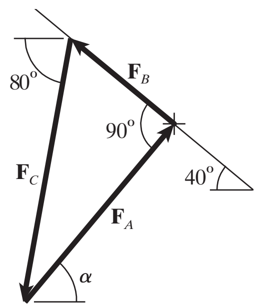
By measuring, we estimate that $|\mathbf{F}_A| = 140$ lb, $\alpha = 42°$.

$\boxed{|\mathbf{F}_A| = 140 \text{ lb}, \alpha = 42°.}$

Problem 2.6 Three forces act on the ring. Their sum is zero: $\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{0}$. The magnitude $|\mathbf{F}_C| = 200$ lb. Graphically determine the value of the angle α for which the magnitude of $\mathbf{F}_A$ is a minimum. If α has that value, what are the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$?



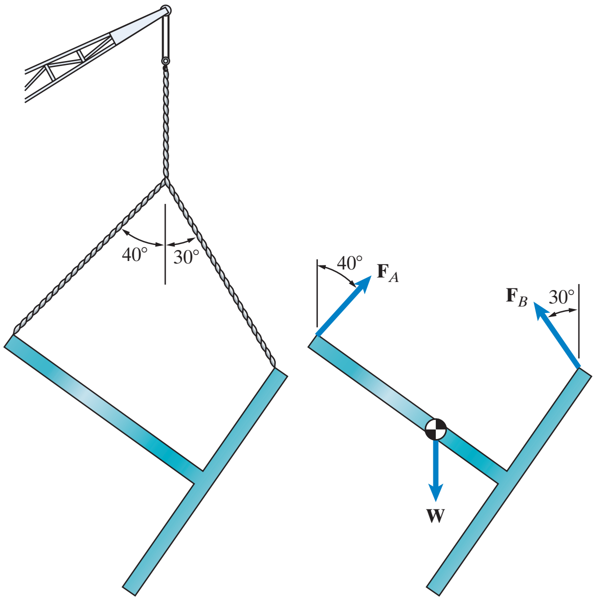
Solution: The magnitude and direction of $\mathbf{F}_C$ are known. The direction of $\mathbf{F}_B$ is known. If the sum of the three forces equals zero, the three force vectors form a closed triangle when placed head-to-tail. Choosing a scale and drawing the force triangle, we can see that the magnitude of $\mathbf{F}_A$ is a minimum when it is perpendicular to the line of action of $\mathbf{F}_B$. Constructing the triangle in this way,



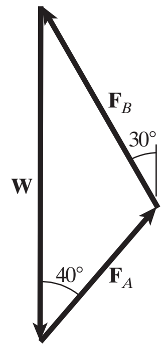
and measuring, we estimate that $\alpha = 50°$, $|\mathbf{F}_A| = 173$ lb, $|\mathbf{F}_B| = 100$ lb.

$\boxed{\alpha = 50°, |\mathbf{F}_A| = 173 \text{ lb}, |\mathbf{F}_B| = 100 \text{ lb}.}$

Problem 2.7 A T-shaped structural member is suspended from cables. The member is subjected to three forces: the forces $\mathbf{F}_A$ and $\mathbf{F}_B$ exerted by the cables and its weight $\mathbf{W}$. The weight of the member is $|\mathbf{W}| = 1000$ lb. The sum of the forces is zero. Graphically determine the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$.



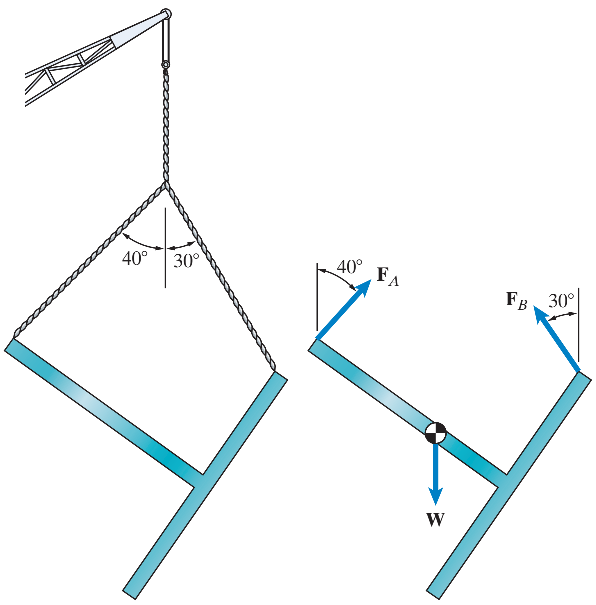
Solution: The directions of all three vectors and the magnitude of $\mathbf{W}$ are known. If the sum of the three forces equals zero, the three force vectors form a closed triangle when placed head-to-tail. We choose a scale and draw the force triangle:



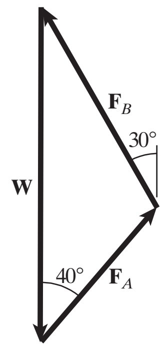
By measuring, we estimate that $|\mathbf{F}_A| = 530$ lb, $|\mathbf{F}_B| = 690$ lb.

$\boxed{|\mathbf{F}_A| = 530 \text{ lb}, |\mathbf{F}_B| = 690 \text{ lb.}}$

Problem 2.8 A T-shaped structural member is suspended from cables. The member is subjected to three forces: the forces $\mathbf{F}_A$ and $\mathbf{F}_B$ exerted by the cables and its weight $\mathbf{W}$. The sum of the forces is zero. Suppose that the magnitude of the force exerted by either cable must not exceed 1200 lb. Graphically determine the largest magnitude of the weight $\mathbf{W}$ that can be suspended in this way.



Solution: The directions of all three vectors are known. If the sum of the three forces equals zero, the three force vectors form a closed triangle when placed head-to-tail. We draw the force triangle:



The force $\mathbf{F}_B$ has the larger magnitude of the two cable forces. It must not exceed 1200 lb. Assuming it has that value and measuring, we estimate that the magnitude of $\mathbf{W}$ is 1740 lb.

$\boxed{|\mathbf{W}| = 1740 \text{ lb.}}$

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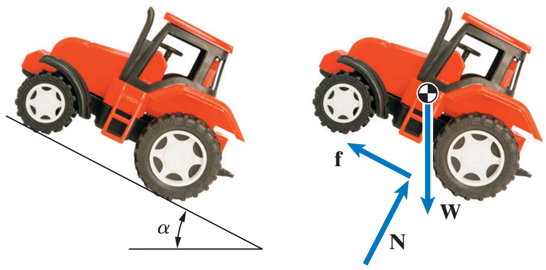
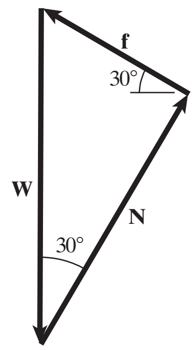
Problem 2.9 The 2700-kg tractor is at rest on the inclined surface. The angle $\alpha = 30°$ relative to the horizontal. The forces exerted on the tractor's tires by the inclined surface are represented by the normal force **N**, which is perpendicular to the surface, and the friction force **f**, which is tangential to the surface. The sum of the normal force, the friction force, and the tractor's weight is zero: $\mathbf{N} + \mathbf{f} + \mathbf{W} = \mathbf{0}$. Graphically determine the magnitudes of **N** and **f** in newtons.



Source: Courtesy of Tka4ko/Shutterstock.

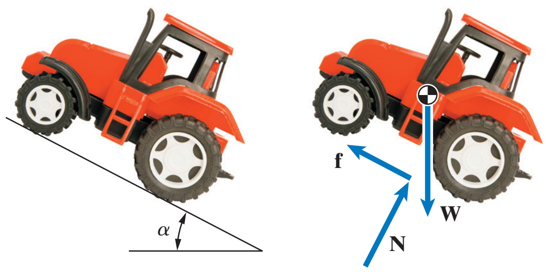
Solution: The tractor's weight is $(2700 \text{ kg})(9.81 \text{ m/s}^2) = 26.5 \text{ kN}$. The directions of all three vectors and the magnitude of **W** are known. If the sum of the three forces equals zero, the three force vectors form a closed triangle when placed head-to-tail. We choose a scale and draw the force triangle:



By measuring, we estimate that $|\mathbf{N}| = 23.0 \text{ kN}$, $|\mathbf{f}| = 13.5 \text{ kN}$.

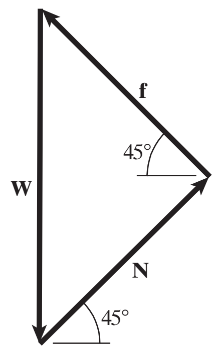
$\boxed{|\mathbf{N}| = 23.0 \text{ kN}, |\mathbf{f}| = 13.5 \text{ kN}.}$

Problem 2.10 The 2700-kg tractor is at rest on the inclined surface. The forces exerted on the tractor's tires by the inclined surface are represented by the normal force **N**, which is perpendicular to the surface, and the friction force **f**, which is tangential to the surface. The sum of the normal force, the friction force, and the tractor's weight is zero: $\mathbf{N} + \mathbf{f} + \mathbf{W} = \mathbf{0}$. Graphically determine the value of the angle α relative to the horizontal for which the magnitudes of **N** and **f** are equal. What is their magnitude?



Source: Courtesy of Tka4ko/Shutterstock.

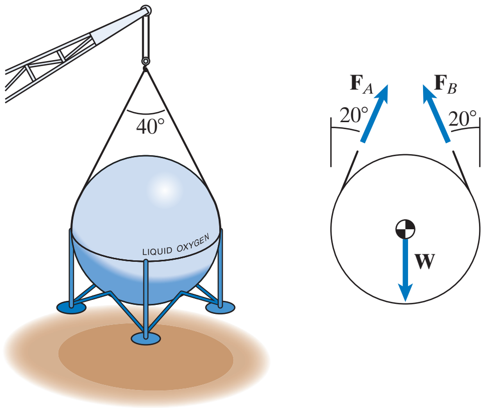
Solution: The tractor's weight is $(2700 \text{ kg})(9.81 \text{ m/s}^2) = 26.5 \text{ kN}$. If the sum of the three forces equals zero, the three force vectors form a closed triangle when placed head-to-tail. The vector f is normal (perpendicular) to the vector N, so these two vectors can be of equal magnitude only if they have the following configuration:



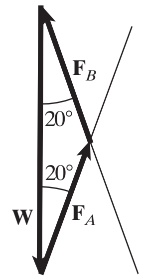
We see immediately that $\alpha = 45°$. Measuring, we estimate that $|\mathbf{N}| = |\mathbf{f}| = 18.8 \text{ kN}$.

$\boxed{\alpha = 45°, |\mathbf{N}| = |\mathbf{f}| = 18.8 \text{ kN}.}$

Problem 2.11 A spherical storage tank is suspended from cables. The tank is subjected to three forces: the forces $\mathbf{F}_A$ and $\mathbf{F}_B$ exerted by the cables and its weight $\mathbf{W}$. The weight of the tank is $|\mathbf{W}| = 800$ lb. The vector sum of the forces acting on the tank equals zero. Graphically determine the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$.



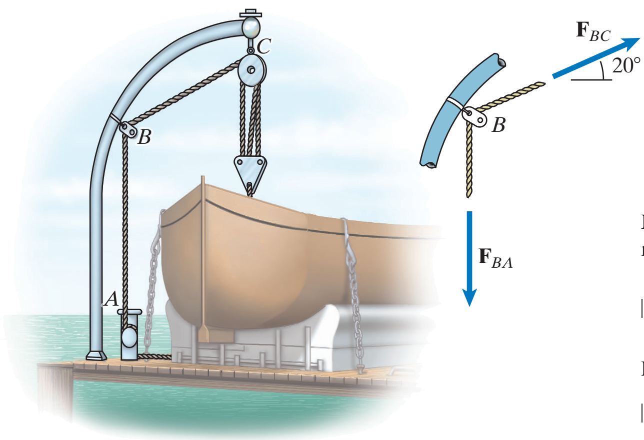
Solution: Measuring a vertical distance of 8 units and drawing two lines at 20° to where they intersect, we obtain the diagram



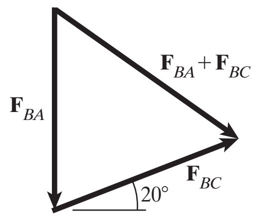
Measuring the magnitudes, we estimate that $|\mathbf{F}_A| = |\mathbf{F}_B| = 424$ lb.

$\boxed{|\mathbf{F}_A| = |\mathbf{F}_B| = 424 \text{ lb.}}$

Problem 2.12 The rope ABC exerts forces $\mathbf{F}_{BA}$ and $\mathbf{F}_{BC}$ of equal magnitude on the block at B. The magnitude of the total force exerted on the block by the two forces is 200 lb. Graphically determine $|\mathbf{F}_{BA}|$.



Solution: We know the magnitudes of the forces $\mathbf{F}_{BA}$ and $\mathbf{F}_{BC}$ are equal, but we don't know its value. Choose a convenient length, call it L_{BA}, and draw both vectors with that length in their correct directions to graphically determine the sum of the forces $\mathbf{F}_{BA} + \mathbf{F}_{BC}$:



Now measure the length of the sum $\mathbf{F}_{BA} + \mathbf{F}_{BC}$, call it L_{SUM}. Then the magnitude of $\mathbf{F}_{BA}$ is

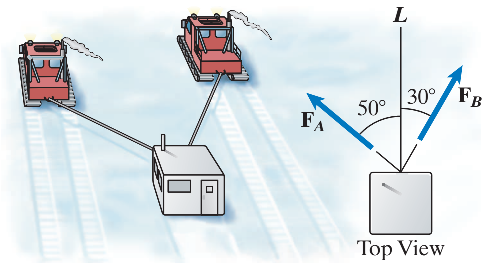
$$|\mathbf{F}_{BA}| = \frac{L_{BA}}{L_{\text{SUM}}}(200 \text{ lb}).$$

From our measurements we obtain

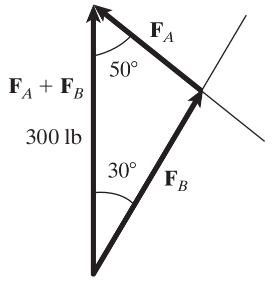
$$|\mathbf{F}_{BA}| = 174 \text{ lb.}$$

$\boxed{|\mathbf{F}_{BA}| = 174 \text{ lb.}}$

Problem 2.13 Two snowcats tow an emergency shelter to a new location near McMurdo Station, Antarctica. (The top view is shown. The cables are horizontal.) The total force $\mathbf{F}_A + \mathbf{F}_B$ exerted on the shelter is in the direction of the line L and has a magnitude of 300 lb. Graphically determine the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$.



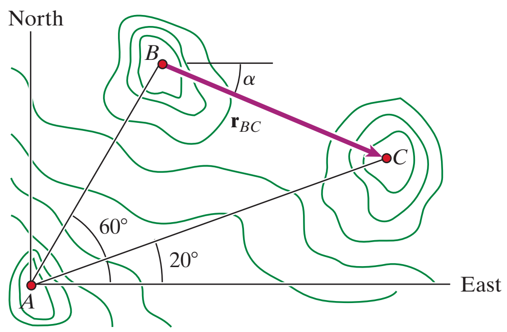
Solution: Measuring a vertical distance of 3 units and drawing two lines at 30° and 50° as shown to identify where they intersect, we obtain the diagram



Measuring the magnitudes, we estimate that $|\mathbf{F}_A| = 150$ lb, $|\mathbf{F}_B| = 234$ lb.

$\boxed{|\mathbf{F}_A| = 150 \text{ lb}, |\mathbf{F}_B| = 234 \text{ lb.}}$

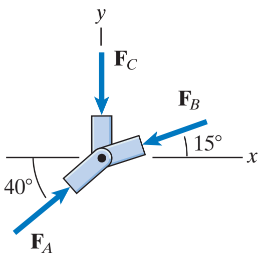
Problem 2.14 A surveyor determines that the horizontal distance from A to B is 400 m and the horizontal distance from A to C is 600 m. Graphically determine the magnitude of the vector $\mathbf{r}_{BC}$ and the angle α.



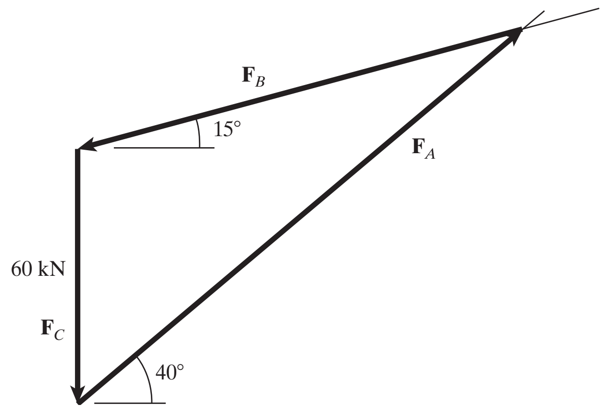
Solution: Using a convenient scale for the given distances, draw lines from point A at 20° and at 60° relative to the East axis. Put point B at 400 m along the 60° line and put point C at 600 m along the 20° line. Then measure the magnitude of the vector $\mathbf{r}_{BC}$ and the angle α. The results are

$\boxed{|\mathbf{r}_{BC}| = 390 \text{ m}, \alpha = 21.2°.}$

Problem 2.15 The figure shows three forces acting on a joint of a structure. The magnitude of $\mathbf{F}_C$ is 60 kN, and $\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{0}$. Graphically determine the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$.



Solution: Measuring a vertical distance of 6 units and drawing two lines at 40° and 15° as shown to identify where they intersect, we obtain the diagram



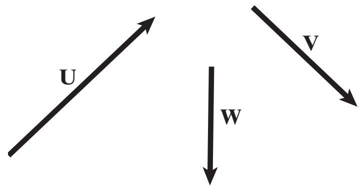
Measuring the magnitudes, we estimate that $|\mathbf{F}_A| = 138$ kN, $|\mathbf{F}_B| = 110$ kN.

$\boxed{|\mathbf{F}_A| = 138 \text{ kN}, |\mathbf{F}_B| = 110 \text{ kN.}}$

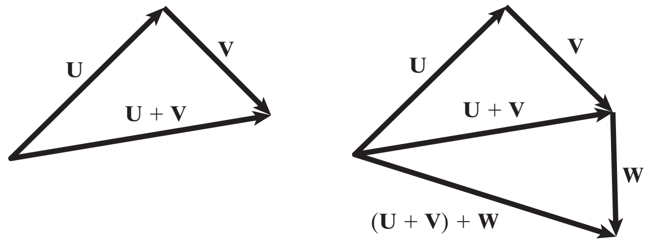
Problem 2.16 By drawing sketches of the vectors, explain why

$$\mathbf{U} + (\mathbf{V} + \mathbf{W}) = (\mathbf{U} + \mathbf{V}) + \mathbf{W}.$$

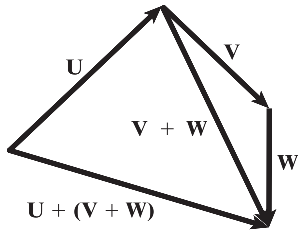
Solution: Consider three arbitrary vectors:



First we add **U** and **V**, then add **W**:



From this diagram we can also see that $\mathbf{U} + (\mathbf{V} + \mathbf{W}) = (\mathbf{U} + \mathbf{V}) + \mathbf{W}$:



Problem 2.17 Two forces are given in terms of their components by $\mathbf{F}_A = 60\mathbf{i} - 20\mathbf{j}$ (lb), $\mathbf{F}_B = 30\mathbf{i} + 40\mathbf{j}$ (lb). (a) What are the magnitudes of the forces? (b) What is the magnitude of their sum $\mathbf{F}_A + \mathbf{F}_B$?

Strategy: The magnitude of a vector in terms of its components is given by Eq. (2.8).

Solution:

(a)

$$\begin{aligned} |\mathbf{F}_A| &= \sqrt{(60\text{ lb})^2 + (-20\text{ lb})^2} \\ &= 63.2\text{ lb.} \\ |\mathbf{F}_B| &= \sqrt{(30\text{ lb})^2 + (40\text{ lb})^2} \\ &= 50.0\text{ lb.} \end{aligned}$$

(b)

$$\begin{aligned} \mathbf{F}_A + \mathbf{F}_B &= (60 + 30)\mathbf{i} + (-20 + 40)\mathbf{j}\text{ (lb)} \\ &= 90\mathbf{i} + 20\mathbf{j}\text{ (lb).} \\ |\mathbf{F}_A + \mathbf{F}_B| &= \sqrt{(90\text{ lb})^2 + (20\text{ lb})^2} \\ &= 92.2\text{ lb.} \end{aligned}$$

$$\boxed{\text{(a) } |\mathbf{F}_A| = 63.2\text{ lb}, |\mathbf{F}_B| = 50.0\text{ lb. (b) } |\mathbf{F}_A + \mathbf{F}_B| = 92.2\text{ lb.}}$$

Problem 2.18 Consider three vectors $\mathbf{A} = 12\mathbf{i} + 8\mathbf{j}$, $\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j}$, and $\mathbf{C} = C_x\mathbf{i} + C_y\mathbf{j}$. Their sum is zero, $\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{0}$, and the components of **B** and **C** satisfy the relations $B_x = -2B_y$ and $C_x = 3C_y$. What are the magnitudes of **B** and **C**?

Solution: If the sum of the three vectors is zero, that implies that each component of their sum is zero. That yields the two equations

$$\begin{aligned} 12 + B_x + C_x &= 0, \\ 8 + B_y + C_y &= 0. \end{aligned}$$

Solving these equations together with the two given equations, we obtain

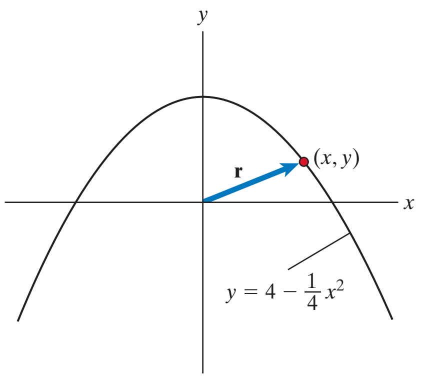
$$\begin{aligned} \mathbf{B} &= 4.8\mathbf{i} - 2.4\mathbf{j}, \\ \mathbf{C} &= -16.8\mathbf{i} - 5.6\mathbf{j}. \end{aligned}$$

Therefore

$$\begin{aligned} |\mathbf{B}| &= \sqrt{(4.8)^2 + (-2.4)^2} \\ &= 5.37, \\ |\mathbf{C}| &= \sqrt{(-16.8)^2 + (-5.6)^2} \\ &= 17.7. \end{aligned}$$

$$\boxed{|\mathbf{B}| = 5.37, |\mathbf{C}| = 17.7.}$$

Problem 2.19 The vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ extends to a point on the curve described by the equation shown. Determine the values of x and y for which the magnitude of the vector is a minimum. What is the minimum magnitude?



Solution: The magnitude of $\mathbf{r}$ is

$$|\mathbf{r}| = \sqrt{x^2 + y^2}.$$

Substituting the relation $y = 4 - (1/4)x^2$ into this relation gives

$$|\mathbf{r}| = \sqrt{16 - x^2 + \frac{1}{16}x^4}.$$

We want to find the value of x for which the magnitude of $\mathbf{r}$ is a minimum. We can simplify things a bit by seeking the value of x for which the square of the magnitude of $\mathbf{r}$ is a minimum:

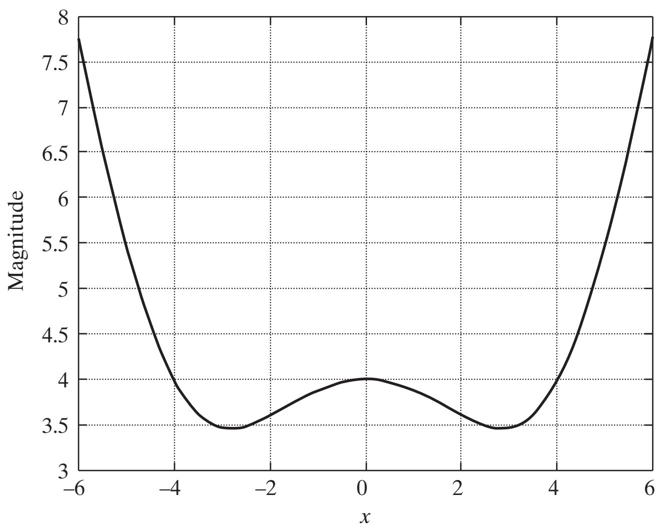
$$|\mathbf{r}|^2 = 16 - x^2 + \frac{1}{16}x^4.$$

We equate the derivative of this expression to zero,

$$-2x + \frac{1}{4}x^3 = 0,$$

obtaining the roots $x = 0$ and $x = \pm\sqrt{8}$. Substituting these values into the expression for the magnitude in terms of x, we find that $|\mathbf{r}| = 4$ at $x = 0$ and $|\mathbf{r}| = 3.46$ at $x = \pm\sqrt{8}$. So the minimum magnitude occurs at $x = \pm\sqrt{8}$. At both of these points, $y = 2$.

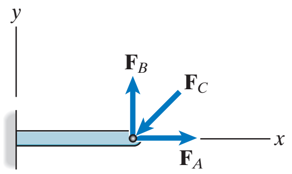
If we examine a graph of the magnitude as a function of x,



we see that this is an interesting case, $x = 0$ is a stationary point but is not the minimum magnitude.

$\boxed{x = \pm\sqrt{8},\ y = 2,\ \text{magnitude} = 3.46.}$

Problem 2.20 The forces $\mathbf{F}_A = 90\mathbf{i}$ (kN) and $\mathbf{F}_B = 60\mathbf{j}$ (kN). The force $\mathbf{F}_C = c(-2\mathbf{i} - \mathbf{j})$, where c is a parameter. Determine the value of c so that the magnitude of the total force exerted on the beam by the three forces is a minimum. What is the minimum magnitude?



Solution: The total force is

$$\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = (90 - 2c)\mathbf{i} + (60 - c)\mathbf{j} \text{ (kN)}.$$

Its magnitude is

$$\begin{aligned} |\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C| &= \sqrt{(90 - 2c)^2 + (60 - c)^2} \\ &= \sqrt{8100 - 360c + 4c^2 + 3600 - 120c + c^2} \\ &= \sqrt{11{,}700 - 480c + 5c^2}. \end{aligned}$$

We can seek the value of c that makes the square of the magnitude a minimum. We set

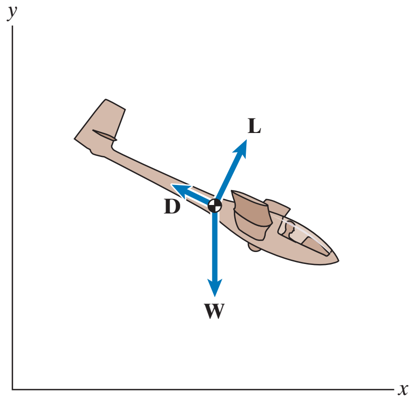
$$\frac{d}{dc}(11{,}700 - 480c + 5c^2) = -480 + 10c = 0,$$

obtaining $c = 48$ kN. Substituting this value into the expression for the magnitude gives

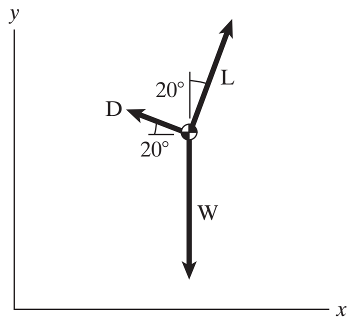
$$|\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C| = 13.4 \text{ kN}.$$

$\boxed{c = 48 \text{ kN, magnitude} = 13.4 \text{ kN.}}$

Problem 2.21 The forces acting on the sailplane are its weight $\mathbf{W} = -500\mathbf{j}$ (lb), the lift $\mathbf{L}$, and the drag $\mathbf{D}$. The angle between the lift vector $\mathbf{L}$ and the vertical y-axis is 20°, and the drag vector is perpendicular to the lift vector. At the instant shown, the sum of the forces on the sailplane is zero: $\mathbf{L} + \mathbf{D} + \mathbf{W} = \mathbf{0}$. Determine the magnitudes of $\mathbf{L}$ and $\mathbf{D}$.



Solution:



Let L and D denote the magnitudes of the lift and drag. Then

$$\mathbf{L} = L\sin 20^\circ\mathbf{i} + L\cos 20^\circ\mathbf{j},$$
$$\mathbf{D} = -D\cos 20^\circ\mathbf{i} + D\sin 20^\circ\mathbf{j}.$$

We have

$$\begin{aligned}\mathbf{L} + \mathbf{D} + \mathbf{W} &= (L\sin 20^\circ - D\cos 20^\circ)\mathbf{i} \\ &\quad + (L\cos 20^\circ + D\sin 20^\circ - 500 \text{ lb})\mathbf{j} \\ &= \mathbf{0}.\end{aligned}$$

Each component must equal zero, resulting in two equations:

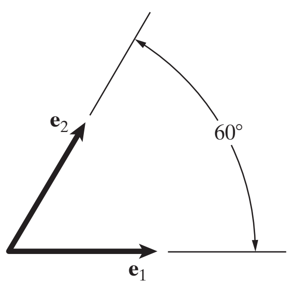
$$L\sin 20^\circ - D\cos 20^\circ = 0,$$
$$L\cos 20^\circ + D\sin 20^\circ = 500 \text{ lb}.$$

Solving, we obtain $L = 470$ lb, $D = 171$ lb.

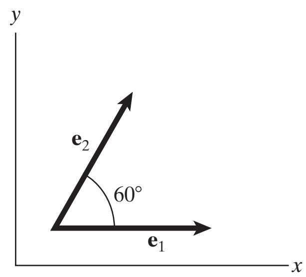
$\boxed{|\mathbf{L}| = 470 \text{ lb}, |\mathbf{D}| = 171 \text{ lb}.}$

Problem 2.22 Two unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are oriented as shown. Determine the magnitude of the vector $\mathbf{U} = 2\mathbf{e}_1 + 3\mathbf{e}_2$.

Strategy: Introduce a cartesian coordinate system and express $\mathbf{e}_1$ and $\mathbf{e}_2$ in terms of their components.



Solution: We introduce the coordinate system shown:



The unit vectors have magnitude one, so we can express them as

$$\mathbf{e}_1 = \mathbf{i},$$
$$\mathbf{e}_2 = \cos 60^\circ\mathbf{i} + \sin 60^\circ\mathbf{j}.$$

With these expressions, the vector

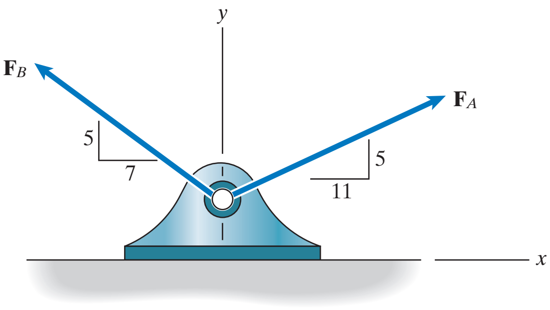
$$\begin{aligned}\mathbf{U} &= 2\mathbf{e}_1 + 3\mathbf{e}_2 \\ &= (2 + 3\cos 60^\circ)\mathbf{i} + 3\sin 60^\circ\mathbf{j}.\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{U}| &= \sqrt{(2 + 3\cos 60^\circ)^2 + (3\sin 60^\circ)^2} \\ &= 4.36.\end{aligned}$$

$\boxed{|\mathbf{U}| = 4.36.}$

Problem 2.23 Two forces act on the support. Their magnitudes are $|\mathbf{F}_A| = 300$ lb, $|\mathbf{F}_B| = 200$ lb. Determine the magnitude $|\mathbf{F}_A + \mathbf{F}_B|$ of the total force exerted on the support by the two forces.



Solution: Using similar triangles to determine the components, the force $\mathbf{F}_A$ in terms of its components is

$$\mathbf{F}_A = \frac{11}{\sqrt{(11)^2 + (5)^2}}(300\text{ lb})\mathbf{i} + \frac{5}{\sqrt{(11)^2 + (5)^2}}(300\text{ lb})\mathbf{j}$$
$$= 273\mathbf{i} + 124\mathbf{j}\text{ (lb)}.$$

The force $\mathbf{F}_B$ in terms of its components is

$$\mathbf{F}_B = -\frac{7}{\sqrt{(7)^2 + (5)^2}}(200\text{ lb})\mathbf{i} + \frac{5}{\sqrt{(7)^2 + (5)^2}}(200\text{ lb})\mathbf{j}$$
$$= -163\mathbf{i} + 116\mathbf{j}\text{ (lb)}.$$

The total force is

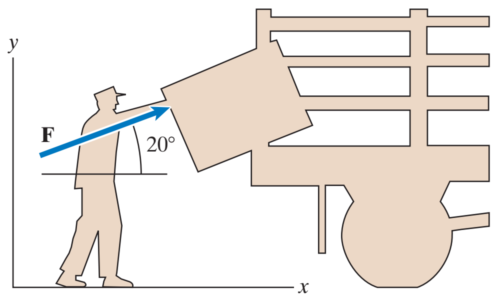
$$\mathbf{F}_A + \mathbf{F}_B = 110\mathbf{i} + 240\mathbf{j}\text{ (lb)},$$

and the magnitude of the total force is

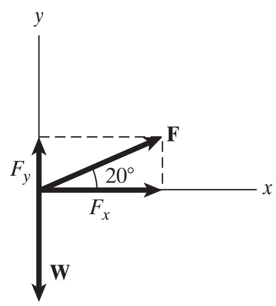
$$|\mathbf{F}_A + \mathbf{F}_B| = \sqrt{(110\text{ lb})^2 + (240\text{ lb})^2}$$
$$= 265\text{ lb}.$$

$$\boxed{|\mathbf{F}_A + \mathbf{F}_B| = 265\text{ lb}.}$$

Problem 2.24 The person exerts a 30-lb force $\mathbf{F}$ to push the crate onto a truck. (a) Express $\mathbf{F}$ in terms of components using the coordinate system shown. (b) The weight of the crate is 120 lb. What is the magnitude of the sum of the two forces exerted by the person and the crate's weight?



Solution:



(a) The force $\mathbf{F}$ in terms of its components is

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$
$$= (30\text{ lb})\cos 20^\circ\,\mathbf{i} + (30\text{ lb})\sin 20^\circ\,\mathbf{j}$$
$$= 28.2\mathbf{i} + 10.3\mathbf{j}\text{ (lb)}.$$

(b) The sum of the forces is

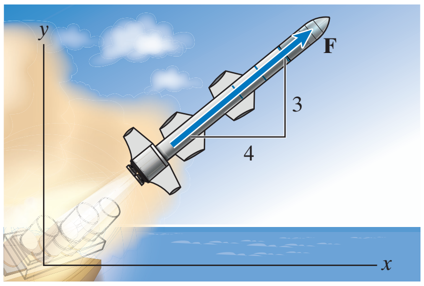
$$\mathbf{F} + \mathbf{W} = 28.2\mathbf{i} + 10.3\mathbf{j} - 120\mathbf{j}\text{ (lb)}$$
$$= 28.2\mathbf{i} - 110\mathbf{j}\text{ (lb)}.$$

Its magnitude is

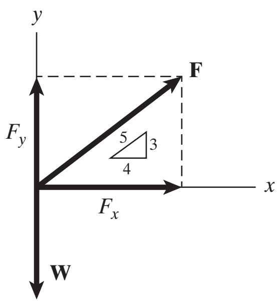
$$|\mathbf{F} + \mathbf{W}| = \sqrt{(28.2\text{ lb})^2 + (-110\text{ lb})^2}$$
$$= 113\text{ lb}.$$

$$\boxed{\text{(a) } \mathbf{F} = 28.2\mathbf{i} - 110\mathbf{j}\text{ (lb). (b) 113 lb.}}$$

Problem 2.25 The missile's engine exerts a 130-kN thrust **F**. (a) Express **F** in terms of components using the coordinate system shown. (b) The mass of the missile is 1800 kg. What is the magnitude of the sum of the thrust **F** and the missile's weight?



Solution:



(a) The thrust **F** in terms of its components is

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$
$$= \frac{4}{5}(130\text{ kN})\mathbf{i} + \frac{3}{5}(130\text{ kN})\mathbf{j}$$
$$= 104\mathbf{i} + 78\mathbf{j}\ (\text{kN}).$$

(b) The missile's weight is

$$W = mg = (1800\text{ kg})(9.81\text{ m/s}^2)$$
$$= 17.7\text{ kg}.$$

The sum of the forces is

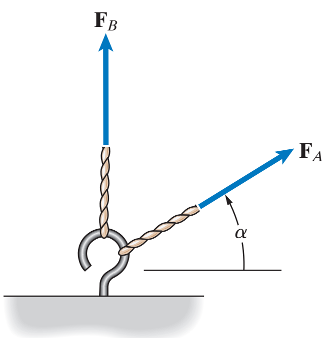
$$\mathbf{F} + \mathbf{W} = 104\mathbf{i} + 78\mathbf{j} - 17.7\mathbf{j}\ (\text{kN})$$
$$= 104\mathbf{i} + 60.3\mathbf{j}\ (\text{kN}).$$

Its magnitude is

$$|\mathbf{F} + \mathbf{W}| = \sqrt{(104\text{ kN})^2 + (60.3\text{ kN})^2}$$
$$= 120\text{ kN}.$$

$\boxed{\text{(a) } \mathbf{F} = 104\mathbf{i} + 78\mathbf{j}\ (\text{kN}).\ \text{(b) } 120\text{ kN}.}$

Problem 2.26 The magnitude of the force $\mathbf{F}_A$ is 8 kN. The magnitude of the vertical force $\mathbf{F}_B$ is 2 kN. For what value of the angle α in the range $0 \le \alpha \le 90°$ is the magnitude of the sum of the two forces equal to 9 kN?



Solution: In terms of the coordinate system shown, the two forces in terms of their components are

$$\mathbf{F}_A = (8\text{ kN})(\cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j}),$$
$$\mathbf{F}_B = (2\text{ kN})\mathbf{j}.$$

The magnitude of their sum is

$$|\mathbf{F}_A + \mathbf{F}_B| = \sqrt{(8\cos\alpha)^2 + (8\sin\alpha + 2)^2}\ (\text{kN})$$
$$= \sqrt{68 + 32\sin\alpha}\ (\text{kN}).$$

We want to determine the value of α for which

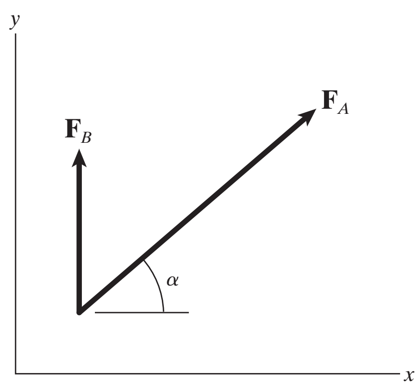
$$|\mathbf{F}_A + \mathbf{F}_B| = \sqrt{68 + 32\sin\alpha}\ (\text{kN}) = 9\text{ kN}.$$

Squaring both sides gives the equation

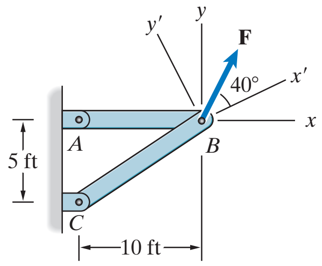
$$68 + 32\sin\alpha = 81.$$

Solving, we obtain $\alpha = 24.0°$.

$\boxed{\alpha = 24.0°.}$



Problem 2.27 The x-axis is parallel to the bar AB. The x'-axis is parallel to the bar BC. The magnitude of the vector $\mathbf{F}$ is 200 lb. (a) Express $\mathbf{F}$ in terms of its components in terms of the x–y coordinate system. Use these components to determine the magnitude of $\mathbf{F}$. (b) Express $\mathbf{F}$ in terms of its components in terms of the x'–y' coordinate system. Use these components to determine the magnitude of $\mathbf{F}$.



Solution:

(a) The angle between bar AB and bar BC is

$$\arctan\left(\frac{5}{10}\right) = 26.6°.$$

Therefore, the angle between the vector $\mathbf{F}$ and the x-axis is $26.6° + 40° = 66.6°$. The vector $\mathbf{F}$ expressed in terms of the x-y coordinate system is

$$\begin{aligned}\mathbf{F} &= (200\text{ lb})\cos 66.6°\mathbf{i} + (200\text{ lb})\sin 66.6°\mathbf{j}\\ &= 79.5\mathbf{i} + 183.5\mathbf{j}\text{ (lb)}.\end{aligned}$$

Using these components to determine the magnitude of $\mathbf{F}$ yields

$$\begin{aligned}|\mathbf{F}| &= \sqrt{(79.5\text{ lb})^2 + (183.5\text{ lb})^2}\\ &= 200\text{ lb}.\end{aligned}$$

(b) The vector $\mathbf{F}$ expressed in terms of the x'-y' coordinate system is

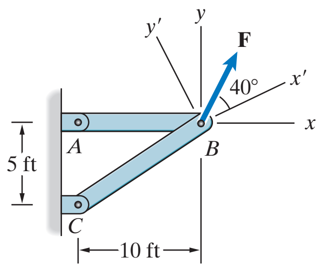
$$\begin{aligned}\mathbf{F} &= (200\text{ lb})\cos 40°\mathbf{i}' + (200\text{ lb})\sin 40°\mathbf{j}'\\ &= 153.2\mathbf{i}' + 128.6\mathbf{j}'\text{ (lb)}.\end{aligned}$$

Using these components to determine the magnitude of $\mathbf{F}$ yields

$$\begin{aligned}|\mathbf{F}| &= \sqrt{(153.2\text{ lb})^2 + (128.6\text{ lb})^2}\\ &= 200\text{ lb}.\end{aligned}$$

$$\boxed{\begin{array}{l}\text{(a) } \mathbf{F} = 79.5\mathbf{i} + 183.5\mathbf{j}\text{ (lb)}, |\mathbf{F}| = 200\text{ lb.}\\ \text{(b) } \mathbf{F} = 153.2\mathbf{i}' + 128.6\mathbf{j}'\text{ (lb)}, |\mathbf{F}| = 200\text{ lb.}\end{array}}$$

Problem 2.28 Suppose that the magnitude of the component of the vector $\mathbf{F}$ parallel to the x'-axis is 150 lb. (a) What is the magnitude of $\mathbf{F}$? (b) Express $\mathbf{F}$ in terms of its components in terms of the x–y coordinate system.



Solution:

(a) The component of $\mathbf{F}$ parallel to the x'-axis is $|\mathbf{F}|\cos 40°$. Solving

$$|\mathbf{F}|\cos 40° = 150\text{ lb}$$

for the magnitude of $\mathbf{F}$ yields $|\mathbf{F}| = 196$ lb.

(b) The angle between bar AB and bar BC is

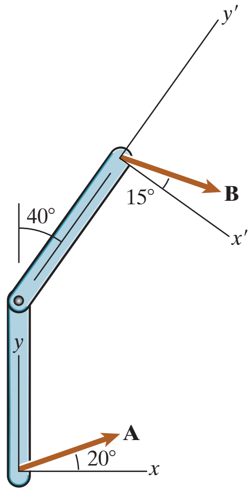
$$\arctan\left(\frac{5}{10}\right) = 26.6°.$$

Therefore the angle between the vector $\mathbf{F}$ and the x-axis is $26.6° + 40° = 66.6°$. The vector $\mathbf{F}$ expressed in terms of the x-y coordinate system is

$$\begin{aligned}\mathbf{F} &= (196\text{ lb})\cos 66.6°\mathbf{i} + (196\text{ lb})\sin 66.6°\mathbf{j}\\ &= 77.9\mathbf{i} + 180\mathbf{j}\text{ (lb)}.\end{aligned}$$

$$\boxed{\text{(a) } |\mathbf{F}| = 196\text{ lb. (b) } \mathbf{F} = 77.9\mathbf{i} + 180\mathbf{j}\text{ (lb).}}$$

Problem 2.29 The magnitudes $|\mathbf{A}| = 12$ and $|\mathbf{B}| = 10$. (a) Express $\mathbf{A}$ in terms of its components in terms of the x'–y' coordinate system. (b) Express $\mathbf{B}$ in terms of its components in terms of the x–y coordinate system.



Solution:

(a) The angle between the vector $\mathbf{A}$ and the x'-axis is $20° + 40° = 60°$. The vector $\mathbf{A}$ expressed in terms of the x'-y' coordinate system is

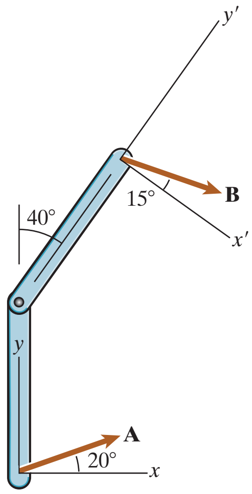
$$\mathbf{A} = 12\cos 60°\mathbf{i}' + 12\sin 60°\mathbf{j}'$$
$$= 6\mathbf{i}' + 10.4\mathbf{j}'.$$

(b) The angle between the vector $\mathbf{B}$ and the x-axis is $15° - 40° = -25°$. The vector $\mathbf{B}$ expressed in terms of the x-y coordinate system is

$$\mathbf{B} = 10\cos 25°\mathbf{i} - 10\sin 25°\mathbf{j}$$
$$= 9.06\mathbf{i} - 4.23\mathbf{j}.$$

$\boxed{\text{(a) } \mathbf{A} = 6\mathbf{i}' + 10.4\mathbf{j}'. \text{ (b) } \mathbf{B} = 9.06\mathbf{i} - 4.23\mathbf{j}.}$

Problem 2.30 The magnitudes $|\mathbf{A}| = 12$ and $|\mathbf{B}| = 10$. Determine $|\mathbf{A} + \mathbf{B}|$.



Solution: To determine the sum of $\mathbf{A}$ and $\mathbf{B}$, we must express them in terms of their components in terms of the *same* coordinate system. The angle between the vector $\mathbf{A}$ and the x'-axis is $20° + 40° = 60°$. The vector $\mathbf{A}$ expressed in terms of the x'-y' coordinate system is

$$\mathbf{A} = 12\cos 60°\mathbf{i}' + 12\sin 60°\mathbf{j}'$$
$$= 6\mathbf{i}' + 10.4\mathbf{j}'.$$

The vector $\mathbf{B}$ expressed in terms of the x'-y' coordinate system is

$$\mathbf{B} = 10\cos 15°\mathbf{i}' + 10\sin 15°\mathbf{j}'$$
$$= 9.66\mathbf{i}' + 2.59\mathbf{j}'.$$

The sum of $\mathbf{A}$ and $\mathbf{B}$ is

$$\mathbf{A} + \mathbf{B} = (6\mathbf{i}' + 10.4\mathbf{j}') + (9.66\mathbf{i}' + 2.59\mathbf{j}')$$
$$= 15.7\mathbf{i}' + 13.0\mathbf{j}'.$$

Therefore

$$|\mathbf{A} + \mathbf{B}| = \sqrt{(15.7)^2 + (13.0)^2}$$
$$= 20.3.$$

$\boxed{|\mathbf{A} + \mathbf{B}| = 20.3.}$

Problem 2.31 In Practice Example 2.3, the cable AB exerts a 900-N force on the top of the tower. Suppose that the attachment point B is moved in the horizontal direction farther from the tower, and assume that the magnitude of the force $\mathbf{F}$ the cable exerts on the top of the tower is proportional to the length of the cable. (a) What is the distance from the tower to point B if the magnitude of the force is 1000 N? (b) Express the 1000-N force $\mathbf{F}$ in terms of components using the coordinate system shown.

Solution: In the new problem assume that point B is located a distance d away from the base. The lengths in the original problem and in the new problem are given by

$$L_{\text{original}} = \sqrt{(40\text{ m})^2 + (80\text{ m})^2} = \sqrt{8000\text{ m}^2}$$
$$L_{\text{new}} = \sqrt{d^2 + (80\text{ m})^2}$$

(a) The force is proportional to the length. Therefore

$$1000\text{ N} = (900\text{ N})\frac{\sqrt{d^2 + (80\text{ m})^2}}{\sqrt{8000\text{ m}^2}}$$
$$d = \sqrt{(8000\text{ m}^2)\left(\frac{1000\text{ N}}{900\text{ N}}\right)^2 - (80\text{ m})^2} = 59.0\text{ m}.$$

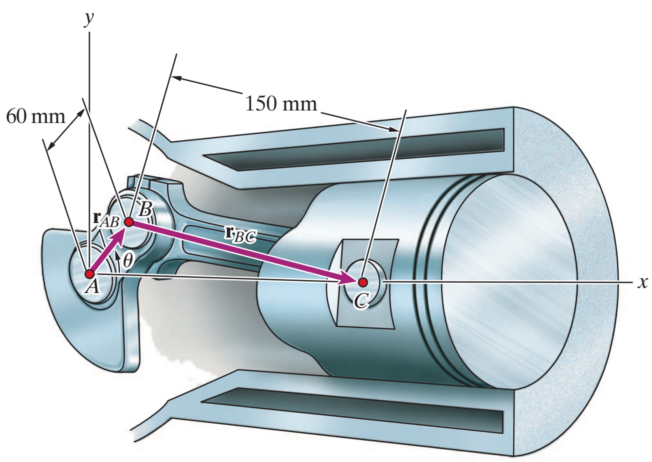
$\boxed{d = 59.0\text{ m}}$

(b) The force F is then

$$\mathbf{F} = (1000\text{ N})\left(\frac{d}{\sqrt{d^2 + (80\text{ m})^2}}\mathbf{i} - \frac{80\text{ m}}{\sqrt{d^2 + (80\text{ m})^2}}\mathbf{j}\right)$$
$$= 593\mathbf{i} - 805\mathbf{j}\text{ (N)}.$$

$\boxed{\mathbf{F} = 593\mathbf{i} - 805\mathbf{j}\text{ (N)}.}$

Problem 2.32 Determine the position vector $\mathbf{r}_{AB}$ in terms of its components if (a) $\theta = 30°$; (b) $\theta = 225°$.

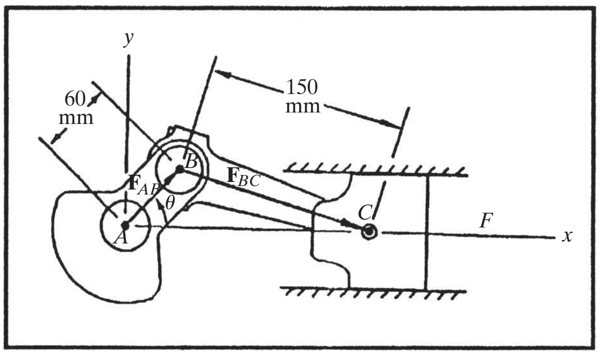


Solution:

(a) $\mathbf{r}_{AB} = (60)\cos(30°)\mathbf{i} + (60)\sin(30°)\mathbf{j}$, or
$\mathbf{r}_{AB} = 52.0\mathbf{i} + 30\mathbf{j}$ (mm).

(b) $\mathbf{r}_{AB} = (60)\cos(225°)\mathbf{i} + (60)\sin(225°)\mathbf{j}$ or
$\mathbf{r}_{AB} = -42.4\mathbf{i} - 42.4\mathbf{j}$ (mm).

$\boxed{\text{(a) } \mathbf{r}_{AB} = 52.0\mathbf{i} + 30\mathbf{j}\text{ (mm). (b) } \mathbf{r}_{AB} = -42.4\mathbf{i} - 42.4\mathbf{j}\text{ (mm).}}$



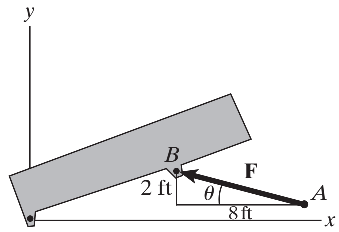
Problem 2.33 In Example 2.4, the coordinates of the fixed point A are (17, 1) ft. The driver lowers the bed of the truck into a new position in which the coordinates of point B are (9, 3) ft. The magnitude of the force $\mathbf{F}$ exerted on the bed by the hydraulic cylinder when the bed is in the new position is 4800 lb. Draw a sketch of the new situation. Express $\mathbf{F}$ in terms of components.

Solution:

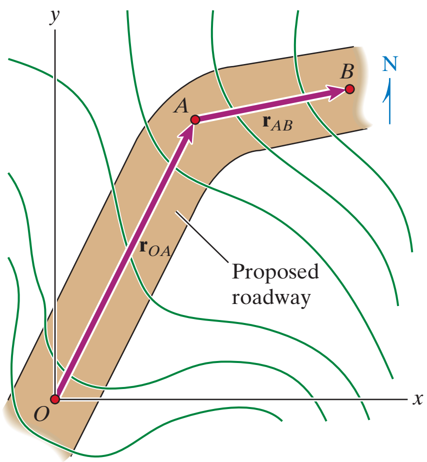
$$\theta = \tan^{-1}\left(\frac{2\text{ ft}}{8\text{ ft}}\right) = 14.04°$$

$$\mathbf{F} = 4800\text{ lb}(-\cos\theta\mathbf{i} + \sin\theta\mathbf{j}).$$

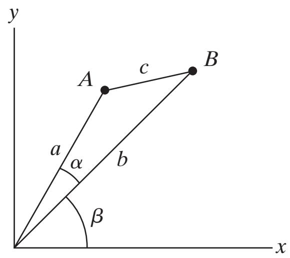
$$\boxed{\mathbf{F} = -4660\mathbf{i} + 1160\mathbf{j}\text{ (lb)}.}$$



Problem 2.34 Using laser instruments, a surveyor determines that the coordinates of point B are $x = 120$ m, $y = 122$ m. He knows that $|\mathbf{r}_{OA}| = 125$ m and $|\mathbf{r}_{AB}| = 62$ m. Determine the vector $\mathbf{r}_{OA}$.



Solution:



We can apply geometry. The angle β = arctan (122 m/120 m) = 45.5°. The side $b = \sqrt{(122\text{ m})^2 + (120\text{ m})^2} = 171$ m, and we are told that $a = 125$ m, $c = 62$ m. Applying the law of cosines (Appendix A),

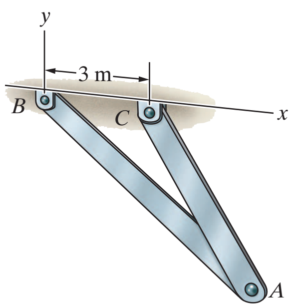
$$c^2 = a^2 + b^2 - 2ab\cos\alpha,$$

we obtain $\cos\alpha = 0.956$, so $\alpha = 16.3°$. We can now determine the position vector of A:

$$\begin{aligned}\mathbf{r}_{OA} &= (125\text{ m})\cos(\alpha + \beta)\mathbf{i} + (125\text{ m})\sin(\alpha + \beta)\mathbf{j}\\ &= 59.1\mathbf{i} + 110\mathbf{j}\text{ (m)}.\end{aligned}$$

$$\boxed{\mathbf{r}_{OA} = 59.1\mathbf{i} + 110\mathbf{j}\text{ (m)}.}$$

Problem 2.35 The magnitude of the position vector $\mathbf{r}_{BA}$ from point B to point A is 6 m and the magnitude of the position vector $\mathbf{r}_{CA}$ from point C to point A is 4 m. What are the components of $\mathbf{r}_{BA}$?

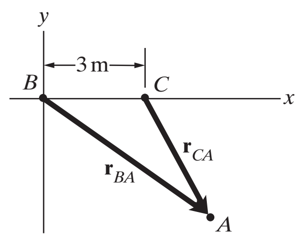


Solution: The coordinates are: $A(x_A, y_A)$, $B(0, 0)$, $C(3\text{ m}, 0)$ Thus

$$\mathbf{r}_{BA} = (x_A - 0)\mathbf{i} + (y_A - 0)\mathbf{j} \Rightarrow (6\text{ m})^2 = x_A^2 + y_A^2$$
$$\mathbf{r}_{CA} = (x_A - 3\text{ m})\mathbf{i} + (y_A - 0)\mathbf{j} \Rightarrow (4\text{ m})^2 = (x_A - 3\text{ m})^2 + y_A^2$$

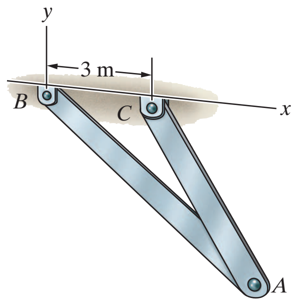
Solving these two equations, we find $x_A = 4.833$ m, $y_A = \pm 3.555$ m. We choose the “-” sign and find

$$\boxed{\mathbf{r}_{BA} = 4.83\mathbf{i} - 3.56\mathbf{j}\text{ (m)}.}$$



Problem 2.36 In Problem 2.35, determine the components of a unit vector $\mathbf{e}_{CA}$ that points from point C toward point A.

Strategy: Determine the components of $\mathbf{r}_{CA}$ and then divide the vector $\mathbf{r}_{CA}$ by its magnitude.



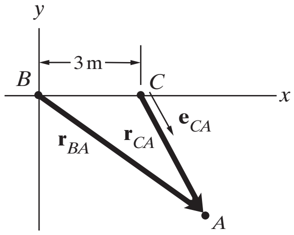
Solution: From the solution to the previous problem, we have

$$\mathbf{r}_{CA} = 1.83i - 3.56j \text{ (m)}, \quad |\mathbf{r}_{CA}| = 4 \text{ m}.$$

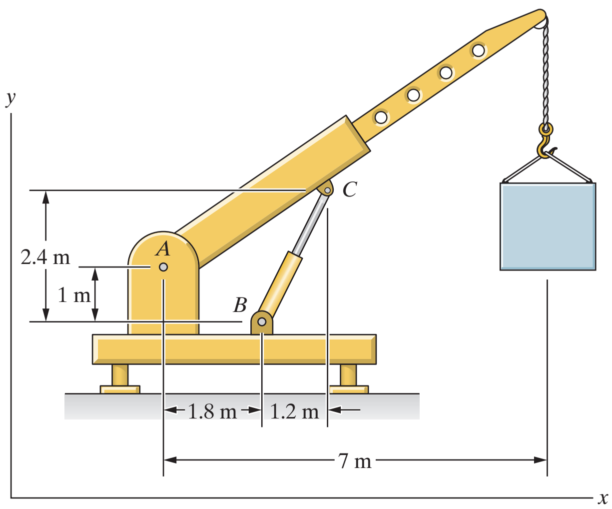
Then

$$\mathbf{e}_{CA} = \frac{\mathbf{r}_{CA}}{|\mathbf{r}_{CA}|} = 0.458\mathbf{i} - 0.889\mathbf{j}.$$

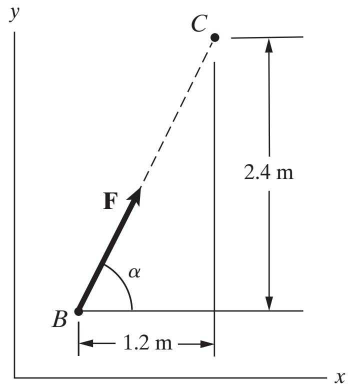
$\boxed{\mathbf{e}_{CA} = 0.458\mathbf{i} - 0.889\mathbf{j}.}$



Problem 2.37 The hydraulic cylinder BC exerts a force $\mathbf{F}$ on the boom of the crane at C. The force points from B toward C, and its magnitude is $|\mathbf{F}| = 300$ kN. Determine the components of $\mathbf{F}$.



Solution:



The angle $\alpha = \arctan(2.4/1.2) = 63.4°$, so

$$\mathbf{F} = (300 \text{ kN})(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$$
$$= 134\mathbf{i} + 268\mathbf{j} \text{ (kN)}.$$

Alternatively, the distance from B to C is

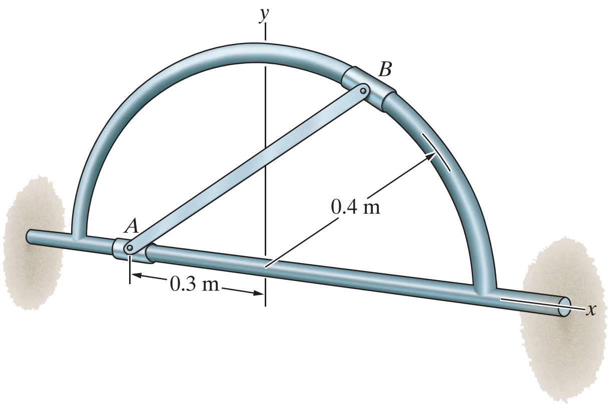
$$\sqrt{(1.2 \text{ m})^2 + (2.4 \text{ m})^2} = 2.68 \text{ m},$$

so

$$\mathbf{F} = \frac{1.2 \text{ m}}{2.68 \text{ m}}(300 \text{ kN})\mathbf{i} + \frac{2.4 \text{ m}}{2.68 \text{ m}}(300 \text{ kN})\mathbf{j}$$
$$= 134\mathbf{i} + 268\mathbf{j} \text{ (kN)}.$$

$\boxed{\mathbf{F} = 134\mathbf{i} + 268\mathbf{j} \text{ (kN)}.}$

Problem 2.38 The length of the bar AB is 0.6 m. Determine the components of a unit vector $\mathbf{e}_{AB}$ that points from point A toward point B.



Solution: We need to find the coordinates of point $B(x, y)$

We have the two equations

$$(0.3\text{ m} + x)^2 + y^2 = (0.6\text{ m})^2$$
$$x^2 + y^2 = (0.4\text{ m})^2$$

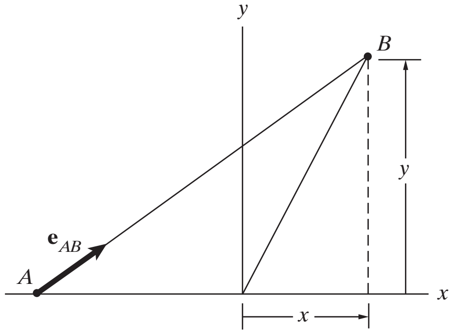
Solving we find

$x = 0.183$ m, $y = 0.356$ m.

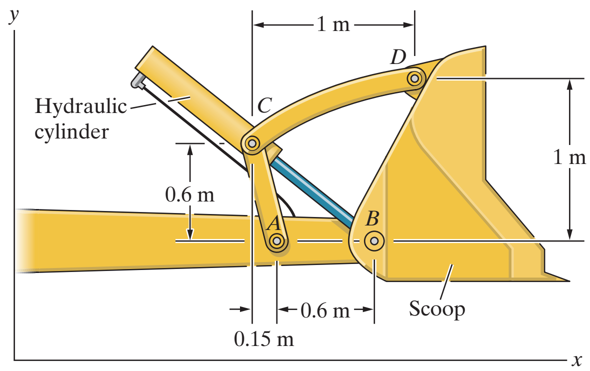
Thus

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = \frac{[(0.183\text{ m}) - (-0.3\text{ m})]\mathbf{i} + (0.356\text{ m})\mathbf{j}}{\sqrt{[(0.183\text{ m}) - (-0.3\text{ m})]^2 + (0.356\text{ m})^2}}$$
$$= 0.806\mathbf{i} + 0.593\mathbf{j}.$$

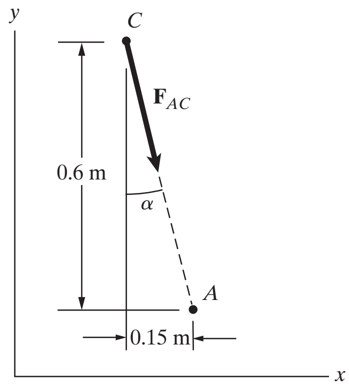
$\boxed{\mathbf{e}_{AB} = 0.806\mathbf{i} + 0.593\mathbf{j}.}$



Problem 2.39 The link AC exerts a force $\mathbf{F}_{AC}$ on the hydraulic cylinder at C. The force points from C toward A, and its magnitude is $|\mathbf{F}_{AC}| = 650$ N. Determine the components of $\mathbf{F}_{AC}$.



Solution:



The angle $\alpha = \arctan(0.15/0.6) = 14.0°$, so

$$\mathbf{F}_{AC} = (650\text{ N})(\sin\alpha\mathbf{i} - \cos\alpha\mathbf{j})$$
$$= 158\mathbf{i} - 631\mathbf{j}\text{ (N)}.$$

Alternatively, the distance from A to C is

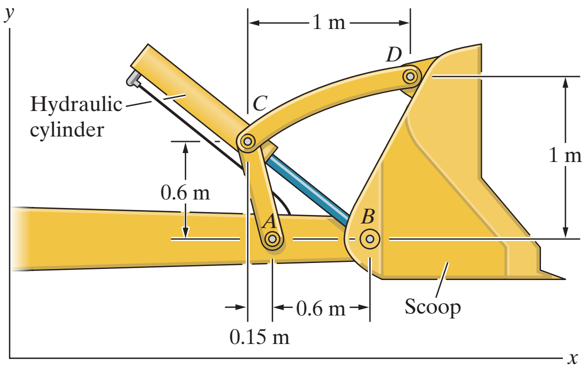
$$\sqrt{(0.15\text{ m})^2 + (0.6\text{ m})^2} = 0.618\text{ m},$$

so

$$\mathbf{F}_{AC} = \frac{0.15\text{ m}}{0.618\text{ m}}(650\text{ N})\mathbf{i} - \frac{0.6\text{ m}}{0.618\text{ m}}(650\text{ N})\mathbf{j}$$
$$= 158\mathbf{i} - 631\mathbf{j}\text{ (N)}.$$

$\boxed{\mathbf{F}_{AC} = 158\mathbf{i} - 631\mathbf{j}\text{ (N)}.}$

Problem 2.40 The link CD exerts a force $\mathbf{F}_{CD}$ on the hydraulic cylinder at C. The force points from C toward D, and its magnitude is $|\mathbf{F}_{CD}| = 600$ N. Determine the components of $\mathbf{F}_{CD}$.



Solution: The angle $\alpha = \arctan(0.4/1.0) = 21.8°$, so

$$\mathbf{F}_{CD} = (600 \text{ N})(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$$
$$= 557\mathbf{i} + 223\mathbf{j} \text{ (N)}.$$

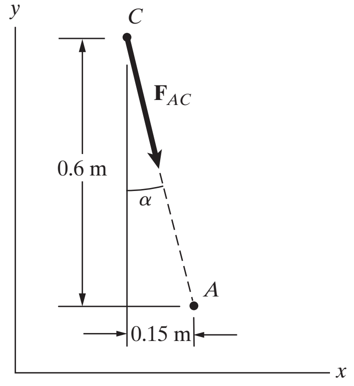
Alternatively, the distance from C to D is

$$\sqrt{(1 \text{ m})^2 + (0.4 \text{ m})^2} = 1.077 \text{ m},$$

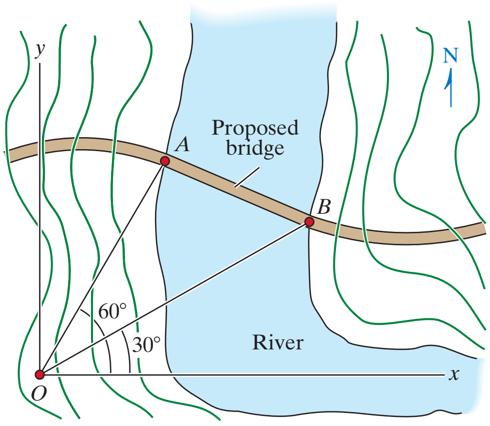
so

$$\mathbf{F}_{CD} = \frac{1.0 \text{ m}}{1.077 \text{ m}}(600 \text{ N})\mathbf{i} + \frac{0.4 \text{ m}}{1.077 \text{ m}}(600 \text{ N})\mathbf{j}$$
$$= 557\mathbf{i} + 223\mathbf{j} \text{ (N)}.$$

$$\boxed{\mathbf{F}_{CD} = 557\mathbf{i} + 223\mathbf{j} \text{ (N)}.}$$



Problem 2.41 A surveyor finds that the length of the line OA is 1500 m and the length of the line OB is 2000 m. (a) Determine the components of the position vector from point A to point B. (b) Determine the components of a unit vector that points from point A toward point B.



Solution: We need to find the coordinates of points A and B

$$\mathbf{r}_{OA} = 1500\cos 60°\mathbf{i} + 1500\sin 60°\mathbf{j}$$
$$\mathbf{r}_{OA} = 750\mathbf{i} + 1299\mathbf{j} \text{ (m)}$$

Point A is at (750, 1299) (m)

$$\mathbf{r}_{OB} = 2000\cos 30°\mathbf{i} + 2000\sin 30°\mathbf{j} \text{ (m)}$$
$$\mathbf{r}_{OB} = 1732\mathbf{i} + 1000\mathbf{j} \text{ (m)}$$

Point B is at (1732, 1000) (m)

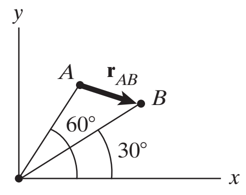
(a) The vector from A to B is

$$\mathbf{r}_{AB} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j}$$
$$\mathbf{r}_{AB} = 982\mathbf{i} - 299\mathbf{j} \text{ (m)}.$$

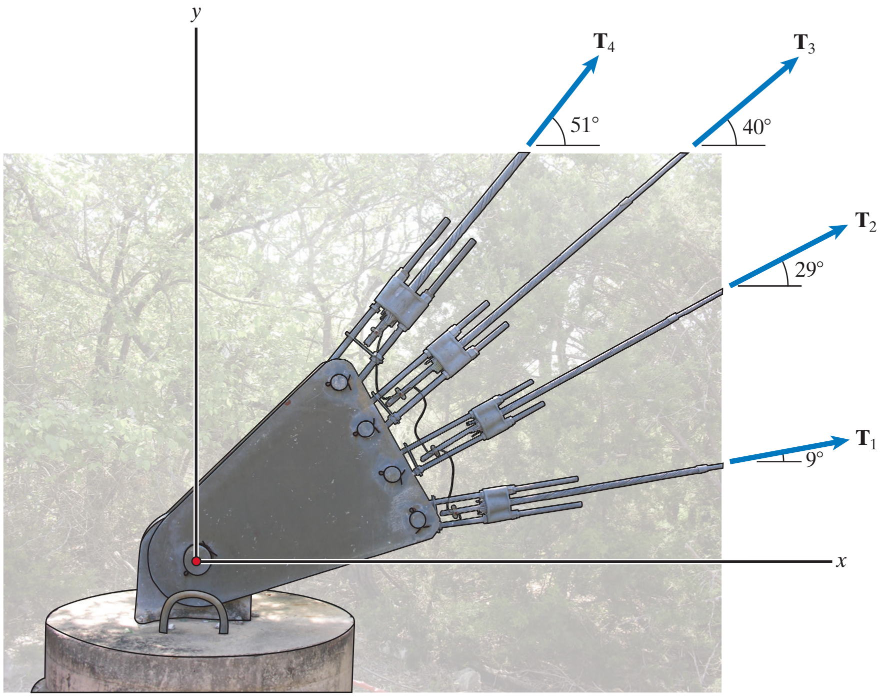
(b) The unit vector $\mathbf{e}_{AB}$ is

$$\mathbf{e}_{AB} = \frac{r_{AB}}{|\mathbf{r}_{AB}|} = \frac{982\mathbf{i} - 299\mathbf{j}}{1026.6}$$
$$\mathbf{e}_{AB} = 0.957\mathbf{i} - 0.291\mathbf{j}.$$

$$\boxed{\text{(a) } \mathbf{r}_{AB} = 982\mathbf{i} - 299\mathbf{j} \text{ (m). (b) } \mathbf{e}_{AB} = 0.957\mathbf{i} - 0.291\mathbf{j}.}$$



Problem 2.42 The magnitudes of the forces exerted by the cables are $|\mathbf{T}_1| = 2800$ lb, $|\mathbf{T}_2| = 3200$ lb, $|\mathbf{T}_3| = 4000$ lb, and $|\mathbf{T}_4| = 5000$ lb. What is the magnitude of the total force exerted by the four cables?



Solution: The x-component of the total force is

$$T_x = |\mathbf{T}_1|\cos 9^\circ + |\mathbf{T}_2|\cos 29^\circ\, |\mathbf{T}_3|\cos 40^\circ + |\mathbf{T}_4|\cos 51^\circ$$
$$T_x = (2800 \text{ lb})\cos 9^\circ + (3200 \text{ lb})\cos 29^\circ + (4000 \text{ lb})\cos 40^\circ + (5000 \text{ lb})\cos 51^\circ$$
$$T_x = 11{,}800 \text{ lb}$$

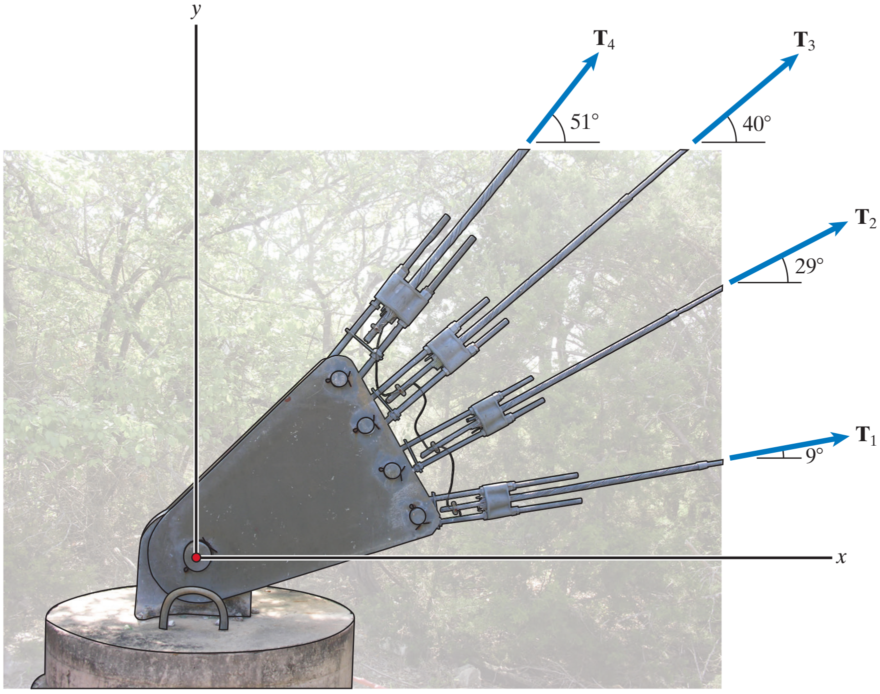
The y-component of the total force is

$$T_y = |\mathbf{T}_1|\sin 9^\circ + |\mathbf{T}_2|\sin 29^\circ + |\mathbf{T}_3|\sin 40^\circ + |\mathbf{T}_4|\sin 51^\circ$$
$$T_y = (2800 \text{ lb})\sin 9^\circ + (3200 \text{ lb})\sin 29^\circ + (4000 \text{ lb})\sin 40^\circ + (5000 \text{ lb})\sin 51^\circ$$
$$T_y = 8450 \text{ lb}$$

The magnitude of the total force is

$$|\mathbf{T}| = \sqrt{T_x^2 + T_y^2} = \sqrt{(11{,}800 \text{ lb})^2 + (8450 \text{ lb})^2} = 14{,}500 \text{ lb} \quad \boxed{|\mathbf{T}| = 14{,}500 \text{ lb}}$$

Problem 2.43 The tensions in the four cables are $|\mathbf{T}_1| = T$, $|\mathbf{T}_2| = 1.5T$, $|\mathbf{T}_3| = 2.5T$, and $|\mathbf{T}_4| = 3T$. Determine the value of T so that the four cables exert a total force of 24 kN magnitude on the support.



Solution: The sum of the x components of the forces is

$$T\cos 9^\circ + 1.5T\cos 29^\circ + 2.5T\cos 40^\circ + 3T\cos 51^\circ = 6.10\,T.$$

The sum of the y components of the forces is

$$T\sin 9^\circ + 1.5T\sin 29^\circ + 2.5T\sin 40^\circ + 3T\sin 51^\circ = 4.82T.$$

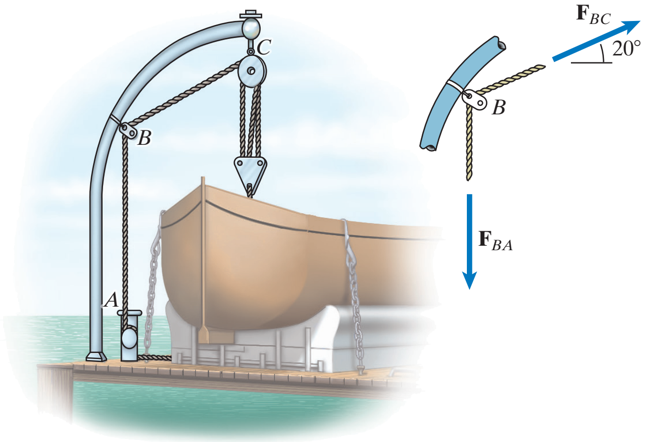
The magnitude of the total force is

$$\sqrt{(6.10T)^2 + (4.82T)^2} = 7.78T.$$

Equating this expression to 24 kN, we obtain

$$\boxed{T = 3.09\text{ kN.}}$$

Problem 2.44 The rope ABC exerts forces $\mathbf{F}_{BA}$ and $\mathbf{F}_{BC}$ on the block at B. Their magnitudes are equal: $|\mathbf{F}_{BA}| = |\mathbf{F}_{BC}|$. The magnitude of the total force exerted on the block at B by the rope is $|\mathbf{F}_{BA} + \mathbf{F}_{BC}| = 920$ N. Determine $|\mathbf{F}_{BA}|$ by expressing the forces $\mathbf{F}_{BA}$ and $\mathbf{F}_{BC}$ in terms of components.



Solution:

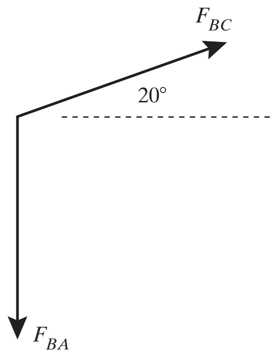
$$\mathbf{F}_{BC} = F(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$$
$$\mathbf{F}_{BA} = F(-\mathbf{j})$$
$$\mathbf{F}_{BC} + \mathbf{F}_{BA} = F(\cos 20°\mathbf{i} + [\sin 20° - 1]\mathbf{j})$$

Therefore

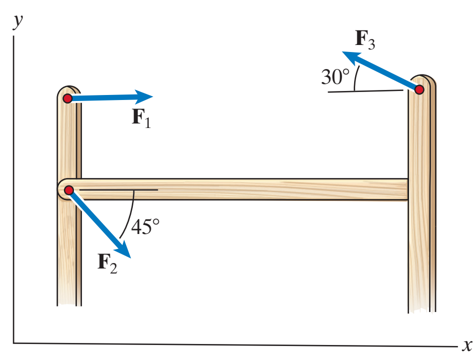
$$(920\text{ N})^2 = F^2[(\cos 20°)^2 + (\sin 20° - 1)^2].$$

Solving, we obtain $F = 802$ N. We see that

$$\boxed{|\mathbf{F}_{BA}| = 802\text{ N.}}$$



Problem 2.45 The magnitude of the horizontal force $\mathbf{F}_1$ is 5 kN and $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$. What are the magnitudes of $\mathbf{F}_2$ and $\mathbf{F}_3$?



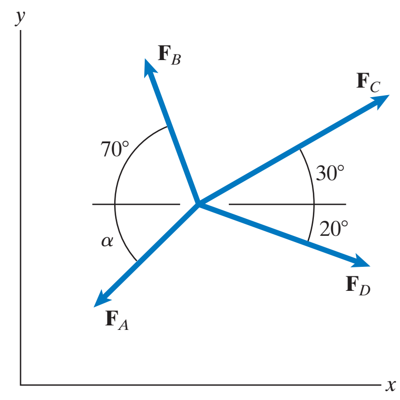
Solution: Using components we have

$$\sum F_x: \quad 5\text{ kN} + F_2\cos 45° - F_3\cos 30° = 0$$
$$\sum F_y: \quad -F_2\sin 45° + F_3\sin 30° = 0$$

Solving simultaneously yields:

$$\Rightarrow \boxed{F_2 = 9.66\text{ kN}, F_3 = 13.66\text{ kN}}$$

Problem 2.46 Four groups engage in a tug-of-war. The magnitudes of the forces exerted by groups B, C, and D are $|\mathbf{F}_B| = 800$ lb, $|\mathbf{F}_C| = 1000$ lb, and $|\mathbf{F}_D| = 900$ lb. If the vector sum of the four forces equals zero, what are the magnitude of $\mathbf{F}_A$ and the angle α?



Solution: The forces in terms of their components are

$$\mathbf{F}_A = |\mathbf{F}_A|(-\cos\alpha\,\mathbf{i} - \sin\alpha\,\mathbf{j}),$$
$$\mathbf{F}_B = (800\text{ lb})(-\cos 70°\,\mathbf{i} + \sin 70°\mathbf{j}),$$
$$\mathbf{F}_C = (1000\text{ lb})(\cos 30°\,\mathbf{i} + \sin 30°\,\mathbf{j}),$$
$$\mathbf{F}_D = (900\text{ lb})(\cos 20°\,\mathbf{i} - \sin 20°\,\mathbf{j}).$$

The sum of the forces is zero, which means that the sums of the x and y components are zero. This yields two equations:

$$-|\mathbf{F}_A|\cos\alpha - (800\text{ lb})\cos 70° + (1000\text{ lb})\cos 30° + (900\text{ lb})\cos 20° = 0,$$
$$-|\mathbf{F}_A|\sin\alpha + (800\text{ lb})\sin 70° + (1000\text{ lb})\sin 30° - (900\text{ lb})\sin 20° = 0,$$

or

$$|\mathbf{F}_A|\cos\alpha = 1440\text{ lb},$$
$$|\mathbf{F}_A|\sin\alpha = 944\text{ lb}.$$

Squaring both sides of these two equations and summing them,

$$|\mathbf{F}_A|^2(\sin^2\alpha + \cos^2\alpha) = |\mathbf{F}_A|^2 = (944\text{ lb})^2 + (1440\text{ lb})^2,$$

We obtain $|\mathbf{F}_A| = 1720$ lb. Then from either of the two equations, we obtain $\alpha = 33.3°$.

$\boxed{|\mathbf{F}_A| = 1720\text{ lb}, \alpha = 33.3°.}$

Problem 2.47 In Example 2.5, suppose that the attachment point of cable A is moved so that the angle between the cable and the wall increases from 40° to 55°. Draw a sketch showing the forces exerted on the hook by the two cables. If you want the total force $\mathbf{F}_A + \mathbf{F}_B$ to have a magnitude of 200 lb and be in the direction perpendicular to the wall, what are the necessary magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$?

Solution: Denote the forces exerted by the cables by $\mathbf{F}_A$ and $\mathbf{F}_B$. We want the sum of the forces to be $\mathbf{F}_A + \mathbf{F}_B = 200\mathbf{i}$ (lb). Expressing $\mathbf{F}_A$ and $\mathbf{F}_B$ in terms of components, we want

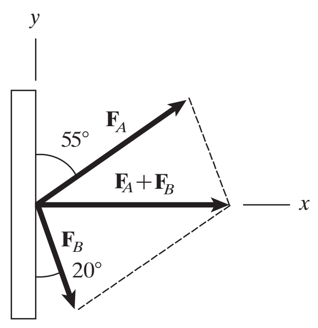
$$|\mathbf{F}_A|\sin 55°\mathbf{i} + |\mathbf{F}_A|\cos 55°\mathbf{j} + |\mathbf{F}_B|\sin 20°\mathbf{i} - |\mathbf{F}_B|\cos 20°\mathbf{j} = 200\mathbf{i}\text{ (lb)}.$$

This yields the two equations

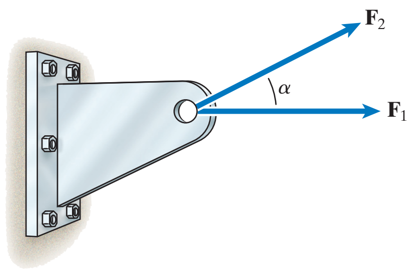
$$|\mathbf{F}_A|\sin 55° + |\mathbf{F}_B|\sin 20° = 200\text{ lb},$$
$$|\mathbf{F}_A|\cos 55° - |\mathbf{F}_B|\cos 20° = 0.$$

Solving, we obtain $|\mathbf{F}_A| = 195$ lb, $|\mathbf{F}_B| = 119$ lb.

$\boxed{|\mathbf{F}_A| = 195\text{ lb}, |\mathbf{F}_B| = 119\text{ lb}.}$



Problem 2.48 The bracket must support the two forces shown, where $|\mathbf{F}_1| = |\mathbf{F}_2| = 2$ kN. An engineer determines that the bracket will safely support a total force of magnitude 3.5 kN in any direction. Assume that $0 \le \alpha \le 90°$. What is the safe range of the angle α?

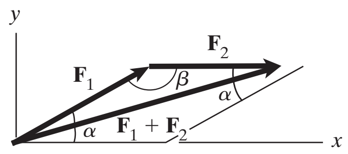


Solution: The sum of the forces is

$$\mathbf{F}_1 + \mathbf{F}_2 = (2 \text{ kN})\mathbf{i} + (2 \text{ kN})\cos\alpha\mathbf{i} + (2 \text{ kN})\sin\alpha\mathbf{j}.$$

The sum Its magnitude is

$$|\mathbf{F}_1 + \mathbf{F}_2| = (2 \text{ kN})\sqrt{(1 + \cos\alpha)^2 + \sin^2\alpha} = (2 \text{ kN})\sqrt{2 + 2\cos\alpha}.$$



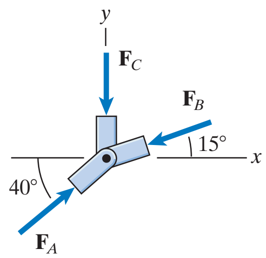
At $\alpha = 0$, the sum $|\mathbf{F}_1 + \mathbf{F}_2| = 4$ kN, exceeding the safe limit. When the angle has increased to $\alpha = 90°$, the sum has decreased to $|\mathbf{F}_1 + \mathbf{F}_2| = (2 \text{ kN})\sqrt{2} = 2.83$ kN, below the limit. At what angle is it equal to the limit? Setting

$$(2 \text{ kN})\sqrt{2 + 2\cos\alpha} = 3.5 \text{ kN}$$

and solving, we obtain $\cos\alpha = 0.531$, which gives $\alpha = 57.9°$. The safe range is

$$\boxed{57.9° \le \alpha \le 90°.}$$

Problem 2.49 The figure shows three forces acting on a joint of a structure. The magnitude of $\mathbf{F}_C$ is 80 kN, and $\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{0}$. What are the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$?



Solution: Denote the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$ by A and B. Writing the vectors in terms of their magnitudes,

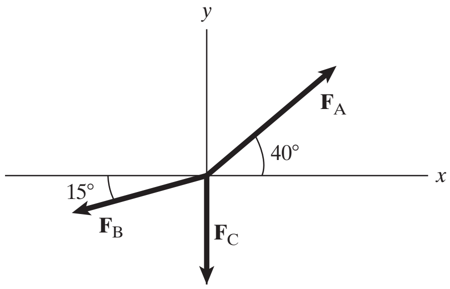
$$\mathbf{F}_A = A\cos 40°\mathbf{i} + A\sin 40°\mathbf{j},$$
$$\mathbf{F}_B = -B\cos 15°\mathbf{i} - B\sin 15°\mathbf{j},$$
$$\mathbf{F}_C = -(80 \text{ kN})\mathbf{j}.$$

We know that

$$\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = (A\cos 40° - B\cos 15°)\mathbf{i} + [A\sin 40° - B\sin 15° - (80 \text{ kN})]\mathbf{j} = \mathbf{0}.$$

If a vector equals zero, each of its components equals zero, so we obtain the two equations

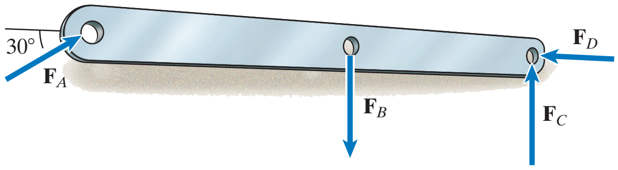
$$A\cos 40° - B\cos 15° = 0,$$
$$A\sin 40° - B\sin 15° - (80 \text{ kN}) = 0.$$



Solving them we obtain $A = |\mathbf{F}_A| = 183$ kN, $B = |\mathbf{F}_B| = 145$ kN.

$$\boxed{|\mathbf{F}_A| = 183 \text{ kN}, |\mathbf{F}_B| = 145 \text{ kN}.}$$

Problem 2.50 Four coplanar forces act on a beam. The forces $\mathbf{F}_B$ and $\mathbf{F}_C$ are vertical. The vector sum of the forces is zero. The magnitudes $|\mathbf{F}_B| = 10$ kN and $|\mathbf{F}_C| = 5$ kN. Determine the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_D$.



Solution: Use the angles and magnitudes to determine the vectors, and then solve for the unknowns. The vectors are:

$$\mathbf{F}_A = |\mathbf{F}_A|(\mathbf{i}\cos 30° + \mathbf{j}\sin 30°) = 0.866|\mathbf{F}_A|\mathbf{i} + 0.5|\mathbf{F}_A|\mathbf{j}$$

$$\mathbf{F}_B = 0\mathbf{i} - 10\mathbf{j},\ \mathbf{F}_C = 0\mathbf{i} + 5\mathbf{j},\ \mathbf{F}_D = -|\mathbf{F}_D|\mathbf{i} + 0\mathbf{j}.$$

Take the sum of each component in the x- and y-directions:

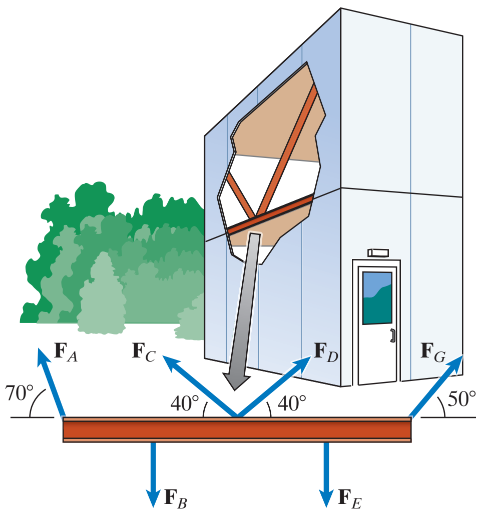
$$\sum \mathbf{F}_x = (0.866|\mathbf{F}_A| - |\mathbf{F}_D|)\mathbf{i} = 0$$

and

$$\sum \mathbf{F}_y = (0.5|\mathbf{F}_A| - (10 - 5))\mathbf{j} = 0.$$

From the second equation we get $\boxed{|\mathbf{F}_A| = 10 \text{ kN}}$. Using this value in the first equation, we get $\boxed{|\mathbf{F}_D| = 8.66 \text{ kN.}}$

Problem 2.51 Six forces act on a beam that forms part of a building's frame. The vector sum of the forces is zero. The magnitudes $|\mathbf{F}_B| = |\mathbf{F}_E| = 20$ kN, $|\mathbf{F}_C| = 16$ kN, and $|\mathbf{F}_D| = 9$ kN. Determine the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_G$.



Solution: Let $F_A, F_B, \ldots$ denote the magnitudes of the forces. The sum of the horizontal components of the forces must equal zero,

$$-F_A\cos 70° - F_C\cos 40° + F_D\cos 40° + F_G\cos 50° = 0,$$

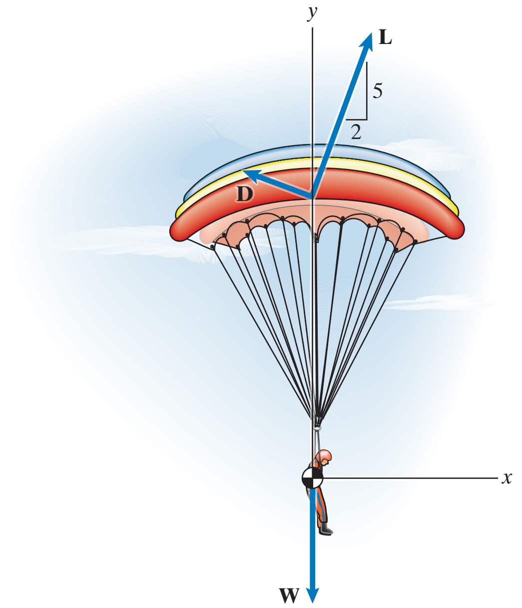
and the sum of the vertical components must equal zero,

$$F_A\sin 70° - F_B + F_C\sin 40° + F_D\sin 40° - F_E + F_G\sin 50° = 0.$$

Substituting the given values of the magnitudes and solving, we obtain $F_A = 13.0$ kN, $F_G = 15.3$ kN.

$\boxed{|\mathbf{F}_A| = 13.0 \text{ kN},\ |\mathbf{F}_G| = 15.3 \text{ kN.}}$

Problem 2.52 The total weight of the man and parasail is $|\mathbf{W}| = 230$ lb. The drag force $\mathbf{D}$ is perpendicular to the lift force $\mathbf{L}$. If the vector sum of the three forces is zero, what are the magnitudes of $\mathbf{L}$ and $\mathbf{D}$?



Solution: Let L and D be the magnitudes of the lift and drag forces. We can use similar triangles to express the vectors $\mathbf{L}$ and $\mathbf{D}$ in terms of components. Then the sum of the forces is zero. Breaking into components we have

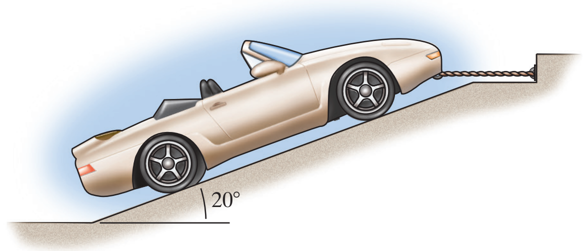
$$\frac{2}{\sqrt{2^2 + 5^2}}L - \frac{5}{\sqrt{2^2 + 5^2}}D = 0$$

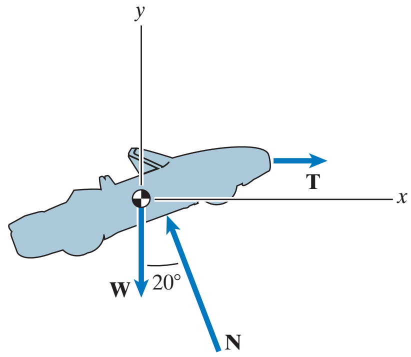
$$\frac{5}{\sqrt{2^2 + 5^2}}L - \frac{2}{\sqrt{2^2 + 5^2}}D - 230 \text{ lb} = 0$$

Solving we find

$$\boxed{|\mathbf{D}| = 85.4 \text{ lb}, |\mathbf{L}| = 214 \text{ lb}}$$

Problem 2.53 The three forces acting on the car are shown. The force $\mathbf{T}$ is parallel to the x-axis and the magnitude of the force $\mathbf{W}$ is 14 kN. If $\mathbf{T} + \mathbf{W} + \mathbf{N} = \mathbf{0}$, what are the magnitudes of the forces $\mathbf{T}$ and $\mathbf{N}$?





Solution: Denote the magnitudes of the forces by T, W, N. The sum of the x components is equal to zero,

$$\Sigma F_x = T - N\sin 20° = 0,$$

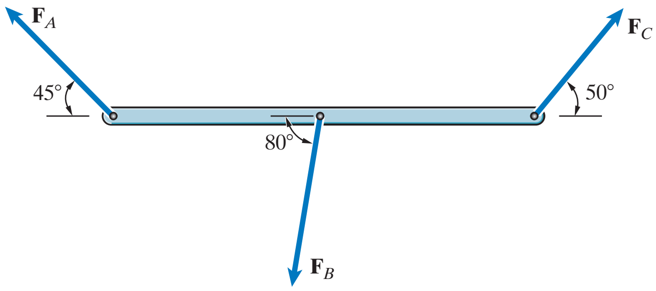
and the sum of the y components is zero,

$$\Sigma F_y = -W + N\cos 20° = 0.$$

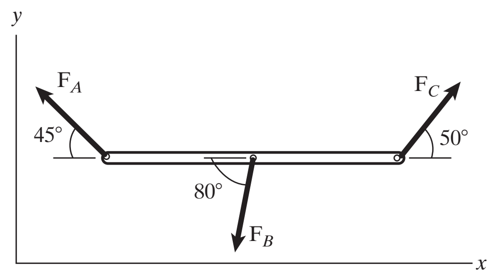
Setting $W = 14$ kN and solving, we obtain $T = 5.10$ N, $N = 14.9$ N.

$$\boxed{|\mathbf{T}| = 5.10 \text{ N}, |\mathbf{N}| = 14.9 \text{ N}.}$$

Problem 2.54 The sum of the three forces acting on the beam is zero: $\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{0}$. The magnitude $|\mathbf{F}_A| = 800$ lb. Determine the magnitudes of $\mathbf{F}_C$ and $\mathbf{F}_B$.



Solution:



Let F_A, F_B, F_C denote the magnitudes of the forces. The sum of the x components of the forces is

$$\Sigma F_x = -F_A \cos 45^\circ - F_B \cos 80^\circ + F_C \cos 50^\circ = 0.$$

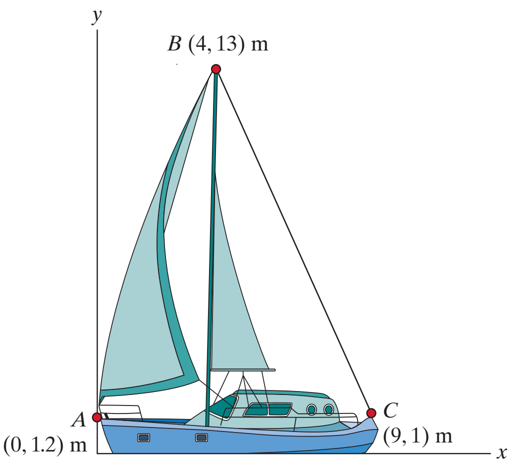
The sum of the x components of the forces is

$$\Sigma F_y = F_A \sin 45^\circ - F_B \sin 80^\circ + F_C \sin 50^\circ = 0.$$

Solving these two equations, we obtain $|\mathbf{F}_B| = 1590$ lb, $|\mathbf{F}_C| = 1310$ lb.

$$\boxed{|\mathbf{F}_B| = 1590 \text{ lb}, |\mathbf{F}_C| = 1310 \text{ lb.}}$$

Problem 2.55 The total force exerted on the top of the mast B by the sailboat's forestay AB and backstay BC is $180\mathbf{i} - 820\mathbf{j}$ (N). What are the magnitudes of the forces exerted at B by the cables AB and BC?



Solution: We first identify the forces:

$$\mathbf{F}_{AB} = T_{AB} \frac{(-4.0 \text{ m}\mathbf{i} - 11.8 \text{ m}\mathbf{j})}{\sqrt{(-4.0 \text{ m})^2 + (11.8 \text{ m})^2}}$$

$$\mathbf{F}_{BC} = T_{BC} \frac{(5.0 \text{ m}\mathbf{i} - 12.0 \text{ m}\mathbf{j})}{\sqrt{(5.0 \text{ m})^2 + (-12.0 \text{ m})^2}}$$

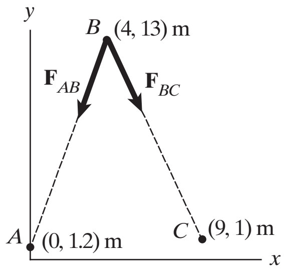
Then if we add the force we find

$$\sum F_x : -\frac{4}{\sqrt{155.24}} T_{AB} + \frac{5}{\sqrt{169}} T_{BC} = 180 \text{ N}$$

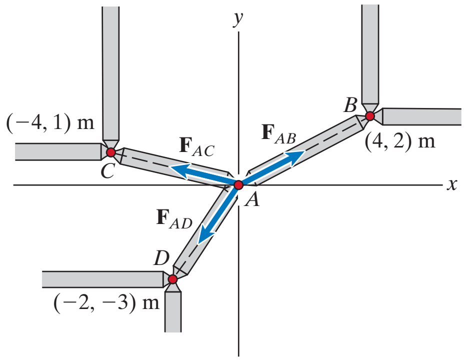
$$\sum F_y : -\frac{11.8}{\sqrt{155.24}} T_{AB} - \frac{12}{\sqrt{169}} T_{BC} = -820 \text{ N}$$

Solving simultaneously yields:

$$\Rightarrow \boxed{T_{AB} = 226 \text{ N}, T_{AC} = 657 \text{ N}}$$



Problem 2.56 The structure shown forms part of a truss designed by an architectural engineer to support the roof of an orchestra shell. The members *AB*, *AC*, and *AD* exert forces $\mathbf{F}_{AB}$, $\mathbf{F}_{AC}$, and $\mathbf{F}_{AD}$ on the joint *A*. The magnitude $|\mathbf{F}_{AB}| = 4$ kN. If the vector sum of the three forces equals zero, what are the magnitudes of $\mathbf{F}_{AC}$ and $\mathbf{F}_{AD}$?



Solution: Determine the unit vectors parallel to each force:

$$\mathbf{e}_{AD} = \frac{-2}{\sqrt{2^2+3^2}}\mathbf{i} + \frac{-3}{\sqrt{2^2+3^2}}\mathbf{j} = -0.5547\mathbf{i} - 0.8320\mathbf{j}$$

$$\mathbf{e}_{AC} = \frac{-4}{\sqrt{4^2+1^2}}\mathbf{i} + \frac{1}{\sqrt{4^2+1^2}}\mathbf{j} = -0.9701\mathbf{i} + 0.2425\mathbf{j}$$

$$\mathbf{e}_{AB} = \frac{4}{\sqrt{4^2+2^2}}\mathbf{i} + \frac{2}{\sqrt{4^2+2^2}}\mathbf{j} = 0.89443\mathbf{i} + 0.4472\mathbf{j}$$

The forces are $\mathbf{F}_{AD} = |\mathbf{F}_{AD}|\mathbf{e}_{AD}$, $\mathbf{F}_{AC} = |\mathbf{F}_{AC}|\mathbf{e}_{AC}$,

$\mathbf{F}_{AB} = |\mathbf{F}_{AB}|\mathbf{e}_{AB} = 3.578\mathbf{i} + 1.789\mathbf{j}$. Since the vector sum of the forces vanishes, the *x*- and *y*-components vanish separately:

$$\sum \mathbf{F}_x = (-0.5547|\mathbf{F}_{AD}| - 0.9701|\mathbf{F}_{AC}| + 3.578)\mathbf{i} = 0,$$

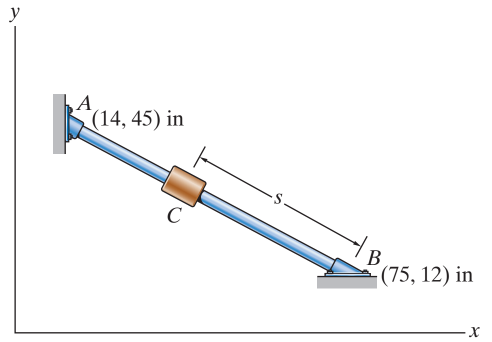
and

$$\sum \mathbf{F}_y = (-0.8320|\mathbf{F}_{AD}| + 0.2425|\mathbf{F}_{AC}| + 1.789)\mathbf{j} = 0$$

These simultaneous equations in two unknowns can be solved by any standard procedure. An HP-28S hand held calculator was used here:

The results: $\boxed{|\mathbf{F}_{AC}| = 2.11 \text{ kN}, |\mathbf{F}_{AD}| = 2.76 \text{ kN}.}$

Problem 2.57 The distance $s = 45$ in. (a) Determine the unit vector $\mathbf{e}_{BA}$ that points from *B* toward *A*. (b) Use the unit vector you obtained in (a) to determine the coordinates of the collar *C*.



Solution:

(a) The position vector from point *B* to point *A* is [Fig. (a)]

$$\mathbf{r}_{BA} = (14 \text{ in} - 75 \text{ in})\mathbf{i} + (45 \text{ in} - 12 \text{ in})\mathbf{j} = -61\mathbf{i} + 33\mathbf{j} \text{ in}.$$

Dividing this vector by its magnitude, we obtain the desired unit vector:

$$\mathbf{e}_{BA} = \frac{\mathbf{r}_{BA}}{|\mathbf{r}_{BA}|} = \frac{-61\mathbf{i} + 33\mathbf{j} \text{ in}}{\sqrt{(-61 \text{ in})^2 + (33 \text{ in})^2}} = -0.880\mathbf{i} + 0.476\mathbf{j}.$$

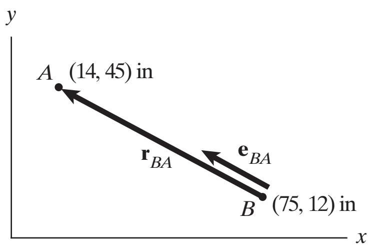


Figure (a)

(b) We can write the position vector from point *B* to the collar *C* [Fig. (b)] as

$$\mathbf{r}_{BC} = s\mathbf{e}_{BA} = (45 \text{ in})(-0.880\mathbf{i} + 0.476\mathbf{j}).$$

The position vector of point *B* relative to the origin is

$$\mathbf{r}_B = 75\mathbf{i} + 12\mathbf{j} \text{ in}.$$

Then the position vector of point *C* relative to the origin is

$$\mathbf{r}_C = \mathbf{r}_B + \mathbf{r}_{BC} = 35.4\mathbf{i} + 33.4\mathbf{j} \text{ in}.$$

$\boxed{(35.4 \text{ in}, 33.4 \text{ in}).}$

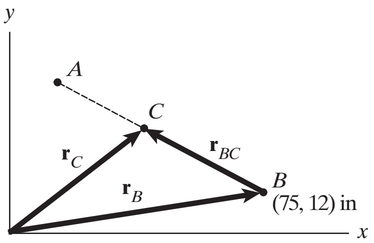
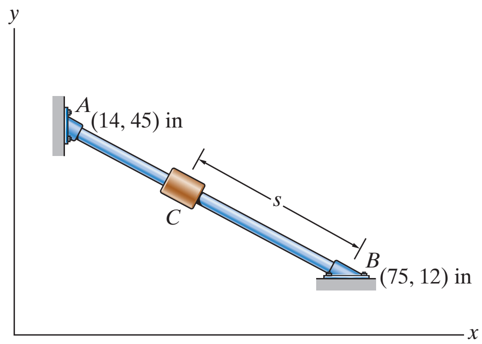


Figure (b)

Problem 2.58 Determine the x and y coordinates of the collar C as functions of the distance s.



Solution:

The position vector from point B to point A is [Fig. (a)]

$$\mathbf{r}_{BA} = (14 \text{ in} - 75 \text{ in})\mathbf{i} + (45 \text{ in} - 12 \text{ in})\mathbf{j} = -61\mathbf{i} + 33\mathbf{j} \text{ in.}$$

Dividing this vector by its magnitude, we obtain a unit vector that points from point B toward point A:

$$\mathbf{e}_{BA} = \frac{\mathbf{r}_{BA}}{|\mathbf{r}_{BA}|} = \frac{-61\mathbf{i} + 33\mathbf{j} \text{ in}}{\sqrt{(-61 \text{ in})^2 + (33 \text{ in})^2}} = -0.880\mathbf{i} + 0.476\mathbf{j}.$$

We can write the position vector from point B to the collar C [Fig. (b)] as

$$\mathbf{r}_{BC} = s\mathbf{e}_{BA} = s(-0.880\mathbf{i} + 0.476\mathbf{j}).$$

The position vector of point B relative to the origin is

$$\mathbf{r}_B = 75\mathbf{i} + 12\mathbf{j} \text{ in.}$$

Then the position vector of point C relative to the origin is

$$\mathbf{r}_C = \mathbf{r}_B + \mathbf{r}_{BC} = (75 - 0.880s)\mathbf{i} + (12 + 0.476s)\mathbf{j} \text{ in.}$$

$$\boxed{x = 75 - 0.880s \text{ in}, \; y = 12 + 0.476s \text{ in.}}$$

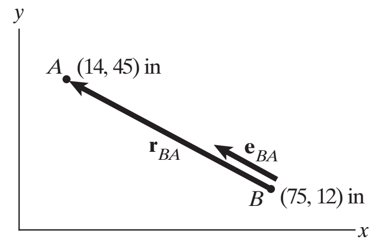


Figure (a)

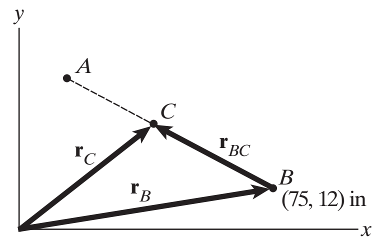


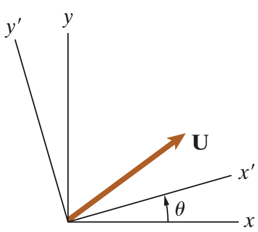
Figure (b)

Problem 2.59 The vector $\mathbf{U}$ can be expressed in terms of its components in terms of either the x–y coordinate system or the x'–y' coordinate system:

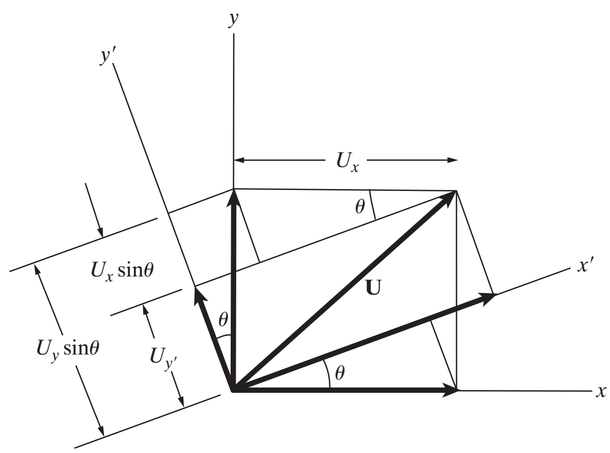
$$\mathbf{U} = U_x\mathbf{i} + U_y\mathbf{j}$$
$$= U_{x'}\mathbf{i}' + U_{y'}\mathbf{j}'.$$

It can be shown that the component $U_{x'}$ is given in terms of the components U_x and U_y and the angle θ by $U_{x'} = U_x\cos\theta + U_y\sin\theta$. Determine the component $U_{y'}$ in terms of the components U_x and U_y and the angle θ.

Strategy: Draw a diagram showing the components U_x, U_y, $U_{x'}$, and $U_{y'}$ and apply trigonometry to determine $U_{y'}$.



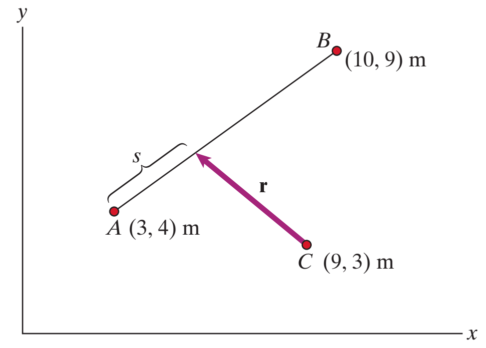
Solution:



From the diagram, $U_{y'} = U_y\cos\theta - U_x\sin\theta$.

$$\boxed{U_{y'} = U_y\cos\theta - U_x\sin\theta.}$$

Problem 2.60 Let $\mathbf{r}$ be the position vector from point C to the point that is a distance s meters from point A along the straight line between A and B. Express $\mathbf{r}$ in terms of components. (Your answer will be in terms of s.)



Solution: First define the unit vector that points from A to B.

$$\mathbf{r}_{B/A} = (10 \text{ m} - 3 \text{ m})\mathbf{i} + (9 \text{ m} - 4 \text{ m})\mathbf{j} = 7\mathbf{i} + 5\mathbf{j} \text{ (m)}.$$

$$|\mathbf{r}_{B/A}| = \sqrt{(7 \text{ m})^2 + (5 \text{ m})^2} = \sqrt{74} \text{ m}$$

$$\mathbf{e}_{B/A} = \frac{1}{\sqrt{74}}(7\mathbf{i} + 5\mathbf{j})$$

Let P be the point that is a distance s along the line from A to B. The coordinates of point P are

$$x_p = 3 \text{ m} + s\left(\frac{7}{\sqrt{74}}\right) = (3 + 0.814s) \text{ m}$$

$$y_p = 4 \text{ m} + s\left(\frac{5}{\sqrt{74}}\right) = (4 + 0.581s) \text{ m}.$$

The vector $\mathbf{r}$ that points from C to P is then

$$\mathbf{r} = [(3 + 0.814s) - 9]\mathbf{i} + [(4 + 0.581s) - 3]\mathbf{j} \text{ (m)}.$$

$$\boxed{\mathbf{r} = (0.814s - 6)\mathbf{i} + (0.581s + 1)\mathbf{j} \text{ (m)}.}$$

Problem 2.61 A vector $\mathbf{U} = 3\mathbf{i} - 4\mathbf{j} - 12\mathbf{k}$. What is its magnitude?

Strategy: The magnitude of a vector is given in terms of its components by Eq. (2.14).

Solution: Use definition given in Eq. (14). The vector magnitude is

$$\boxed{|\mathbf{U}| = \sqrt{3^2 + (-4)^2 + (-12)^2} = 13}$$

Problem 2.62 Two forces are given in terms of their components by $\mathbf{F}_A = 60\mathbf{i} - 20\mathbf{j} + 30\mathbf{k}$ (lb), $\mathbf{F}_B = 30\mathbf{i} + 40\mathbf{j} - 10\mathbf{k}$ (lb). (a) What are the magnitudes of the forces? (b) What is the magnitude of their sum $\mathbf{F}_A + \mathbf{F}_B$?

Strategy: The magnitude of a vector in terms of its components is given by Eq. (2.14).

Solution:

(a) The magnitudes of the vectors are

$$|\mathbf{F}_A| = \sqrt{(60 \text{ lb})^2 + (-20 \text{ lb})^2 + (30 \text{ lb})^2} = 70.0 \text{ lb},$$

$$|\mathbf{F}_B| = \sqrt{(30 \text{ lb})^2 + (40 \text{ lb})^2 + (-10 \text{ lb})^2} = 51.0 \text{ lb}.$$

(b) The sum of the vectors is

$$\mathbf{F}_A + \mathbf{F}_B = 90\mathbf{i} + 20\mathbf{j} + 20\mathbf{k} \text{ (lb)}.$$

The magnitude of the sum is

$$|\mathbf{F}_A + \mathbf{F}_B| = \sqrt{(90 \text{ lb})^2 + (20 \text{ lb})^2 + (20 \text{ lb})^2} = 94.3 \text{ lb}.$$

$$\boxed{\text{(a) } |\mathbf{F}_A| = 70.0 \text{ lb}, |\mathbf{F}_B| = 51.0 \text{ lb. (b) } |\mathbf{F}_A + \mathbf{F}_B| = 94.3 \text{ lb.}}$$

Problem 2.63 Consider three vectors $\mathbf{A} = 12\mathbf{i} + 8\mathbf{j} - 16\mathbf{k}$, $\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$, and $\mathbf{C} = C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k}$. Their sum is zero, $\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{0}$, and the components of **B** and **C** satisfy the relations $B_x = -2B_y$, $B_y = -B_z$, and $C_x = 3C_y$. What are the magnitudes of **B** and **C**?

Solution: The sum of the vectors is zero, which yields the three equations

$$\Sigma F_x = 12 + B_x + C_x = 0,$$
$$\Sigma F_y = 8 + B_y + C_y = 0,$$
$$\Sigma F_z = -16 + B_z + C_z = 0.$$

We also have the three given equations

$$B_x = -2B_y,$$
$$B_y = -B_z,$$
$$C_x = 3C_y.$$

Solving these six equations, we obtain $B_x = 4.8$, $B_y = -2.4$, $B_z = 2.4$, $C_x = -16.8$, $C_y = -5.6$, $C_z = 13.6$.
The magnitudes of the vectors are

$$|\mathbf{B}| = \sqrt{B_x{}^2 + B_y{}^2 + B_z{}^2}$$
$$= \sqrt{(4.8)^2 + (-2.4\text{ lb})^2 + (2.4)^2}$$
$$= 5.88,$$
$$|\mathbf{C}| = \sqrt{C_x{}^2 + C_y{}^2 + C_z{}^2}$$
$$= \sqrt{(-16.8)^2 + (-5.6)^2 + (13.6)^2}$$
$$= 22.3.$$

$$\boxed{|\mathbf{B}| = 5.88,\ |\mathbf{C}| = 22.3.}$$

Problem 2.64 A vector $\mathbf{U} = U_x\mathbf{i} + U_y\mathbf{j} + U_z\mathbf{k}$. Its magnitude $|\mathbf{U}| = 30$. Its components are related by the equations $U_y = -2U_x$ and $U_z = 4U_y$. Determine the components.

Solution: Substitute the relations between the components, determine the magnitude, and solve for the unknowns. Thus

$$\mathbf{U} = U_x\mathbf{i} + (-2U_x)\mathbf{j} + (4(-2U_x))\mathbf{k} = U_x(1\mathbf{i} - 2\mathbf{j} - 8\mathbf{k})$$

where U_x can be factored out since it is a scalar. Take the magnitude, noting that the absolute value of $|U_x|$ must be taken:

$$30 = |U_x|\sqrt{1^2 + 2^2 + 8^2} = |U_x|(8.31).$$

Solving, we get $|U_x| = 3.612$, or $U_x = \pm 3.61$. The two possible vectors are

$$\mathbf{U} = +3.61\mathbf{i} + (-2(3.61))\mathbf{j} + (4(-2)(3.61))\mathbf{k}$$
$$\boxed{= 3.61\mathbf{i} - 7.22\mathbf{j} - 28.9\mathbf{k}}$$
$$\mathbf{U} = -3.61\mathbf{i} + (-2(-3.61)\mathbf{j} + 4(-2)(-3.61)\mathbf{k}$$
$$\boxed{= -3.61\mathbf{i} + 7.22\mathbf{j} + 28.9\mathbf{k}}$$

Problem 2.65 An object is acted on by two forces $\mathbf{F}_1 = 20\mathbf{i} + 30\mathbf{j} - 24\mathbf{k}$ (kN) and $\mathbf{F}_2 = -60\mathbf{i} + 20\mathbf{j} + 40\mathbf{k}$ (kN). What is the magnitude of the total force acting on the object?

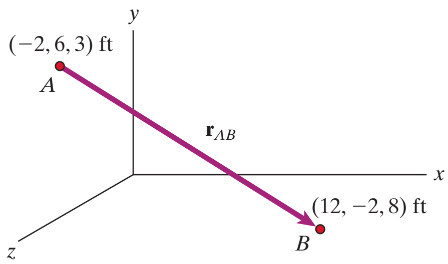
Solution: The sum of the forces is

$$\mathbf{F}_1 + \mathbf{F}_2 = (20\text{ kN} - 60\text{ kN})\mathbf{i} + (30\text{ kN} + 20\text{ kN})\mathbf{j}$$
$$+ (-24\text{ kN} + 40\text{ kN})\mathbf{k}$$
$$= -40\mathbf{i} + 50\mathbf{j} + 16\mathbf{k}\text{ (kN)},$$

So $|\mathbf{F}_1 + \mathbf{F}_2| = \sqrt{(-40\text{ kN})^2 + (50\text{ kN})^2 + (16\text{ kN})^2} = 66$ kN.

$\boxed{66\text{ kN.}}$

Problem 2.66 The position vector $\mathbf{r}_{AB}$ extends from point A to point B. (a) Express $\mathbf{r}_{AB}$ in terms of components. (b) Determine its magnitude $|\mathbf{r}_{AB}|$.



Solution:

(a) The components of $\mathbf{r}_{AB}$ are

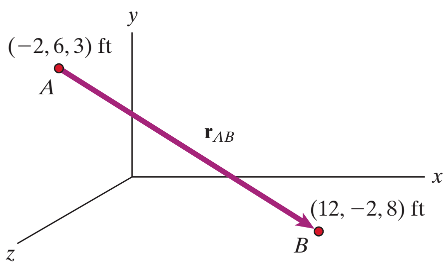
$$\mathbf{r}_{AB} = (12\text{ ft} - (-2\text{ ft}))\mathbf{i} + (-2\text{ ft} - 6\text{ ft})\mathbf{j} + (8\text{ ft} - 3\text{ ft})\mathbf{k}$$
$$= 14\mathbf{i} - 8\mathbf{j} + 5\mathbf{k}\ (\text{ft}).$$

(b) Its magnitude is

$$|\mathbf{r}_{AB}| = \sqrt{(14\text{ ft})^2 + (-8\text{ ft})^2 + (5\text{ ft})^2}$$
$$= 16.9\text{ ft}.$$

$\boxed{\text{(a) } \mathbf{r}_{AB} = 14\mathbf{i} - 8\mathbf{j} + 5\mathbf{k}\ (\text{ft}).\ \text{(b) } |\mathbf{r}_{AB}| = 16.9\text{ ft}.}$

Problem 2.67 The position vector $\mathbf{r}_{AB}$ extends from point A to point B. (a) Determine the direction cosines of $\mathbf{r}_{AB}$. (b) Determine the components of a unit vector that has the same direction as $\mathbf{r}_{AB}$.



Solution:

(a) The components of $\mathbf{r}_{AB}$ are

$$\mathbf{r}_{AB} = (12\text{ ft} - (-2\text{ ft}))\mathbf{i} + (-2\text{ ft} - 6\text{ ft})\mathbf{j} + (8\text{ ft} - 3\text{ ft})\mathbf{k}$$
$$= 14\mathbf{i} - 8\mathbf{j} + 5\mathbf{k}\ (\text{ft}),$$

and its magnitude is

$$|\mathbf{r}_{AB}| = \sqrt{(14\text{ ft})^2 + (-8\text{ ft})^2 + (5\text{ ft})^2}$$
$$= 16.9\text{ ft}.$$

Its direction cosines are

$$\cos\theta_x = \frac{14\text{ ft}}{16.9\text{ ft}} = 0.829,$$
$$\cos\theta_y = \frac{-8\text{ ft}}{16.9\text{ ft}} = -0.474,$$
$$\cos\theta_z = \frac{5\text{ ft}}{16.9\text{ ft}} = 0.296.$$

(b) The direction cosines are the components of the desired unit vector. (Notice that the direction cosines are the components of $\mathbf{r}_{AB}$ divided by its magnitude.) Therefore,

$$\mathbf{e} = 0.829\mathbf{i} - 0.474\mathbf{j} + 0.296\mathbf{k}.$$

$\boxed{\begin{array}{l}\text{(a) } \cos\theta_x = 0.829,\ \cos\theta_y = -0.474,\ \cos\theta_z = 0.296.\\ \text{(b) } \mathbf{e} = 0.829\mathbf{i} - 0.474\mathbf{j} + 0.296\mathbf{k}.\end{array}}$

Problem 2.68 A force vector is given in terms of its components by $\mathbf{F} = 10\mathbf{i} - 20\mathbf{j} - 20\mathbf{k}$ (N). (a) What are the direction cosines of $\mathbf{F}$? (b) Determine the components of a unit vector $\mathbf{e}$ that has the same direction as $\mathbf{F}$.

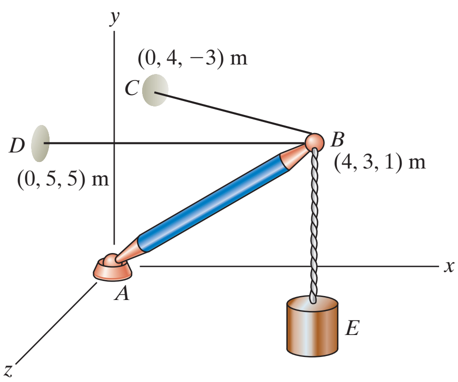
Solution:

$$\mathbf{F} = (10\mathbf{i} - 20\mathbf{j} - 20\mathbf{k})\text{ N}$$

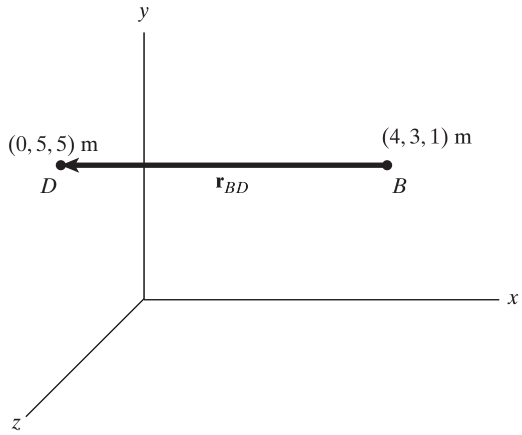
$$F = \sqrt{(10\text{ N})^2 + (-20\text{ N})^2 + (-20\text{ N})^2} = 30\text{ N}$$

$$\boxed{\begin{array}{l}\text{(a) } \cos\theta_x = 0.333,\ \cos\theta_y = -0.667,\ \cos\theta_z = -0.667. \\ \text{(b) } \mathbf{e} = 0.333\mathbf{i} - 0.667\mathbf{j} - 0.667\mathbf{k}.\end{array}}$$

Problem 2.69 Express the position vector $\mathbf{r}_{BD}$ from point B to point D in terms of components. What are its direction cosines?



Solution:



The vector $\mathbf{r}_{BD}$ from point B to point D is

$$\begin{aligned}\mathbf{r}_{BD} &= (0 - 4\text{ m})\mathbf{i} + (5\text{ m} - 3\text{ m})\mathbf{j} + (5\text{ m} - 1\text{ m}) \\ &= -4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}\text{ (m)}.\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{BD}| &= \sqrt{(-4\text{ m})^2 + (2\text{ m})^2 + (4\text{ m})^2} \\ &= 6\text{ m}.\end{aligned}$$

Its direction cosines are

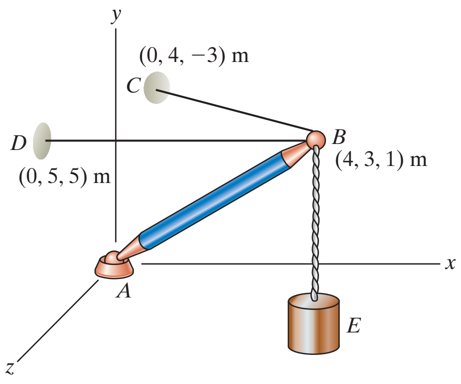
$$\cos\theta_x = \frac{-4\text{ m}}{6\text{ m}} = -0.667,$$

$$\cos\theta_y = \frac{2\text{ m}}{6\text{ m}} = 0.333,$$

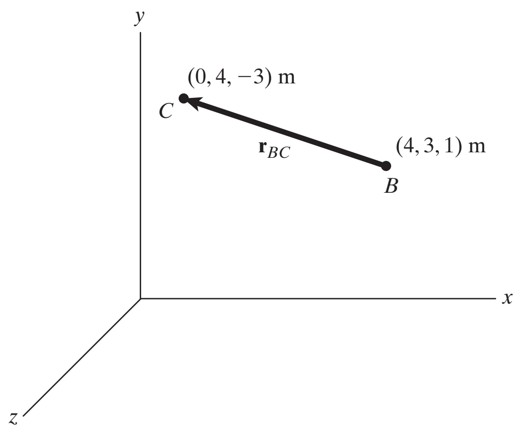
$$\cos\theta_z = \frac{4\text{ m}}{6\text{ m}} = 0.667.$$

$$\boxed{\begin{array}{l}\mathbf{r}_{BD} = -4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}\text{ (m)},\ \cos\theta_x = -0.667, \\ \cos\theta_y = 0.333,\ \cos\theta_z = 0.667.\end{array}}$$

Problem 2.70 The cable from B to C exerts a force on the bar AB at B. The force points from B toward C and its magnitude is 80 N. Express the force in terms of components.



Solution:



The vector $\mathbf{r}_{BC}$ from point B to point C is

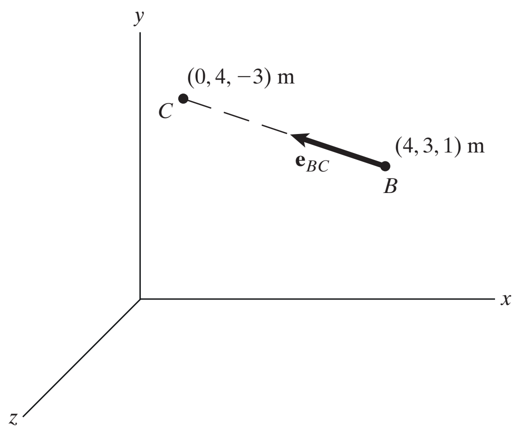
$$\begin{aligned}\mathbf{r}_{BC} &= (0 - 4\text{ m})\mathbf{i} + (4\text{ m} - 3\text{ m})\mathbf{j} + (-3\text{ m} - 1\text{ m}) \\ &= -4\mathbf{i} + \mathbf{j} - 4\mathbf{k}\ (\text{m}).\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{r}_{BC}| &= \sqrt{(-4\text{ m})^2 + (1\text{ m})^2 + (-4\text{ m})^2} \\ &= 5.74\text{ m}.\end{aligned}$$

By dividing the vector by its magnitude, we obtain a unit vector that points from point B toward point C:

$$\begin{aligned}\mathbf{e}_{BC} &= \frac{-4\mathbf{i} + \mathbf{j} - 4\mathbf{k}\ (\text{m})}{5.74\text{ m}} \\ &= -0.696\mathbf{i} + 0.174\mathbf{j} - 0.696\mathbf{k}.\end{aligned}$$

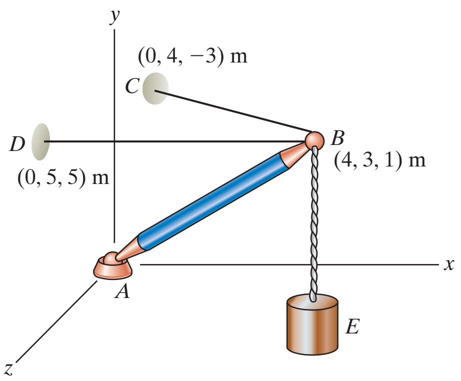


We obtain the force in terms of its components by multiplying the unit vector by 80 N:

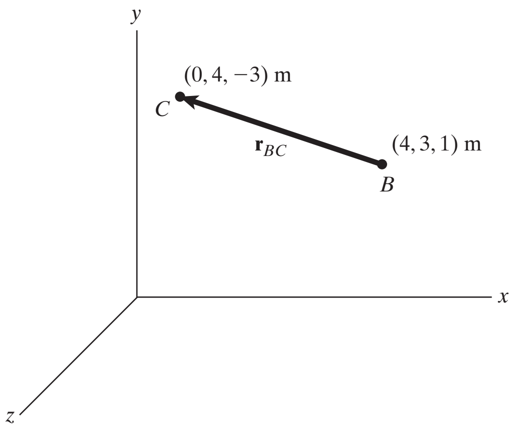
$$(80\text{ N})\mathbf{e}_{BC} = -0.55.7\mathbf{i} + 13.9\mathbf{j} - 55.7\mathbf{k}\ (\text{N}).$$

$$\boxed{-0.55.7\mathbf{i} + 13.9\mathbf{j} - 55.7\mathbf{k}\ (\text{N}).}$$

Problem 2.71* The suspended weight E exerts an 800-N downward force on bar AB at B. The cable from B to C exerts a force on the bar at B that points from B toward C, but its magnitude is unknown. The cable from B to D exerts a force on the bar that points from B toward D, and its magnitude is also unknown. If the weight of bar AB is negligible, the sum of the three forces exerted on it at B is a vector that is parallel to the bar. Use this condition to determine the magnitudes of the forces exerted by cables BC and BD.



Solution:



The vector $\mathbf{r}_{BC}$ from point B to point C is

$$\begin{aligned}\mathbf{r}_{BC} &= (0 - 4\text{ m})\mathbf{i} + (4\text{ m} - 3\text{ m})\mathbf{j} + (-3\text{ m} - 1\text{ m})\\ &= -4\mathbf{i} + \mathbf{j} - 4\mathbf{k}\text{ (m)}.\end{aligned}$$

Its magnitude is

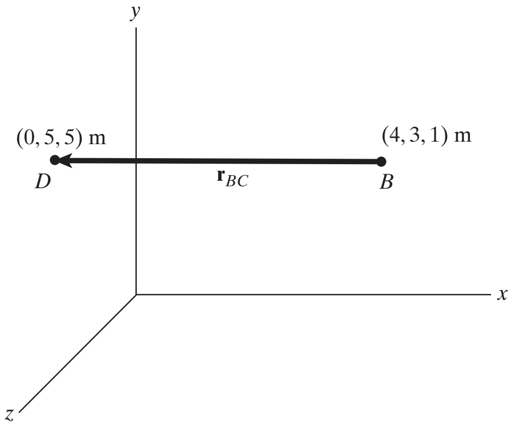
$$\begin{aligned}|\mathbf{r}_{BC}| &= \sqrt{(-4\text{ m})^2 + (1\text{ m})^2 + (-4\text{ m})^2}\\ &= 5.74\text{ m}.\end{aligned}$$

By dividing $\mathbf{r}_{BC}$ by its magnitude, we obtain a unit vector that points from point B toward point C:

$$\begin{aligned}\mathbf{e}_{BC} &= \frac{-4\mathbf{i} + \mathbf{j} - 4\mathbf{k}\text{ (m)}}{5.74\text{ m}}\\ &= -0.696\mathbf{i} + 0.174\mathbf{j} - 0.696\mathbf{k}.\end{aligned}$$

Let $\mathbf{F}_{BC}$ be the force exerted at B by the cable BC. It can be expressed as

$$\begin{aligned}\mathbf{F}_{BC} &= |\mathbf{F}_{BC}|\mathbf{e}_{BC}\\ &= |\mathbf{F}_{BC}|(-0.696\mathbf{i} + 0.174\mathbf{j} - 0.696\mathbf{k}).\end{aligned}$$



The vector $\mathbf{r}_{BD}$ from point B to point D is

$$\begin{aligned}\mathbf{r}_{BD} &= (0 - 4\text{ m})\mathbf{i} + (5\text{ m} - 3\text{ m})\mathbf{j} + (5\text{ m} - 1\text{ m})\mathbf{k}\\ &= -4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}\text{ (m)}.\end{aligned}$$

Its magnitude is

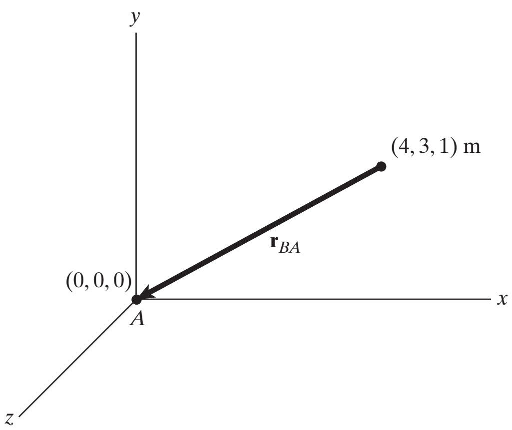
$$\begin{aligned}|\mathbf{r}_{BD}| &= \sqrt{(-4\text{ m})^2 + (2\text{ m})^2 + (4\text{ m})^2}\\ &= 6\text{ m}.\end{aligned}$$

Dividing $\mathbf{r}_{BD}$ by its magnitude, we obtain a unit vector that points from point B toward point D:

$$\begin{aligned}\mathbf{e}_{BD} &= \frac{-4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}\text{ (m)}}{6\text{ m}}\\ &= -0.667\mathbf{i} + 0.333\mathbf{j} + 0.667\mathbf{k}.\end{aligned}$$

Let $\mathbf{F}_{BD}$ be the force exerted at B by the cable BD. It can be expressed as

$$\begin{aligned}\mathbf{F}_{BD} &= |\mathbf{F}_{BD}|\mathbf{e}_{BD}\\ &= |\mathbf{F}_{BD}|(-0.667\mathbf{i} + 0.333\mathbf{j} + 0.667\mathbf{k}).\end{aligned}$$



The vector $\mathbf{r}_{BA}$ from point B to point A is

$$\begin{aligned}\mathbf{r}_{BA} &= (0 - 4\text{ m})\mathbf{i} + (0 - 3\text{ m})\mathbf{j} + (0 - 1\text{ m})\mathbf{k}\\ &= -4\mathbf{i} - 3\mathbf{j} - \mathbf{k}\text{ (m)}.\end{aligned}$$

2.71 (*Continued*)

Its magnitude is

$$|\mathbf{r}_{BA}| = \sqrt{(-4\text{ m})^2 + (-3\text{ m})^2 + (-1\text{ m})^2} = 5.10\text{ m}.$$

Dividing $\mathbf{r}_{BA}$ by its magnitude, we obtain a unit vector that points from point B toward point A:

$$\mathbf{e}_{BA} = \frac{-4\mathbf{i} - 3\mathbf{j} - \mathbf{k}\ (\text{m})}{5.10\text{ m}} = -0.784\mathbf{i} - 0.588\mathbf{j} - 0.196\mathbf{k}.$$

We can write the given condition on the forces at B as

$$\mathbf{F}_{BC} + \mathbf{F}_{BD} - (800\text{ N})\mathbf{j} = F\mathbf{e}_{BA},$$

where F is an unknown constant. Notice that this condition yields three equations. Its $\mathbf{i}$ component is

$$-0.696|\mathbf{F}_{BC}| - 0.667|\mathbf{F}_{BD}| = -0.784F,$$

its $\mathbf{j}$ component is

$$0.174|\mathbf{F}_{BC}| + 0.333|\mathbf{F}_{BD}| - 800\text{ N} = -0.588F,$$

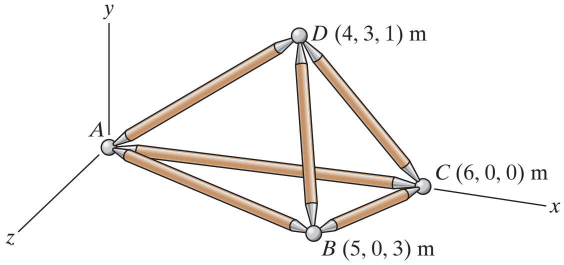
and its $\mathbf{k}$ component is

$$-0.696|\mathbf{F}_{BC}| + 0.667|\mathbf{F}_{BD}| = -0.196F.$$

Solving these three equations yields $|\mathbf{F}_{BC}| = 657$ N, $|\mathbf{F}_{BD}| = 411$ N, and $F = 932$ N.

$\boxed{BC: 657\text{ N}, BD: 411\text{ N}.}$

Problem 2.72 Determine the components of the position vector $\mathbf{r}_{BD}$ from point B to point D. Use your result to determine the distance from B to D.



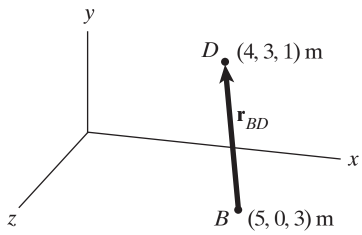
Solution: The position vector from point B to point D is

$$\mathbf{r}_{BD} = (4\text{ m} - 5\text{ m})\mathbf{i} + (3\text{ m} - 0)\mathbf{j} + (1\text{ m} - 3\text{ m})\mathbf{k} = -\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\ (\text{m}).$$

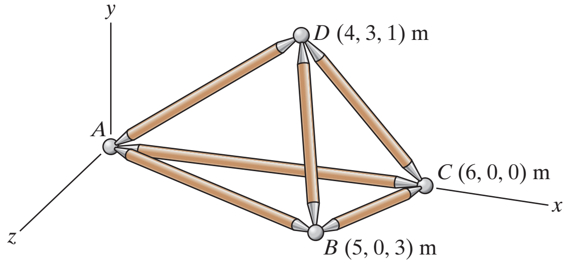
The distance from B to D is the magnitude

$$|\mathbf{r}_{BD}| = \sqrt{(-1\text{ m})^2 + (3\text{ m})^2 + (-2\text{ m})^2} = 3.74\text{ m}.$$

$\boxed{3.74\text{ m}.}$



Problem 2.73 What are the direction cosines of the position vector $\mathbf{r}_{BD}$ from point B to point D?



Solution: The position vector from point B to point D is

$$\mathbf{r}_{BD} = (4\text{ m} - 5\text{ m})\mathbf{i} + (3\text{ m} - 0)\mathbf{j} + (1\text{ m} - 3\text{ m})\mathbf{k} = -\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\ (\text{m}).$$

Its magnitude is

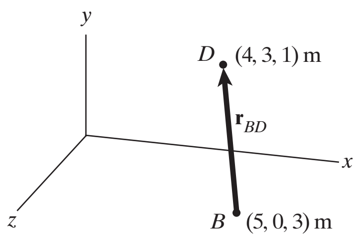
$$|\mathbf{r}_{BD}| = \sqrt{(-1\text{ m})^2 + (3\text{ m})^2 + (-2\text{ m})^2} = 3.74\text{ m}.$$

Using the definitions of the direction cosines,

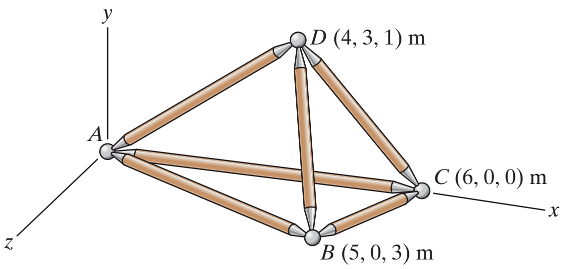
$$\cos\theta_x = \frac{-1\text{ m}}{3.74\text{ m}} = -0.267, \quad \cos\theta_y = \frac{3\text{ m}}{3.74\text{ m}} = 0.802,$$

$$\cos\theta_z = \frac{-2\text{ m}}{3.74\text{ m}} = -0.535.$$

$\boxed{\cos\theta_x = -0.267, \quad \cos\theta_y = 0.802, \quad \cos\theta_z = -0.535.}$



Problem 2.74 Determine the components of the unit vector $\mathbf{e}_{CD}$ that points from point C toward point D.



Solution: The position vector from point C to point D is

$$\mathbf{r}_{CD} = (4\text{ m} - 6\text{ m})\mathbf{i} + (3\text{ m} - 0)\mathbf{j} + (1\text{ m} - 0)\mathbf{k}$$
$$= -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}\ (\text{m}).$$

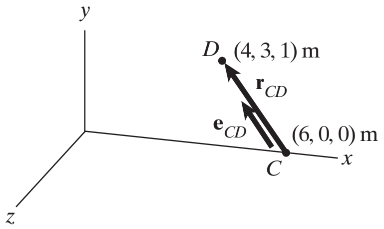
Its magnitude is

$$|\mathbf{r}_{CD}| = \sqrt{(-2\text{ m})^2 + (3\text{ m})^2 + (1\text{ m})^2} = 3.74\text{ m}.$$

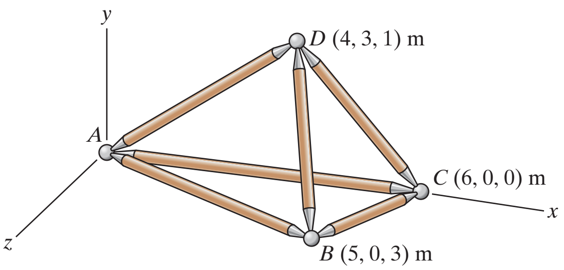
We obtain the desired unit vector by dividing this vector by its magnitude:

$$e_{CD} = \frac{\mathbf{r}_{CD}}{|\mathbf{r}_{CD}|} = -0.535\mathbf{i} + 0.802\mathbf{j} + 0.267\mathbf{k}.$$

$$\boxed{\mathbf{e}_{CD} = -0.535\mathbf{i} + 0.802\mathbf{j} + 0.267\mathbf{k}.}$$



Problem 2.75 What are the direction cosines of the unit vector $\mathbf{e}_{CD}$ that points from point C toward point D?



Solution: Because the magnitude of a unit vector is 1, we can see from the definitions of the direction cosines that the direction cosines of a unit vector are its components. The position vector from point C to point D is

$$\mathbf{r}_{CD} = (4\text{ m} - 6\text{ m})\mathbf{i} + (3\text{ m} - 0)\mathbf{j} + (1\text{ m} - 0)\mathbf{k}$$
$$= -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}\ (\text{m}).$$

Its magnitude is

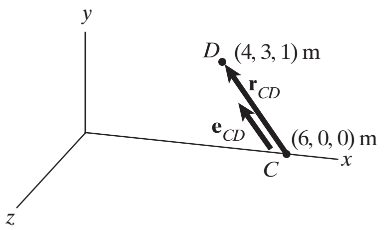
$$|\mathbf{r}_{CD}| = \sqrt{(-2\text{ m})^2 + (3\text{ m})^2 + (1\text{ m})^2} = 3.74\text{ m}.$$

We obtain the unit vector by dividing this vector by its magnitude:

$$e_{CD} = \frac{\mathbf{r}_{CD}}{|\mathbf{r}_{CD}|} = -0.535\mathbf{i} + 0.802\mathbf{j} + 0.267\mathbf{k}.$$

We see that $\cos\theta_x = -0.535,\ \cos\theta_y = 0.802,\ \cos\theta_z = 0.267.$

$$\boxed{\cos\theta_x = -0.535,\ \cos\theta_y = 0.802,\ \cos\theta_z = 0.267.}$$



Problem 2.76 In Example 2.7, suppose that the caisson shifts on the ground to a new position. The magnitude of the force **F** remains 600 lb. In the new position, the angle between the force **F** and the x-axis is 60° and the angle between **F** and the z-axis is 70°. Express **F** in terms of components.

Solution: We need to find the angle θ_y between the force **F** and the y-axis. We know that

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos\theta_y = \pm\sqrt{1 - \cos^2\theta_x - \cos^2\theta_z} = \pm\sqrt{1 - \cos^2 60° - \cos^2 70°} = \pm 0.7956$$

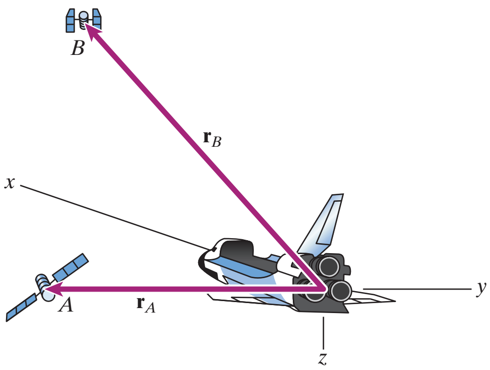
$$\theta_y = \pm\cos^{-1}(0.7956) = 37.3° \text{ or } 142.7°$$

We will choose $\theta_y = 37.3°$ because the picture shows the force pointing up. Now

$$F_x = (600 \text{ lb})\cos 60° = 300 \text{ lb}$$
$$F_y = (600 \text{ lb})\cos 37.3° = 477 \text{ lb}$$
$$F_z = (600 \text{ lb})\cos 70° = 205 \text{ lb}$$

Thus $\boxed{\mathbf{F} = 300\mathbf{i} + 477\mathbf{j} + 205\mathbf{k} \text{ (lb)}.}$

Problem 2.77 Astronauts on the space shuttle use radar to determine the magnitudes and direction cosines of the position vectors of two satellites A and B. The vector $\mathbf{r}_A$ from the shuttle to satellite A has magnitude 2 km and direction cosines $\cos\theta_x = 0.768$, $\cos\theta_y = 0.384$, $\cos\theta_z = 0.512$. The vector $\mathbf{r}_B$ from the shuttle to satellite B has magnitude 4 km and direction cosines $\cos\theta_x = 0.743$, $\cos\theta_y = 0.557$, $\cos\theta_z = -0.371$. What is the distance between the satellites?



Solution: The two position vectors are:

$$\mathbf{r}_A = 2(0.768\mathbf{i} + 0.384\mathbf{j} + 0.512\mathbf{k}) = 1.536\mathbf{i} + 0.768\mathbf{j} + 1.024\mathbf{k} \text{ (km)}$$

$$\mathbf{r}_B = 4(0.743\mathbf{i} + 0.557\mathbf{j} - 0.371\mathbf{k}) = 2.972\mathbf{i} + 2.228\mathbf{j} - 1.484\mathbf{k} \text{ (km)}$$

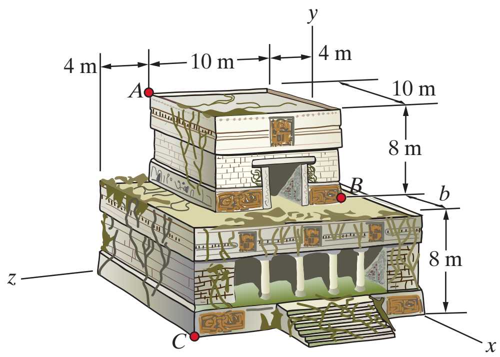
The distance is the magnitude of the difference:

$$|\mathbf{r}_A - \mathbf{r}_B|$$

$$= \sqrt{(1.536 - 2.927)^2 + (0.768 - 2.228)^2 + (1.024 - (-1.484))^2}$$

$$\boxed{= 3.24 \text{ (km)}}$$

Problem 2.78 Archaeologists measure a pre-Columbian ceremonial structure and obtain the dimensions shown. Determine (a) the magnitude and (b) the direction cosines of the position vector from point A to point B.



Solution:

(a) The coordinates are A (0, 16, 14) m and B (10, 8, 4) m.

$$\mathbf{r}_{AB} = ([10 - 0]\mathbf{i} + [8 - 16]\mathbf{j} + [4 - 14]\mathbf{k})\text{ m}$$
$$= (10\mathbf{i} - 8\mathbf{j} - 10\mathbf{k})\text{ m}$$

$$|\mathbf{r}_{AB}| = \sqrt{10^2 + 8^2 + 10^2}\text{ m} = \sqrt{264}\text{ m} = 16.2\text{ m}$$

$$\boxed{|\mathbf{r}_{AB}| = 16.2\text{ m}}$$

(b)
$$\boxed{\begin{aligned}\cos\theta_x &= \frac{10}{\sqrt{264}} = 0.615\\ \cos\theta_y &= \frac{-8}{\sqrt{264}} = -0.492\\ \cos\theta_z &= \frac{-10}{\sqrt{264}} = -0.615\end{aligned}}$$

Problem 2.79 Consider the structure described in Problem 2.78. After returning to the United States, an archaeologist discovers that a graduate student has erased the only data file containing the dimension b. But from recorded GPS data he is able to calculate that the distance from point B to point C is 16.61 m. (a) What is the distance b? (b) Determine the direction cosines of the position vector from B to C.

Solution: We have the coordinates B (10 m, 8 m, 4 m), C (10 m + b, 0 18 m).

$$\mathbf{r}_{BC} = (10\text{ m} + b - 10\text{ m})\mathbf{i} + (0 - 8\text{ m})\mathbf{j} + (18\text{ m} - 4\text{ m})\mathbf{k}$$

$$\mathbf{r}_{BC} = (b)\mathbf{i} + (-8\text{ m})\mathbf{j} + (14\text{ m})\mathbf{k}$$

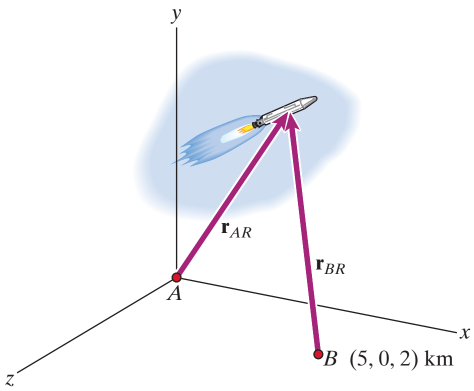
(a) We have $(16.61\text{ m})^2 = b^2 + (-8\text{ m})^2 + (14\text{ m})^2$

$$\Rightarrow \boxed{b = 3.99\text{ m}}$$

(b) The direction cosines of $\mathbf{r}_{BC}$ are

$$\boxed{\begin{aligned}\cos\theta_x &= \frac{3.99\text{ m}}{16.61\text{ m}} = 0.240\\ \cos\theta_y &= \frac{-8\text{ m}}{16.61\text{ m}} = -0.482\\ \cos\theta_z &= \frac{14\text{ m}}{16.61\text{ m}} = 0.843\end{aligned}}$$

Problem 2.80 Observers at A and B use theodolites to measure the direction from their positions to a rocket in flight. If the coordinates of the rocket's position at a given instant are (4, 4, 2) km, determine the direction cosines of the vectors $\mathbf{r}_{AR}$ and $\mathbf{r}_{BR}$ that the observers would measure at that instant.



Solution: The vector $\mathbf{r}_{AR}$ is given by

$$\mathbf{r}_{AR} = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} \text{ km}$$

and the magnitude of $\mathbf{r}_{AR}$ is given by

$$|\mathbf{r}_{AR}| = \sqrt{(4)^2 + (4)^2 + (2)^2} \text{ km} = 6 \text{ km}.$$

The unit vector along AR is given by

$$\mathbf{u}_{AR} = \mathbf{r}_{AR}/|\mathbf{r}_{AR}|.$$

Thus, $\mathbf{u}_{AR} = 0.667\mathbf{i} + 0.667\mathbf{j} + 0.333\mathbf{k}$

and the direction cosines are

$$\boxed{\cos\theta_x = 0.667, \ \cos\theta_y = 0.667, \text{ and } \cos\theta_z = 0.333.}$$

The vector $\mathbf{r}_{BR}$ is given by

$$\begin{aligned}\mathbf{r}_{BR} &= (x_R - x_B)\mathbf{i} + (y_R - y_B)\mathbf{j} + (z_R - z_B)\mathbf{k} \text{ km}\\ &= (4 - 5)\mathbf{i} + (4 - 0)\mathbf{j} + (2 - 2)\mathbf{k} \text{ km}\end{aligned}$$

and the magnitude of $\mathbf{r}_{BR}$ is given by

$$|\mathbf{r}_{BR}| = \sqrt{(1)^2 + (4)^2 + (0)^2} \text{ km} = 4.12 \text{ km}.$$

The unit vector along BR is given by

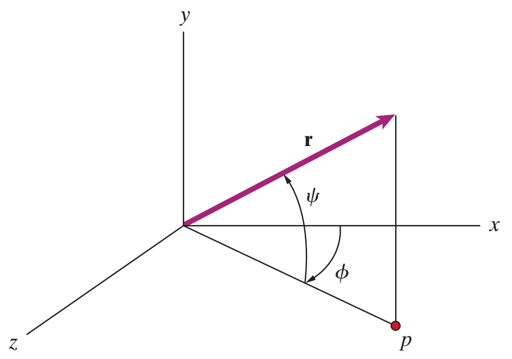
$$\mathbf{e}_{BR} = r_{BR}/|r_{BR}|.$$

Thus, $\mathbf{u}_{BR} = -0.242\mathbf{i} + 0.970\mathbf{j} + 0\mathbf{k}$

and the direction cosines are

$$\boxed{\cos\theta_x = -0.242, \ \cos\theta_y = 0.970, \text{ and } \cos\theta_z = 0.0.}$$

Problem 2.81 Point p lies in the x–z plane. The line from point p to the tip of the vector $\mathbf{r}$ is parallel to the y-axis. If $\mathbf{r} = 12\mathbf{i} + 6\mathbf{j} + 10\mathbf{k}$ (m), what are the angles ϕ and ψ in degrees?



Solution: The coordinates of the tip of the vector $\mathbf{r}$ are (12, 6, 10) m, so the coordinates of point P are (12, 0, 10) m. The length of the line from the origin to point P is

$$\sqrt{(12\text{ m})^2 + (10\text{ m})^2} = 15.6\text{ m}:$$

The tangent of ψ is

$$\tan\psi = \frac{6\text{ m}}{15.6\text{ m}} = 0.384,$$

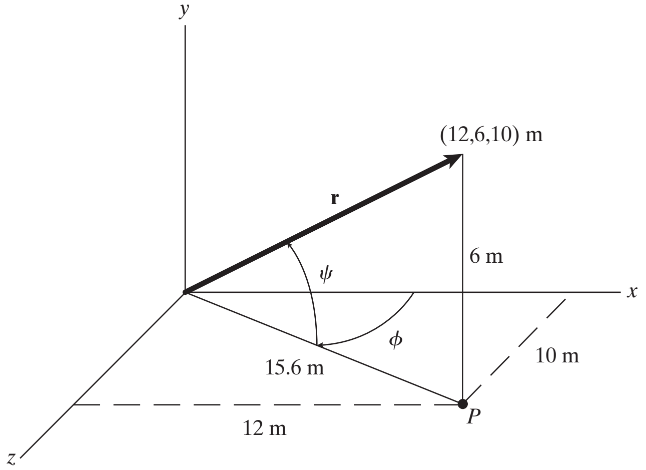
which yields $\theta = 21.0°$.

The tangent of ϕ is

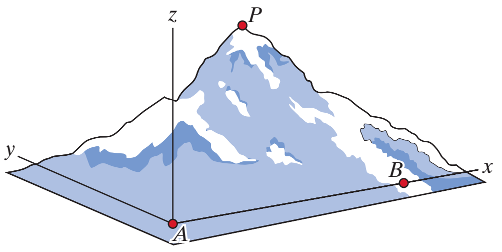
$$\tan\phi = \frac{10\text{ m}}{12\text{ m}} = 0.833,$$

which yields $\phi = 39.8°$.

$\boxed{\phi = 39.8°, \psi = 21.0°.}$



Problem 2.82 The height of Mount Everest was originally measured by a surveyor in the following way. He first measured the altitudes of two points and the horizontal distance between them. For example, suppose that the points A and B are 3000 m above sea level and are 10,000 m apart. He then used a theodolite to measure the direction cosines of the vector $\mathbf{r}_{AP}$ from point A to the top of the mountain P and the vector $\mathbf{r}_{BP}$ from point B to P. Suppose that the direction cosines of $\mathbf{r}_{AP}$ are $\cos\theta_x = 0.5179$, $\cos\theta_y = 0.6906$, and $\cos\theta_z = 0.5048$, and the direction cosines of $\mathbf{r}_{BP}$ are $\cos\theta_x = -0.3743$, $\cos\theta_y = 0.7486$, and $\cos\theta_z = 0.5472$. Using this data, determine the height of Mount Everest above sea level.



Solution: Using the given direction cosines, we have

$$\mathbf{r}_{AP} = |\mathbf{r}_{AP}|(0.5179\mathbf{i} + 0.6906\mathbf{j} + 0.5048\mathbf{k}), \quad (1)$$

$$\mathbf{r}_{BP} = |\mathbf{r}_{BP}|(-0.3743\mathbf{i} + 0.7486\mathbf{j} + 0.5472\mathbf{k}). \quad (2)$$

Let $\mathbf{r}_{AB} = (10{,}000\text{ m})\mathbf{i}$ denote the vector from point A to point B. We can see that $\mathbf{r}_{AP} = \mathbf{r}_{AB} + \mathbf{r}_{BP}$, and using the expression for the vectors in terms of their components, this gives the three equations

$$0.5179|\mathbf{r}_{AP}| = 10{,}000\text{ m} - 0.3743|\mathbf{r}_{BP}|,$$
$$0.6906|\mathbf{r}_{AP}| = 0.7486|\mathbf{r}_{BP}|,$$
$$0.5048|\mathbf{r}_{AP}| = 0.5472|\mathbf{r}_{BP}|.$$

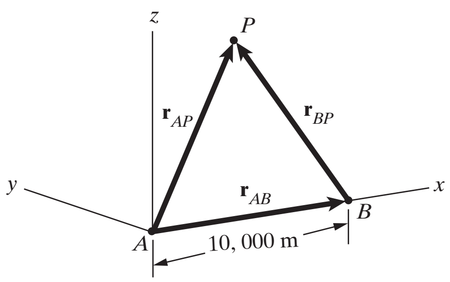
Solving these three equations (the second and third equations are redundant), we obtain

$$|\mathbf{r}_{AP}| = 11{,}585\text{ m}, |\mathbf{r}_{BP}| = 10{,}687\text{ m}.$$

To obtain the height of the mountain, we add 3000 m to the z component of $\mathbf{r}_{AP}$:

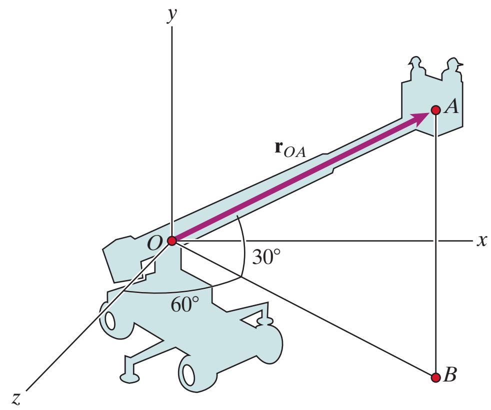
$$3000\text{ m} + 0.5048|\mathbf{r}_{AP}| = 8848\text{ m (29,030 ft)}.$$

$\boxed{8848\text{ m (29,030 ft).}}$



Problem 2.83 The distance from point O to point A is 20 ft. The straight line AB is parallel to the y-axis, and point B is in the x–z plane. Express the vector $\mathbf{r}_{OA}$ in terms of components.

Strategy: You can express $\mathbf{r}_{OA}$ as the sum of a vector from O to B and a vector from B to A. You can then express the vector from O to B as the sum of vector components parallel to the x- and z-axes. See Example 2.8.



Solution: The vector $\mathbf{r}_{OA}$ is the sum of the vector $\mathbf{r}_{OB}$, which lies in the x-z plane, and the vertical vector $\mathbf{r}_{BA}$. Because the distance from O to A is 20 ft, we can see that the magnitudes $|\mathbf{r}_{OB}| = (20 \text{ ft})\cos 30°$ and $|\mathbf{r}_{BA}| = (20 \text{ ft})\sin 30°$. The vector $\mathbf{r}_{OB}$ in terms of its components is

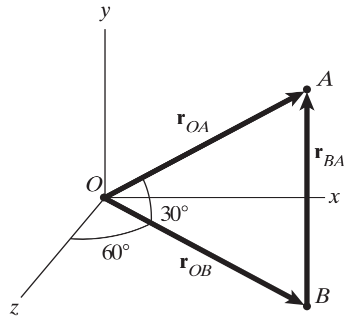
$$\mathbf{r}_{OB} = |\mathbf{r}_{OB}|\sin 60°\mathbf{i} + |\mathbf{r}_{OB}|\cos 60°\mathbf{k},$$

and the vector $\mathbf{r}_{BA}$ in terms of its components is

$$\mathbf{r}_{BA} = |\mathbf{r}_{BA}|\mathbf{j}.$$

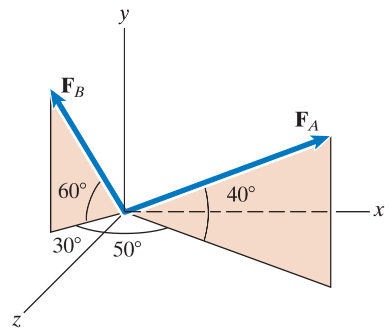
Therefore

$$\begin{aligned}\mathbf{r}_{OA} &= \mathbf{r}_{OB} + \mathbf{r}_{BA}\\ &= |\mathbf{r}_{OB}|\sin 60°\mathbf{i} + |\mathbf{r}_{OB}|\cos 60°\mathbf{k} + |\mathbf{r}_{BA}|\mathbf{j}\\ &= (20 \text{ ft})\cos 30°\sin 60°\mathbf{i} + (20 \text{ ft})\cos 30°\cos 60°\mathbf{k} + (20 \text{ ft})\sin 30°\mathbf{j}\\ &= 15\mathbf{i} + 10\mathbf{j} + 8.66\mathbf{k} \text{ (ft).}\end{aligned}$$



$$\boxed{\mathbf{r}_{OA} = 15\mathbf{i} + 10\mathbf{j} + 8.66\mathbf{k} \text{ (ft).}}$$

Problem 2.84 The magnitudes of the two force vectors are $|\mathbf{F}_A| = 140$ lb and $|\mathbf{F}_B| = 100$ lb. Determine the magnitude of the sum of the forces $\mathbf{F}_A + \mathbf{F}_B$.



Solution: We have the vectors

$$\mathbf{F}_A = 140 \text{ lb}([\cos 40°\sin 50°]\mathbf{i} + [\sin 40°]\mathbf{j} + [\cos 40°\cos 50°]\mathbf{k})$$

$$\mathbf{F}_A = (82.2\mathbf{i} + 90.0\mathbf{j} + 68.9\mathbf{k}) \text{ lb}$$

$$\mathbf{F}_B = 100 \text{ lb}([-\cos 60°\sin 30°]\mathbf{i} + [\sin 60°]\mathbf{j} + [\cos 60°\cos 30°]\mathbf{k})$$

$$\mathbf{F}_B = (-25.0\mathbf{i} + 86.6\mathbf{j} + 43.3\mathbf{k}) \text{ lb}$$

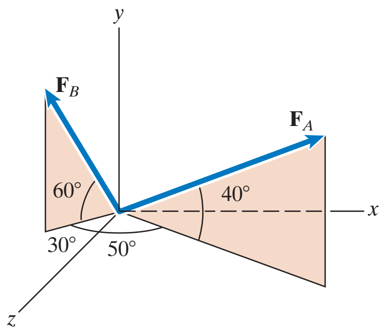
Adding and taking the magnitude we have

$$\mathbf{F}_A + \mathbf{F}_B = (57.2\mathbf{i} + 176.6\mathbf{j} + 112.2\mathbf{k}) \text{ lb}$$

$$|\mathbf{F}_A + \mathbf{F}_B| = \sqrt{(57.2 \text{ lb})^2 + (176.6 \text{ lb})^2 + (112.2 \text{ lb})^2} = 217 \text{ lb}$$

$$\boxed{|\mathbf{F}_A + \mathbf{F}_B| = 217 \text{ lb}}$$

Problem 2.85 Determine the direction cosines of the vectors $\mathbf{F}_A$ and $\mathbf{F}_B$.



Solution: We have the vectors

$$\mathbf{F}_A = 140\text{ lb}([\cos 40^\circ \sin 50^\circ]\mathbf{i} + [\sin 40^\circ]\mathbf{j} + [\cos 40^\circ \cos 50^\circ]\mathbf{k})$$

$$\mathbf{F}_A = (82.2\mathbf{i} + 90.0\mathbf{j} + 68.9\mathbf{k})\text{ lb}$$

$$\mathbf{F}_B = 100\text{ lb}([-\cos 60^\circ \sin 30^\circ]\mathbf{i} + [\sin 60^\circ]\mathbf{j} + [\cos 60^\circ \cos 30^\circ]\mathbf{k})$$

$$\mathbf{F}_B = (-25.0\mathbf{i} + 86.6\mathbf{j} + 43.3\mathbf{k})\text{ lb}$$

The direction cosines for $\mathbf{F}_A$ are

$$\cos\theta_x = \frac{82.2\text{ lb}}{140\text{ lb}} = 0.587,\ \cos\theta_y = \frac{90.0\text{ lb}}{140\text{ lb}} = 0.643,$$

$$\cos\theta_z = \frac{68.9\text{ lb}}{140\text{ lb}} = 0.492$$

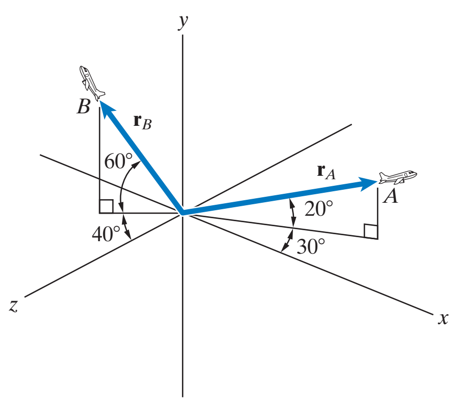
The direction cosines for $\mathbf{F}_B$ are

$$\cos\theta_x = \frac{-25.0\text{ lb}}{100\text{ lb}} = -0.250,\ \cos\theta_y = \frac{86.6\text{ lb}}{100\text{ lb}} = 0.866,$$

$$\cos\theta_z = \frac{43.3\text{ lb}}{100\text{ lb}} = 0.433$$

$$\boxed{\begin{array}{l} \mathbf{F}_A\text{: } \cos\theta_x = 0.587,\ \cos\theta_y = 0.643,\ \cos\theta_z = 0.492 \\ \mathbf{F}_B\text{: } \cos\theta_x = -0.250,\ \cos\theta_y = 0.866,\ \cos\theta_z = 0.433 \end{array}}$$

Problem 2.86 The positions of two airplanes A and B relative to a control tower are determined by radar. Their distances from the control tower are $|\mathbf{r}_A| = 16$ km and $|\mathbf{r}_B| = 12$ km. Determine the components of the position vector $\mathbf{r}_{AB}$ from plane A to plane B (that is, the vector that specifies the position of plane B relative to plane A). What is the straight-line distance between the two planes?

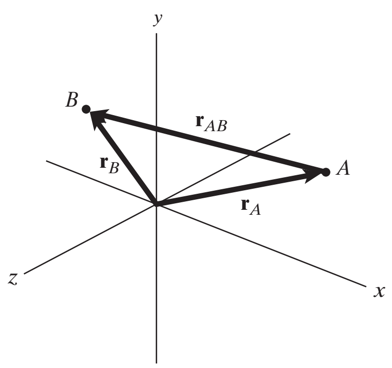


Solution: The components of $\mathbf{r}_A$ are

$$\mathbf{r}_A = (16\text{ km})(\cos 20^\circ \cos 30^\circ \mathbf{i} + \sin 20^\circ \mathbf{j} - \cos 20^\circ \sin 30^\circ \mathbf{k})$$
$$= 13.0\mathbf{i} + 5.47\mathbf{j} - 7.52\mathbf{k}\text{ (km)}.$$

The components of are

$$\mathbf{r}_B = (12\text{ km})(-\cos 60^\circ \sin 40^\circ \mathbf{i} + \sin 60^\circ \mathbf{j} + \cos 60^\circ \cos 40^\circ \mathbf{k})$$
$$= -3.86\mathbf{i} + 10.4\mathbf{j} + 4.60\mathbf{k}\text{ (km)}.$$



The vector from plane A to plane B is

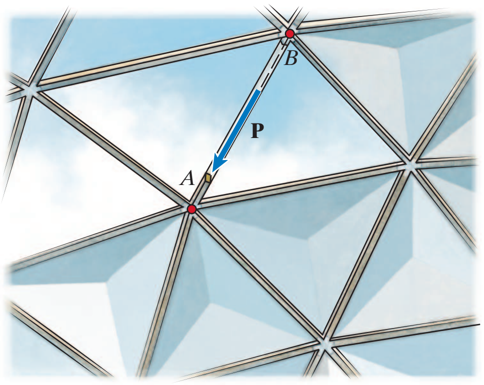
$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A$$
$$= -16.9\mathbf{i} + 4.92\mathbf{j} + 12.1\mathbf{k}\text{ (km)}.$$

The distance between the planes is

$$|\mathbf{r}_{AB}| = \sqrt{(-16.9\text{ km})^2 + (4.92\text{ km})^2 + (12.1\text{ km})^2}$$
$$= 21.3\text{ km}.$$

$$\boxed{\mathbf{r}_{AB} = -16.9\mathbf{i} + 4.92\mathbf{j} + 12.1\mathbf{k}\text{ (km), distance} = 21.3\text{ km.}}$$

Problem 2.87 An engineer calculates that the magnitude of the axial force in one of the beams of a geodesic dome is $|\mathbf{P}| = 7.65$ kN. The cartesian coordinates of the endpoints A and B of the straight beam are (–12.4, 22.0, –18.4) m and (–9.2, 24.4, –15.6) m, respectively. Express the force $\mathbf{P}$ in terms of components.



Solution: The components of the position vector from B to A are

$$\begin{aligned}\mathbf{r}_{BA} &= (x_A - x_B)\mathbf{i} + (y_A - y_B)\mathbf{j} + (z_A - z_B)\mathbf{k}\\ &= (-12.4 + 9.2)\mathbf{i} + (22.0 - 24.4)\mathbf{j} + (-18.4 + 15.6)\mathbf{k}\\ &= -3.2\mathbf{i} - 2.4\mathbf{j} - 2.8\mathbf{k}\ (\text{m}).\end{aligned}$$

Dividing this vector by its magnitude, we obtain a unit vector that points from B toward A:

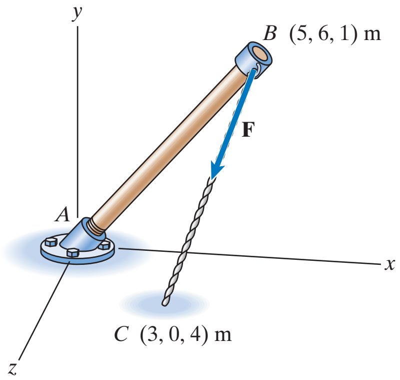
$$\mathbf{e}_{BA} = -0.655\mathbf{i} - 0.492\mathbf{j} - 0.573\mathbf{k}.$$

Therefore

$$\begin{aligned}\mathbf{P} &= |\mathbf{P}|\mathbf{e}_{BA}\\ &= 7.65\ \mathbf{e}_{BA}\\ &= -5.01\mathbf{i} - 3.76\mathbf{j} - 4.39\mathbf{k}\ (\text{kN}).\end{aligned}$$

$$\boxed{\mathbf{P} = -5.01\mathbf{i} - 3.76\mathbf{j} - 4.39\mathbf{k}\ (\text{kN}).}$$

Problem 2.88 The cable BC exerts an 8-kN force $\mathbf{F}$ on the bar AB at B. (a) Determine the components of a unit vector that points from point B toward point C. (b) Express $\mathbf{F}$ in terms of components.



Solution:

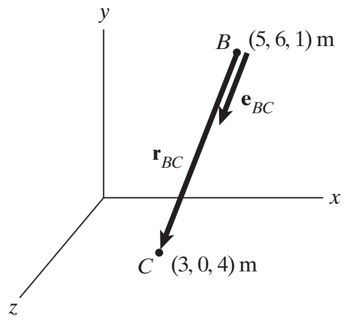
(a) $$\mathbf{e}_{BC} = \frac{\mathbf{r}_{BC}}{|\mathbf{r}_{BC}|} = \frac{(x_C - x_B)\mathbf{i} + (y_C - y_B)\mathbf{j} + (z_C - z_B)\mathbf{k}}{\sqrt{(x_C - x_B)^2 + (y_C - y_B)^2 + (z_C - z_B)^2}}$$

$$\mathbf{e}_{BC} = \frac{-2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}}{\sqrt{2^2 + 6^2 + 3^2}} = -\frac{2}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$$

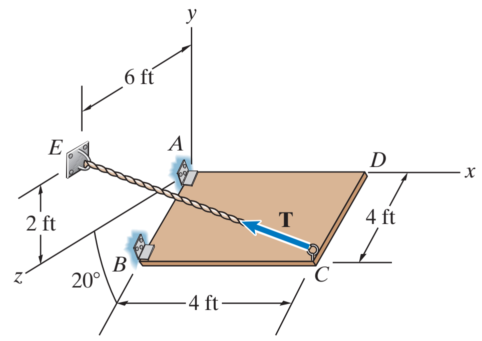
$$\mathbf{e}_{BC} = -0.286\mathbf{i} - 0.857\mathbf{j} + 0.429\mathbf{k}$$

(b) $\mathbf{F} = |\mathbf{F}|\mathbf{e}_{BC} = 8\mathbf{e}_{BC} = -2.29\mathbf{i} - 6.86\mathbf{j} + 3.43\mathbf{k}$ (kN)

$$\boxed{\begin{aligned}&\text{(a) } \mathrm{e}_{BC} = -0.286\mathbf{i} - 0.857\mathbf{j} + 0.429\mathbf{k}.\\ &\text{(b) } \mathrm{e}_{BC} = -2.29\mathbf{i} - 6.86\mathbf{j} + 3.43\mathbf{k}\ (\text{kN}).\end{aligned}}$$



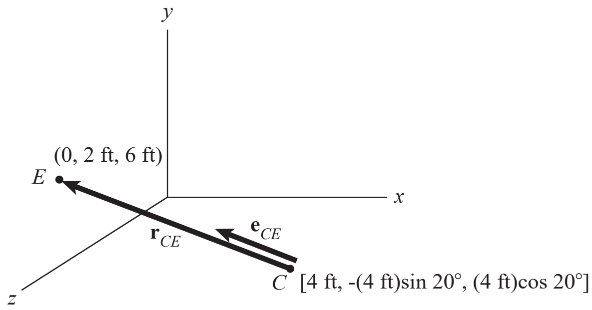
Problem 2.89 A cable extends from point C to point E. It exerts a 50-lb force $\mathbf{T}$ on the plate at C that is directed along the line from C to E. Express $\mathbf{T}$ in terms of components.



Solution: Find the unit vector $\mathbf{e}_{CE}$ and multiply it times the magnitude of the force to get the vector in component form,

$$\mathbf{e}_{CE} = \frac{\mathbf{r}_{CE}}{|\mathbf{r}_{CE}|} = \frac{(x_E - x_C)\mathbf{i} + (y_E - y_C)\mathbf{j} + (z_E - z_C)\mathbf{k}}{\sqrt{(x_E - x_C)^2 + (y_E - y_C)^2 + (z_E - z_C)^2}}$$

The coordinates of point C are $(4, -4\sin 20°, 4\cos 20°)$ or $(4, -1.37, 3.76)$ (ft) The coordinates of point E are $(0, 2, 6)$ (ft)



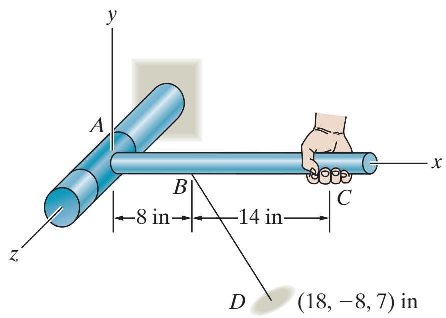
$$\mathbf{e}_{CE} = \frac{(0 - 4)\mathbf{i} + (2 - (-1.37))\mathbf{j} + (6 - 3.76)\mathbf{k}}{\sqrt{4^2 + 3.37^2 + 2.24^2}}$$

$$\mathbf{e}_{CE} = -0.703\mathbf{i} + 0.592\mathbf{j} + 0.394\mathbf{k}$$

$$\mathbf{T} = 50\mathbf{e}_{CE} \text{ (lb)}$$

$$\mathbf{T} = -35.2\mathbf{i} + 29.6\mathbf{j} + 19.7\mathbf{k} \text{ (lb)}$$

Problem 2.90 A cable extends from point B to point D. It exerts a 30-lb force on the bar at B that is directed along the line from B to D. Express the force in terms of components.

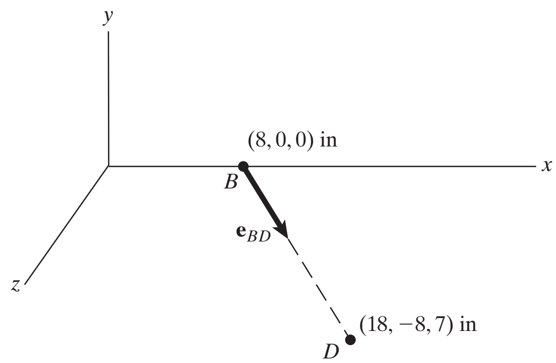


Solution: The vector $\mathbf{r}_{BD}$ from point B to point D is

$$\mathbf{r}_{BD} = (18 \text{ in} - 8 \text{ in})\mathbf{i} + (-8 \text{ in} - 0)\mathbf{j} + (7 \text{ in} - 0)$$
$$= 10\mathbf{i} - 8\mathbf{j} + 7\mathbf{k} \text{ (in)}.$$

Its magnitude is

$$|\mathbf{r}_{BD}| = \sqrt{(10 \text{ in})^2 + (-8 \text{ in})^2 + (7 \text{ in})^2}$$
$$= 14.6 \text{ in}.$$



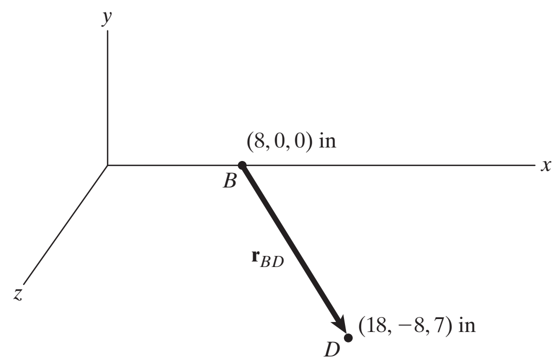
By dividing this vector by its magnitude, we obtain a unit vector that points from point B toward point D:

$$\mathbf{e}_{BD} = \frac{10\mathbf{i} - 8\mathbf{j} + 7\mathbf{k} \text{ (in)}}{14.6 \text{ in}}$$
$$= 0.685\mathbf{i} - 0.548\mathbf{j} + 0.480\mathbf{k}.$$

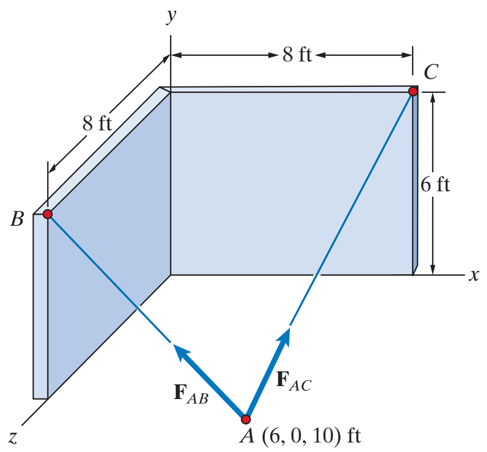
We obtain the force in terms of its components by multiplying this unit vector by 30 lb:

$$(30 \text{ lb})\mathbf{e}_{BD} = 20.6\mathbf{i} - 16.4\mathbf{j} + 14.4\mathbf{k} \text{ (lb)}.$$

$$\boxed{20.6\mathbf{i} - 16.4\mathbf{j} + 14.4\mathbf{k} \text{ (lb)}.}$$



Problem 2.91 The cable AB exerts a 200-lb force $\mathbf{F}_{AB}$ at point A that is directed along the line from A to B. Express $\mathbf{F}_{AB}$ in terms of components.



Solution: The vector from point A to point B is

$$\mathbf{r}_{AB} = (0 - 6\text{ ft})\mathbf{i} + (6\text{ ft} - 0)\mathbf{j} + (8\text{ ft} - 10\text{ ft})\mathbf{k}$$
$$= -6\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}\ (\text{ft}).$$

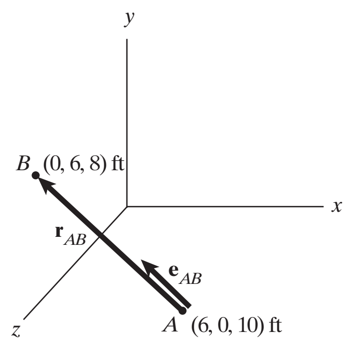
Its magnitude is $|\mathbf{r}_{AB}| = \sqrt{(-6\text{ ft})^2 + (6\text{ ft})^2 + (-2\text{ ft})^2} = 8.72$ ft. The unit vector that points from point A toward point B is given by

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = -0.688\mathbf{i} + 0.688\mathbf{j} - 0.229\mathbf{k}.$$

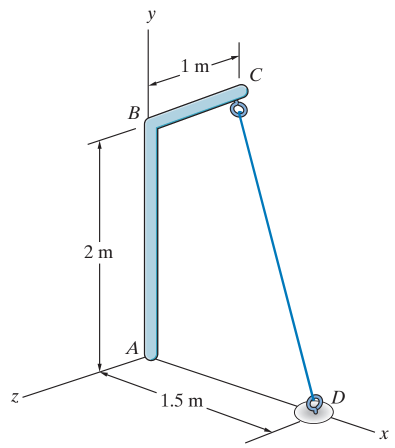
Then the force $\mathbf{F}_{AB}$ in terms of its components is

$$\mathbf{F}_{AB} = (200\text{ lb})\mathbf{e}_{AB} = -138\mathbf{i} + 138\mathbf{j} - 45.9\mathbf{k}\ (\text{lb}).$$

$$\boxed{\mathbf{F}_{AB} = -138\mathbf{i} + 138\mathbf{j} - 45.9\mathbf{k}\ (\text{lb}).}$$



Problem 2.92 The cable CD exerts a 6-kN force on the bar at C that is directed from C toward D. Express the force in terms of components.



Solution:

The vector $\mathbf{r}_{CD}$ from point B to point D is

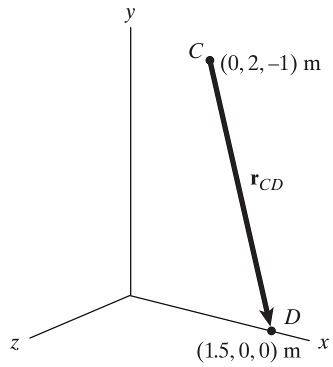
$$\mathbf{r}_{CD} = (1.5\text{ m} - 0)\mathbf{i} + (0 - 2\text{ m})\mathbf{j} + (0 - (-1\text{ m}))\mathbf{k}$$
$$= 1.5\mathbf{i} - 2\mathbf{j} + \mathbf{k}\ (\text{m}).$$

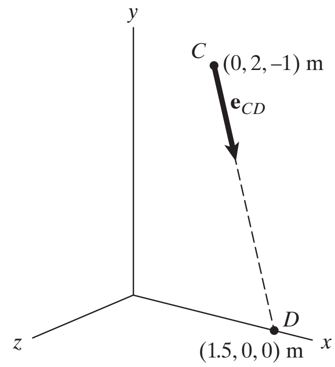
Its magnitude is

$$|\mathbf{r}_{CD}| = \sqrt{(1.5\text{ m})^2 + (-2\text{ m})^2 + (1\text{ m})^2}$$
$$= 2.69\text{ m}.$$

By dividing this vector by its magnitude, we obtain a unit vector that points from point C toward point D:

$$\mathbf{e}_{CD} = \frac{1.5\mathbf{i} - 2\mathbf{j} + \mathbf{k}\ (\text{m})}{2.69\text{ m}}$$
$$= 0.557\mathbf{i} - 0.743\mathbf{j} + 0.371\mathbf{k}.$$



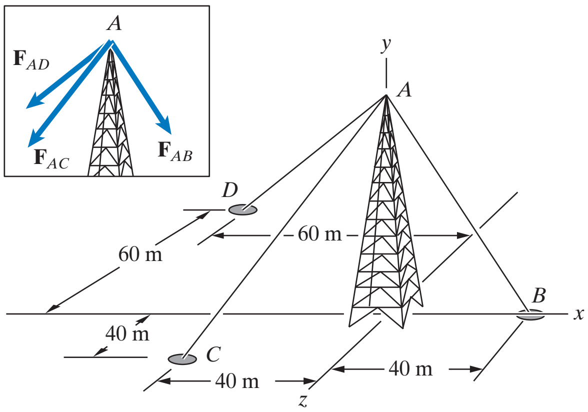


We obtain the force in terms of its components by multiplying this unit vector by 6 kN:

$$(6\text{ kN})\mathbf{e}_{CD} = 3.34\mathbf{i} - 4.46\mathbf{j} + 2.23\mathbf{k}\ (\text{kN}).$$

$$\boxed{3.34\mathbf{i} - 4.46\mathbf{j} + 2.23\mathbf{k}\ (\text{kN}).}$$

Problem 2.93 The 70-m-tall tower is supported by three cables that exert forces $\mathbf{F}_{AB}$, $\mathbf{F}_{AC}$, and $\mathbf{F}_{AD}$ on it. The magnitude of each force is 2 kN. Express the total force exerted on the tower by the three cables in terms of components.



Solution: The coordinates of the points are A (0, 70, 0), B (40, 0, 0), C (−40, 0, 40) D (−60, 0, −60).

The position vectors corresponding to the cables are:

$$\mathbf{r}_{AD} = (-60 - 0)\mathbf{i} + (0 - 70)\mathbf{j} + (-60 - 0)\mathbf{k}$$

$$\mathbf{r}_{AD} = -60\mathbf{i} - 70\mathbf{k} - 60\mathbf{k}$$

$$\mathbf{r}_{AC} = (-40 - 0)\mathbf{i} + (0 - 70)\mathbf{j} + (40 - 0)\mathbf{k}$$

$$\mathbf{r}_{AC} = -40\mathbf{i} - 70\mathbf{j} + 40\mathbf{k}$$

$$\mathbf{r}_{AB} = (40 - 0)\mathbf{i} + (0 - 70)\mathbf{j} + (0 - 0)\mathbf{k}$$

$$\mathbf{r}_{AB} = 40\mathbf{i} - 70\mathbf{j} + 0\mathbf{k}$$

The unit vectors corresponding to these position vectors are:

$$\mathbf{u}_{AD} = \frac{\mathbf{r}_{AD}}{|\mathbf{r}_{AD}|} = \frac{-60}{110}\mathbf{i} - \frac{70}{110}\mathbf{j} - \frac{60}{110}\mathbf{k}$$
$$= -0.5455\mathbf{i} - 0.6364\mathbf{j} - 0.5455\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = -\frac{40}{90}\mathbf{i} - \frac{70}{90}\mathbf{j} + \frac{40}{90}\mathbf{k}$$
$$= -0.4444\mathbf{i} - 0.7778\mathbf{j} + 0.4444\mathbf{k}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = \frac{40}{80.6}\mathbf{i} - \frac{70}{80.6}\mathbf{j} + 0\mathbf{k} = 0.4963\mathbf{i} - 0.8685\mathbf{j} + 0\mathbf{k}$$

The forces are:

$$\mathbf{F}_{AB} = |\mathbf{F}_{AB}|\mathbf{u}_{AB} = 0.9926\mathbf{i} - 1.737\mathbf{j} + 0\mathbf{k}$$

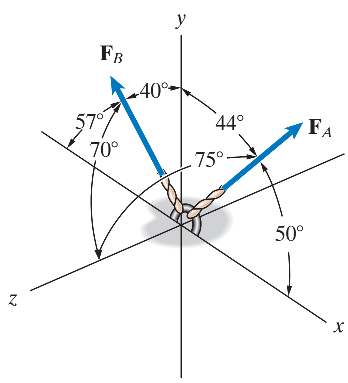
$$\mathbf{F}_{AC} = |\mathbf{F}_{AC}|\mathbf{u}_{AC} = -0.8888\mathbf{i} - 1.5556\mathbf{j} + 0.8888$$

$$\mathbf{F}_{AD} = |\mathbf{F}_{AD}|\mathbf{u}_{AD} = -1.0910\mathbf{i} - 1.2728\mathbf{j} - 1.0910\mathbf{k}$$

The resultant force exerted on the tower by the cables is:

$$\boxed{\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} = -0.9875\mathbf{i} - 4.5648\mathbf{j} - 0.2020\mathbf{k}\ \text{kN}}$$

Problem 2.94 The magnitudes of the two force vectors are $|\mathbf{F}_A| = 4.6$ kN and $|\mathbf{F}_B| = 5.2$ kN. Determine the magnitude of the sum of the forces $\mathbf{F}_A + \mathbf{F}_B$. (Notice that the angle between the vector $\mathbf{F}_B$ and the positive x-axis is $180° - 57° = 123°$.)



Solution: The vectors in terms of their direction cosines are

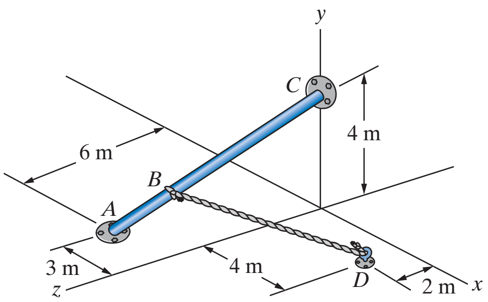
$$\mathbf{F}_A = (4.6\text{ kN})(\cos\theta_{Ax}\mathbf{i} + \cos\theta_{Ay}\mathbf{j} + \cos\theta_{Az}\mathbf{k})$$
$$= (4.6\text{ kN})(\cos 50°\mathbf{i} + \cos 44°\mathbf{j} + \cos 75°\mathbf{k})$$
$$= 2.96\mathbf{i} + 3.31\mathbf{j} + 1.19\mathbf{k}\ (\text{kN}),$$
$$\mathbf{F}_B = (5.2\text{ kN})(\cos\theta_{Bx}\mathbf{i} + \cos\theta_{By}\mathbf{j} + \cos\theta_{Bz}\mathbf{k})$$
$$= (5.2\text{ kN})(\cos 123°\mathbf{i} + \cos 40°\mathbf{j} + \cos 70°\mathbf{k})$$
$$= -2.83\mathbf{i} + 3.98\mathbf{j} + 1.78\mathbf{k}\ (\text{kN}).$$

The magnitude of their sum is

$$|\mathbf{F}_A + \mathbf{F}_B| = \sqrt{(2.96 - 2.83)^2 + (3.31 + 3.98)^2 + (1.19 + 1.78)^2}\ (\text{kN})$$
$$= 7.87\ \text{kN}.$$

$$\boxed{|\mathbf{F}_A + \mathbf{F}_B| = 7.87\ \text{kN}.}$$

Problem 2.95 The rope BD is attached to the bar ABC at B. The distance from A to B is 3 m. Determine the components of the position vector $\mathbf{r}_{BD}$ from point B to point D.



Solution: The vector from A to C is

$$\mathbf{r}_{AC} = 3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}\ (\text{m}).$$

Dividing this vector by its magnitude gives a unit vector that points from A toward C. We can obtain the vector from A to B by multiplying that unit vector by 3 m:

$$\begin{aligned}\mathbf{r}_{AB} &= (3\ \text{m})\frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|}\\ &= (3\ \text{m})\left[\frac{3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}\ (\text{m})}{\sqrt{(3\ \text{m})^2 + (4\ \text{m})^2 + (-6\ \text{m})^2}}\right]\\ &= 1.15\mathbf{i} + 1.54\mathbf{j} - 2.30\mathbf{k}\ (\text{m}).\end{aligned}$$

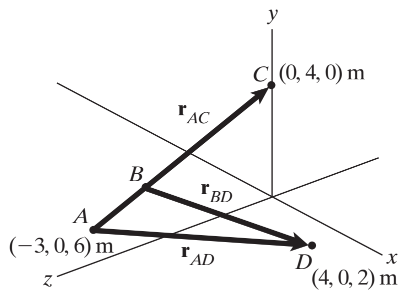
The vector from A to D is

$$\mathbf{r}_{AD} = 7\mathbf{i} - 4\mathbf{k}\ (\text{m}).$$

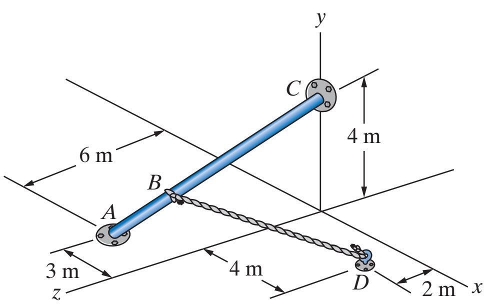
Because $\mathbf{r}_{AD} = \mathbf{r}_{AB} + \mathbf{r}_{BD}$, we obtain

$$\begin{aligned}\mathbf{r}_{BD} &= \mathbf{r}_{AD} - \mathbf{r}_{AB}\\ &= 7\mathbf{i} - 4\mathbf{k} - (1.15\mathbf{i} + 1.54\mathbf{j} - 2.30\mathbf{k})\ (\text{m})\\ &= 5.85\mathbf{i} - 1.54\mathbf{j} - 1.70\mathbf{k}\ (\text{m}).\end{aligned}$$

$$\boxed{\mathbf{r}_{BD} = 5.85\mathbf{i} - 1.54\mathbf{j} - 1.70\mathbf{k}\ (\text{m}).}$$



Problem 2.96 The rope BD is attached to the bar ABC at B. The distance from A to B is 3 m. The rope exerts a 2-kN force on the bar at B that is directed from B toward D. Determine the components of the force.



Solution: The vector from A to C is

$$\mathbf{r}_{AC} = 3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}\ (\text{m}).$$

Dividing this vector by its magnitude gives a unit vector that points from A toward C. We can obtain the vector from A to B by multiplying that unit vector by 3 m:

$$\begin{aligned}\mathbf{r}_{AB} &= (3\ \text{m})\frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|}\\ &= (3\ \text{m})\left[\frac{3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}\ (\text{m})}{\sqrt{(3\ \text{m})^2 + (4\ \text{m})^2 + (-6\ \text{m})^2}}\right]\\ &= 1.15\mathbf{i} + 1.54\mathbf{j} - 2.30\mathbf{k}\ (\text{m}).\end{aligned}$$

The vector from A to D is

$$\mathbf{r}_{AD} = 7\mathbf{i} - 4\mathbf{k}\ (\text{m}).$$

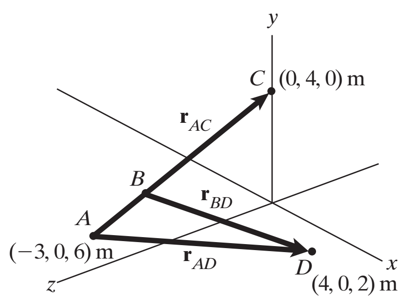
Because $\mathbf{r}_{AD} = \mathbf{r}_{AB} + \mathbf{r}_{BD}$, we obtain

$$\begin{aligned}\mathbf{r}_{BD} &= \mathbf{r}_{AD} - \mathbf{r}_{AB}\\ &= 7\mathbf{i} - 4\mathbf{k} - (1.15\mathbf{i} + 1.54\mathbf{j} - 2.30\mathbf{k})\ \ (\text{m})\\ &= 5.85\mathbf{i} - 1.54\mathbf{j} - 1.70\mathbf{k}\ (\text{m}).\end{aligned}$$

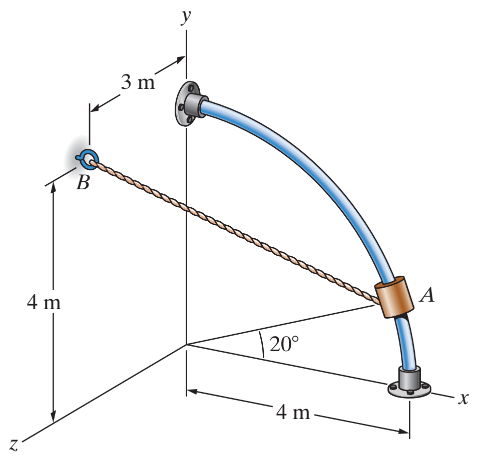
Using this result, the force exerted on the bar at B is

$$\begin{aligned}\mathbf{F} &= (2\ \text{kN})\frac{\mathbf{r}_{BD}}{|\mathbf{r}_{BD}|}\\ &(2\ \text{kN})\left[\frac{5.85\mathbf{i} - 1.54\mathbf{j} - 1.70\mathbf{k}\ (\text{m})}{\sqrt{(5.85\ \text{m})^2 + (-1.54\ \text{m})^2 + (-1.70\ \text{m})^2}}\right]\\ &= 1.86\mathbf{i} - 0.489\mathbf{j} - 0.540\mathbf{k}\ (\text{kN}).\end{aligned}$$

$$\boxed{1.86\mathbf{i} - 0.489\mathbf{j} - 0.540\mathbf{k}\ (\text{kN}).}$$



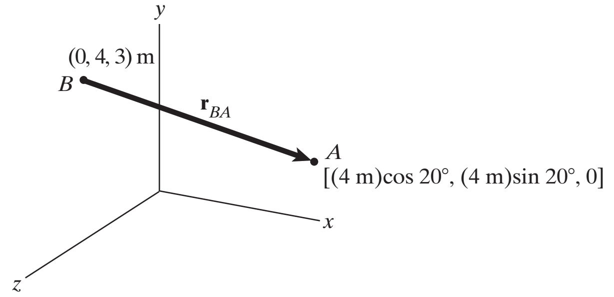
Problem 2.97 The circular bar has a 4-m radius and lies in the x–y plane. Express the position vector from point B to the collar at A in terms of components.



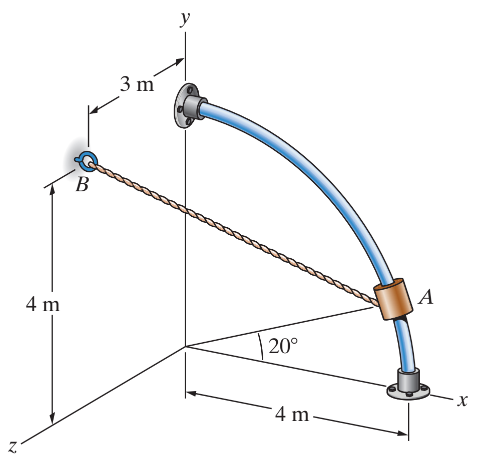
Solution: The coordinates of point A are [(4 m) cos 20°, (4 m) sin 20°, 0], and the coordinates of point B are (0, 4, 3) m, so the vector from B to A is

$$r_{BA} = [(4\text{ m})\cos 20^\circ - 0]\mathbf{i} + [(4\text{ m})\sin 20^\circ - 4\text{ m}]\mathbf{j} + [0 - 3\text{ m}]\mathbf{k}$$
$$= 3.76\mathbf{i} - 2.63\mathbf{j} - 3\mathbf{k}\ (\text{m}).$$

$$\boxed{r_{BA} = 3.76\mathbf{i} - 2.63\mathbf{j} - 3\mathbf{k}\ (\text{m}).}$$



Problem 2.98 The cable AB exerts a 60-N force $\mathbf{T}$ on the collar at A that is directed along the line from A toward B. Express $\mathbf{T}$ in terms of components.



Solution: The coordinates of point A are [(4 m) cos 20°, (4 m) sin 20°, 0], and the coordinates of point B are (0, 4, 3) m, so the vector from A to B is

$$\mathbf{r}_{AB} = [0 - (4\text{ m})\cos 20^\circ]\mathbf{i} + [4\text{ m} - (4\text{ m})\sin 20^\circ]\mathbf{j} + [3\text{ m} - 0]\mathbf{k}$$
$$= -3.76\mathbf{i} + 2.63\mathbf{j} + 3\mathbf{k}\ (\text{m}).$$

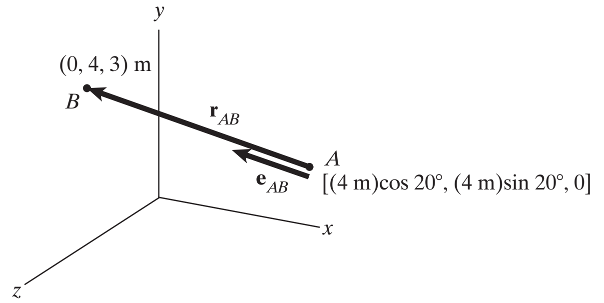
By dividing this vector by its magnitude, we obtain a unit vector that points from A toward B:

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = -0.686\mathbf{i} + 0.480\mathbf{j} + 0.547\mathbf{k}.$$

Then the force $\mathbf{T}$ in terms of its components is

$$\mathbf{T} = (60\text{ N})\mathbf{e}_{AB} = -41.1\mathbf{i} + 28.8\mathbf{j} + 32.8\mathbf{k}\ (\text{N}).$$

$$\boxed{\mathbf{T} = -41.1\mathbf{i} + 28.8\mathbf{j} + 32.8\mathbf{k}\ (\text{N}).}$$



Problem 2.99 In Practice Example 2.11, suppose that the vector $\mathbf{V}$ is changed to $\mathbf{V} = 4\mathbf{i} - 6\mathbf{j} - 10\mathbf{k}$. (a) What is the value of $\mathbf{U}\cdot\mathbf{V}$? (b) What is the angle between $\mathbf{U}$ and $\mathbf{V}$ when they are placed tail to tail?

Solution: From Active Example 2.4 we have the expression for $\mathbf{U}$. Thus

$$\mathbf{U} = 6\mathbf{i} - 5\mathbf{j} - 3\mathbf{k},\ \mathbf{V} = 4\mathbf{i} - 6\mathbf{k} - 10\mathbf{k}$$

$$\mathbf{U}\cdot\mathbf{V} = (6)(4) + (-5)(-6) + (-3)(-10) = 84$$

$$\cos\theta = \frac{\mathbf{U}\cdot\mathbf{V}}{|\mathbf{V}||\mathbf{V}|} = \frac{84}{\sqrt{6^2 + (-5)^2 + (-3)^2}\sqrt{4^2 + (-6)^2 + (-10)^2}} = 0.814$$

$$\theta = \cos^{-1}(0.814) = 35.5^\circ$$

$$\boxed{(a)\ \mathbf{U}\cdot\mathbf{V} = 84,\ (b)\ \theta = 35.5^\circ}$$

Problem 2.100 In Example 2.12, suppose that the coordinates of point B are changed to (6, 4, 4) m. What is the angle θ between the lines AB and AC?

Solution: Using the new coordinates we have

$$\mathbf{r}_{AB} = (2\mathbf{i} + \mathbf{j} + 2\mathbf{k})\text{ m}, \ |\mathbf{r}_{AB}| = 3\text{ m}$$

$$\mathbf{r}_{AC} = (4\mathbf{i} + 5\mathbf{j} + 2\mathbf{k})\text{ m}, \ |\mathbf{r}_{AC}| = 6.71\text{ m}$$

$$\cos\theta = \frac{\mathbf{r}_{AB}\cdot\mathbf{r}_{AC}}{|\mathbf{r}_{AB}||\mathbf{r}_{AC}|} = \frac{((2)(4) + (1)(5) + (2)(2))\text{ m}^2}{(3\text{ m})(6.71\text{ m})} = 0.845$$

$$\theta = \cos^{-1}(0.845) = 32.4^\circ$$

$$\boxed{\theta = 32.4^\circ}$$

Problem 2.101 Consider the vectors

$$\mathbf{U} = 12\mathbf{i} + 4\mathbf{j} - 8\mathbf{k},$$
$$\mathbf{V} = -6\mathbf{i} + 8\mathbf{j} + 14\mathbf{k},$$
$$\mathbf{W} = 8\mathbf{i} - 8\mathbf{j} + 8\mathbf{k}.$$

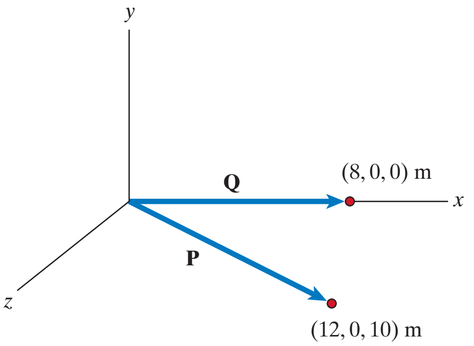
Determine the values of the dot products $\mathbf{U}\cdot\mathbf{V}$, $\mathbf{U}\cdot\mathbf{W}$, and $\mathbf{V}\cdot\mathbf{W}$. What can you deduce about the three vectors from the results?

Solution: Applying Eq. (2.23), the dot products are

$$\begin{aligned}\mathbf{U}\cdot\mathbf{V} &= (12)(-6) + (4)(8) + (-8)(14)\\ &= -152,\\ \mathbf{U}\cdot\mathbf{W} &= (12)(8) + (4)(-8) + (-8)(8)\\ &= 0,\\ \mathbf{V}\cdot\mathbf{W} &= (-6)(8) + (8)(-8) + (14)(8)\\ &= 0.\end{aligned}$$

$\mathbf{U}\cdot\mathbf{V} = -152$, $\mathbf{U}\cdot\mathbf{W} = 0$, $\mathbf{V}\cdot\mathbf{W} = 0$. **U** and **V** are not perpendicular. **W** is perpendicular to both **U** and **V**.

Problem 2.102 Consider the position vectors **P** and **Q**. (a) Determine the dot product $\mathbf{P}\cdot\mathbf{Q}$ by using the definition, Eq. (2.18). (b) Determine the dot product $\mathbf{P}\cdot\mathbf{Q}$ by using Eq. (2.23).



Solution:

(a) The two vectors in terms of their components are

$$\mathbf{P} = 12\mathbf{i} + 10\mathbf{k}\ (\text{m}),$$
$$\mathbf{Q} = 8\mathbf{i}\ (\text{m}).$$

Their magnitudes are

$$|\mathbf{P}| = \sqrt{(12\text{ m})^2 + (10\text{ m})^2} = 15.6\text{ m},$$
$$|\mathbf{Q}| = 8\text{ m}.$$

The angle between the two vectors is

$$\theta = \arctan\left(\frac{10\text{ m}}{12\text{ m}}\right) = 39.8^\circ.$$

From Eq. (2.18),

$$\begin{aligned}\mathbf{P}\cdot\mathbf{Q} &= (15.6\text{ m})(8\text{ m})\cos 39.8^\circ\\ &= 96\text{ m}^2.\end{aligned}$$

(b) From Eq. (2.23),

$$\begin{aligned}\mathbf{P}\cdot\mathbf{Q} &= (12\text{ m})(8\text{ m}) + (0)(0) + (10\text{ m})(0)\\ &= 96\text{ m}^2.\end{aligned}$$

$$\boxed{\text{(a), (b) } \mathbf{P}\cdot\mathbf{Q} = 96\text{ m}^2.}$$

Problem 2.103 Two *perpendicular* vectors are given in terms of their components by $\mathbf{U} = U_x\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ and $\mathbf{V} = 3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. Use the dot product to determine the component U_x.

Solution: When the vectors are perpendicular, $\mathbf{U}\cdot\mathbf{V} \equiv 0$.

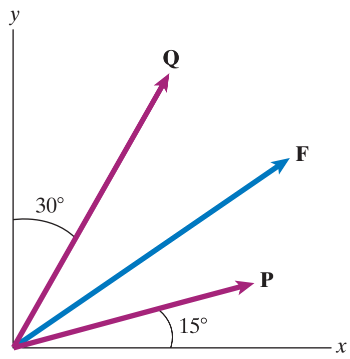
Thus

$$\begin{aligned}\mathbf{U}\cdot\mathbf{V} &= U_xV_x + U_yV_y + U_zV_z = 0\\ &= 3U_x + (-4)(2) + (6)(-3) = 0\end{aligned}$$

$$3U_x = 26$$

$$U_x = 8.67$$

Problem 2.104 The vectors **P**, **Q**, and $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$ lie in the x–y plane. The magnitudes of **P** and **Q** are 4 m and 5 m, respectively. The dot products $\mathbf{P} \cdot \mathbf{F} = 50$ N-m (newton-meters) and $\mathbf{Q} \cdot \mathbf{F} = 60$ N-m. Determine the components F_x and F_y.



Solution:
The components of **P** and **Q** are

$$\mathbf{P} = (4\text{ m})(\cos 15°\mathbf{i} + \sin 15°\mathbf{j}),$$
$$\mathbf{Q} = (5\text{ m})(\sin 30°\mathbf{i} + \cos 30°\mathbf{j}).$$

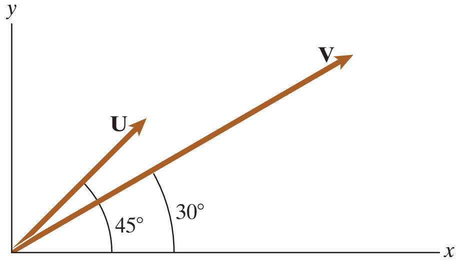
The two given conditions are

$$\mathbf{P} \cdot \mathbf{F} = [(4\text{ m})\cos 15°]F_x + [(4\text{ m})\sin 15°]F_y = 50\text{ N-m},$$
$$\mathbf{Q} \cdot \mathbf{F} = [(5\text{ m})\sin 30°]F_x + [(5\text{ m})\cos 30°]F_y = 60\text{ N-m}.$$

Solving these two equations, we obtain

$$\boxed{F_x = 10.9\text{ N},\ F_y = 7.55\text{ N.}}$$

Problem 2.105 The magnitudes $|\mathbf{U}| = 10$ and $|\mathbf{V}| = 20$. (a) Use Eq. (2.18) to determine $\mathbf{U} \cdot \mathbf{V}$. (b) Use Eq. (2.23) to determine $\mathbf{U} \cdot \mathbf{V}$.



Solution:

(a) The definition of the dot product (Eq. (2.18)) is

$\mathbf{U} \cdot \mathbf{V} = |\mathbf{U}||\mathbf{V}|\cos\theta$. Thus

$$\mathbf{U} \cdot \mathbf{V} = (10)(20)\cos(45° - 30°) = 193.2$$

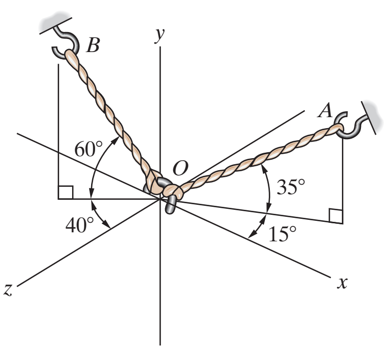
(b) The components of **U** and **V** are

$$\mathbf{U} = 10(\mathbf{i}\cos 45° + \mathbf{j}\sin 45°) = 7.07\mathbf{i} + 7.07\mathbf{j}$$

$$\mathbf{V} = 20(\mathbf{i}\cos 30° + \mathbf{j}\sin 30°) = 17.32\mathbf{i} + 10\mathbf{j}$$

From Eq. (2.23) $\boxed{\mathbf{U} \cdot \mathbf{V} = (7.07)(17.32) + (7.07)(10) = 193.2}$

Problem 2.106 What is the angle between the ropes OA and OB?



Solution: Let $\mathbf{e}_{OA}$ be a unit vector that points from the origin O toward A. In terms of the angles shown, its components are

$$\mathbf{e}_{OA} = \cos 35°\cos 15°\mathbf{i} + \sin 35°\mathbf{j} - \cos 35°\sin 15°\mathbf{k}.$$

The components of a unit vector $\mathbf{e}_{OB}$ that points from O toward B are

$$\mathbf{e}_{OB} = -\cos 60°\sin 40°\mathbf{i} + \sin 60°\mathbf{j} + \cos 60°\cos 40°\mathbf{k}.$$

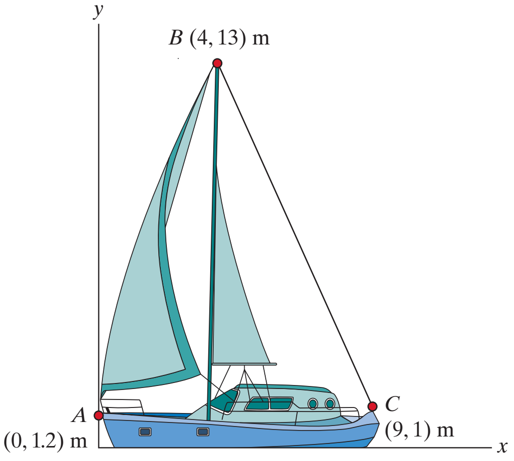
We can use definition of the dot product to determine the angle between OA and OB:

$$\cos\theta = \frac{\mathbf{e}_{OA} \cdot \mathbf{e}_{OB}}{|\mathbf{e}_{OA}||\mathbf{e}_{OB}|}$$
$$= \frac{(\cos 35°\cos 15°)(-\cos 60°\sin 40°) + (\sin 35°)(\sin 60°) + (-\cos 35°\sin 15°)(\cos 60°\cos 40°)}{(1)(1)}$$
$$= 0.161.$$

We obtain $\theta = 80.7°$.

$\boxed{80.7°.}$

Problem 2.107 Use the dot product to determine the angle between the forestay (cable AB) and the backstay (cable BC) of the sailboat.



Solution: The unit vector from B to A is

$$\mathbf{e}_{BA} = \frac{\mathbf{r}_{BA}}{|\mathbf{r}_{BA}|} = -0.321\mathbf{i} - 0.947\mathbf{j}$$

The unit vector from B to C is

$$\mathbf{e}_{BC} = \frac{\mathbf{r}_{BC}}{|\mathbf{r}_{BC}|} = 0.385\mathbf{i} - 0.923\mathbf{j}$$

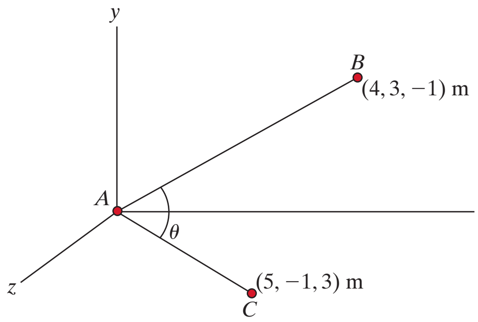
From the definition of the dot product, $\mathbf{e}_{BA} \cdot \mathbf{e}_{BC} = 1 \cdot 1 \cdot \cos\theta$, where θ is the angle between BA and BC. Thus

$$\cos\theta = (-0.321)(0.385) + (-0.947)(-0.923)$$

$$\cos\theta = 0.750$$

$$\theta = 41.3^\circ$$

Problem 2.108 Determine the angle θ between the lines AB and AC (a) by using the law of cosines (see Appendix A); (b) by using the dot product.



Solution:

(a) We have the distances:

$$AB = \sqrt{4^2 + 3^2 + 1^2}\text{ m} = \sqrt{26}\text{ m}$$

$$AC = \sqrt{5^2 + 1^2 + 3^2}\text{ m} = \sqrt{35}\text{ m}$$

$$BC = \sqrt{(5-4)^2 + (-1-3)^2 + (3+1)^2}\text{ m} = \sqrt{33}\text{ m}$$

The law of cosines gives

$$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos\theta$$

$$\cos\theta = \frac{AB^2 + AC^2 - BC^2}{2(AB)(AC)} = 0.464 \Rightarrow \boxed{\theta = 62.3^\circ}$$

(b) Using the dot product

$$\mathbf{r}_{AB} = (4\mathbf{i} + 3\mathbf{j} - \mathbf{k})\text{ m}, \quad \mathbf{r}_{AC} = (5\mathbf{i} - \mathbf{j} + 3\mathbf{k})\text{ m}$$

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AC} = (4\text{ m})(5\text{ m}) + (3\text{ m})(-1\text{ m}) + (-1\text{ m})(3\text{ m}) = 14\text{ m}^2$$

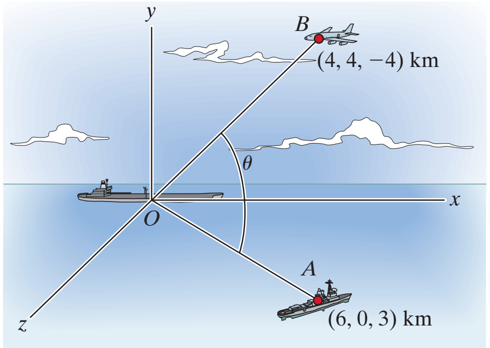
$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AC} = (AB)(AC)\cos\theta$$

Therefore

$$\boxed{\cos\theta = \frac{14\text{ m}^2}{\sqrt{26}\text{ m}\sqrt{35}\text{ m}} = 0.464 \Rightarrow \theta = 62.3^\circ}$$

$\boxed{62.3^\circ.}$

Problem 2.109 The ship O measures the positions of the ship A and the airplane B and obtains the coordinates shown. What is the angle θ between the lines of sight OA and OB?



Solution: From the coordinates, the position vectors are: $\mathbf{r}_{OA} = 6\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}$ and $\mathbf{r}_{OB} = 4\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$ The dot product:

$$\mathbf{r}_{OA} \cdot \mathbf{r}_{OB} = (6)(4) + (0)(4) + (3)(-4) = 12$$

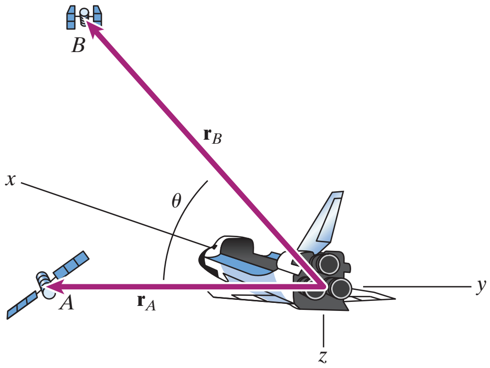
The magnitudes: $|\mathbf{r}_{OA}| = \sqrt{6^2 + 0^2 + 3^2} = 6.71$ km and

$$|\mathbf{r}_{OA}| = \sqrt{4^2 + 4^2 + 4^2} = 6.93 \text{ km.}$$

From Eq. (2.24) $\cos\theta = \dfrac{\mathbf{r}_{OA} \cdot \mathbf{r}_{OB}}{|\mathbf{r}_{OA}||\mathbf{r}_{OB}|} = 0.2581$, from which $\theta = \pm 75°$.

From the problem and the construction, only the positive angle makes sense, hence $\boxed{\theta = 75°}$

Problem 2.110 Astronauts on the space shuttle use radar to determine the magnitudes and direction cosines of the position vectors of two satellites A and B. The vector $\mathbf{r}_A$ from the shuttle to satellite A has magnitude 2 km and direction cosines $\cos\theta_x = 0.768$, $\cos\theta_y = 0.384$, $\cos\theta_z = 0.512$. The vector $\mathbf{r}_B$ from the shuttle to satellite B has magnitude 4 km and direction cosines $\cos\theta_x = 0.743$, $\cos\theta_y = 0.557$, $\cos\theta_z = -0.371$. What is the angle θ between the vectors $\mathbf{r}_A$ and $\mathbf{r}_B$?



Solution: The direction cosines of a vector are the components of a unit vector having the same direction as the vector. We are given the components of unit vectors $\mathbf{e}_A$ and $\mathbf{e}_B$ having the same directions as the vectors $\mathbf{r}_A$ and $\mathbf{r}_B$:

$$\mathbf{e}_A = 0.768\mathbf{i} + 0.384\mathbf{j} + 0.512\mathbf{k},$$
$$\mathbf{e}_B = 0.743\mathbf{i} + 0.557\mathbf{j} - 0.371\mathbf{k}.$$

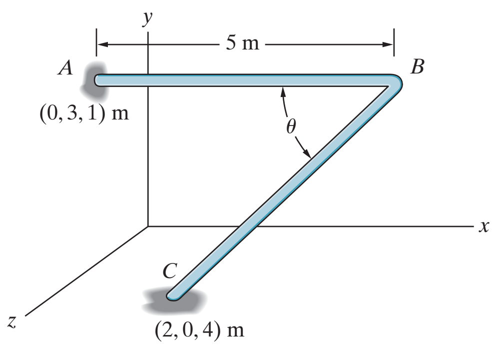
From the definition of the dot product, we have

$$\cos\theta = \frac{\mathbf{e}_A \cdot \mathbf{e}_B}{|\mathbf{e}_A||\mathbf{e}_B|} = \frac{(0.768)(0.743) + (0.384)(0.557) + (0.512)(-0.371)}{1} = 0.595,$$

from which we obtain

$\boxed{\theta = 53.5°.}$

Problem 2.111 Segment AB of the bar is parallel to the x-axis. Determine the angle θ between the straight segments AB and BC.



Solution: The coordinates of point B are (5, 3, 1) m. The vector from B to A is $\mathbf{r}_{BA} = -5\mathbf{i}$ (m). The vector from B to C is

$$\mathbf{r}_{BC} = (2\text{ m} - 5\text{ m})\mathbf{i} + (0 - 3\text{ m})\mathbf{j} + (4\text{ m} - 1\text{ m})\mathbf{k}$$
$$= -3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}\text{ (m)}.$$

The magnitudes of these vectors are

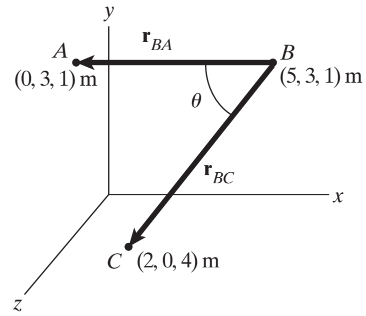
$$|\mathbf{r}_{BA}| = 5\text{ m},$$
$$|\mathbf{r}_{BC}| = \sqrt{(-3\text{ m})^2 + (-3\text{ m})^2 + (3\text{ m})^2}$$
$$= 5.20\text{ m}.$$

The value of their dot product is

$$\mathbf{r}_{BA} \cdot \mathbf{r}_{BC} = (-5\text{ m})(-3\text{ m}) + (0)(-3\text{ m}) + (0)(3\text{ m})$$
$$= 15\text{ m}^2.$$

Using the definition of the dot product,

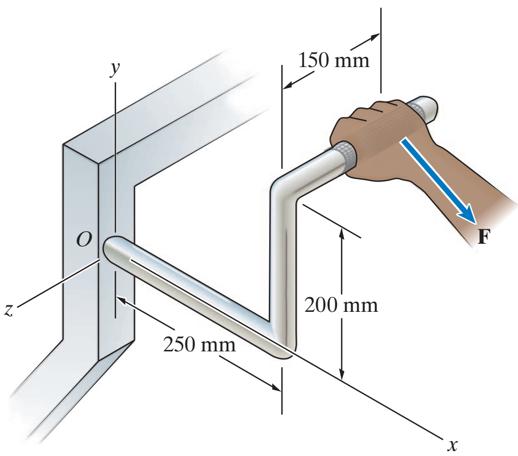
$$\cos\theta = \frac{\mathbf{r}_{BA} \cdot \mathbf{r}_{BC}}{|\mathbf{r}_{BA}||\mathbf{r}_{BC}|}$$
$$= \frac{15\text{ m}^2}{(5\text{ m})(5.20\text{ m})}$$
$$= 0.577.$$



This yields

$$\boxed{\theta = 54.7^\circ.}$$

Problem 2.112 The person exerts a force $\mathbf{F} = 60\mathbf{i} - 40\mathbf{j}$ (N) on the handle of the exercise machine. Use Eq. (2.26) to determine the vector component of $\mathbf{F}$ that is parallel to the line from the origin O to where the person grips the handle.



Solution: The vector $\mathbf{r}$ from the O to where the person grips the handle is

$$\mathbf{r} = (250\mathbf{i} + 200\mathbf{j} - 150\mathbf{k})\text{ mm},$$

$$|\mathbf{r}| = 354\text{ mm}$$

To produce the unit vector that is parallel to this line we divide by the magnitude

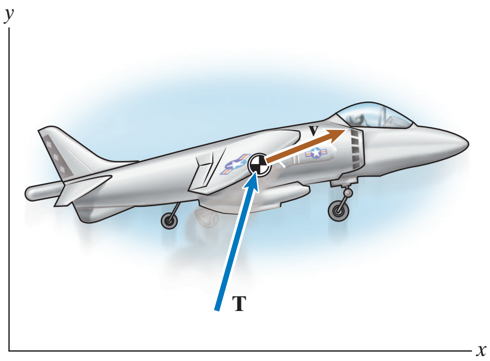
$$\mathbf{e} = \frac{\mathbf{r}}{|\mathbf{r}|} = \frac{(250\mathbf{i} + 200\mathbf{j} - 150\mathbf{k})\text{ mm}}{354\text{ mm}} = (0.707\mathbf{i} + 0.566\mathbf{j} - 0.424\mathbf{k})$$

Using Eq. (2.26), we find that the vector component parallel to the line is

$$\mathbf{F}_p = (\mathbf{e} \cdot \mathbf{F})\mathbf{e} = [(0.707)(60\text{ N}) + (0.566)(-40\text{ N})](0.707\mathbf{i} + 0.566\mathbf{j} - 0.424\mathbf{k})$$

$$\boxed{14.0\mathbf{i} + 11.2\mathbf{j} - 8.40\mathbf{k}\text{ (N).}}$$

Problem 2.113 At the instant shown, the Harrier's thrust vector is $\mathbf{T} = 4.22\mathbf{i} + 21.6\mathbf{j} - 3.30\mathbf{k}$ (kip) and its velocity vector is $\mathbf{v} = 24.4\mathbf{i} + 8.86\mathbf{j} + 2.44\mathbf{k}$ (ft/s). (a) Determine the components of the thrust vector parallel and normal (perpendicular) to the velocity vector. (b) The term $\mathbf{T} \cdot \mathbf{v}$ is the power being transferred to the plane by its engine. What is the power in horsepower?



Solution:

(a) We divide the velocity vector by its magnitude to obtain a unit vector with the same direction:

$$\mathbf{e} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{24.4\mathbf{i} + 8.86\mathbf{j} + 2.44\mathbf{k}\ \text{(ft/s)}}{\sqrt{(24.4\ \text{ft/s})^2 + (8.86\ \text{ft/s})^2 + (2.44\ \text{ft/s})^2}} = 0.936\mathbf{i} + 0.340\mathbf{j} + 0.0936\mathbf{k}.$$

The component of $\mathbf{T}$ parallel to the velocity vector is

$$\mathbf{T}_p = (\mathbf{e} \cdot \mathbf{T})\mathbf{e}.$$

The dot product of $\mathbf{e}$ and $\mathbf{T}$ is

$$\mathbf{e} \cdot \mathbf{T} = (0.936)(4.22\ \text{kip}) + (0.340)(21.6\ \text{kip}) + (0.0936)(-3.30\ \text{kip}) = 11.0\ \text{kip}.$$

The parallel component is

$$\mathbf{T}_p = (\mathbf{e} \cdot \mathbf{T})\mathbf{e} = (11.0\ \text{kip})(0.936\mathbf{i} + 0.340\mathbf{j} + 0.0936\mathbf{k}) = 10.3\mathbf{i} + 3.73\mathbf{j} + 1.03\mathbf{k}\ \text{(kip)}.$$

The component of $\mathbf{T}$ normal to the velocity vector is

$$\mathbf{T}_n = \mathbf{T} - \mathbf{T}_p = -6.06\mathbf{i} + 17.9\mathbf{j} - 4.33\mathbf{k}\ \text{(kip)}.$$

(b) The dot product of $\mathbf{T}$ and $\mathbf{v}$ is

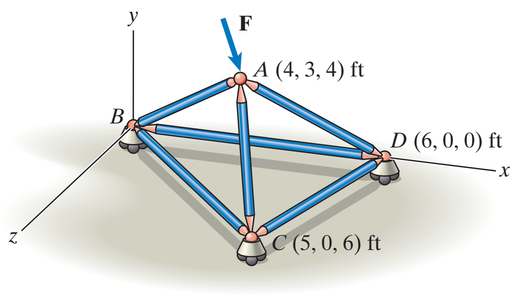
$$\mathbf{T} \cdot \mathbf{v} = (4.22\ \text{kip})(24.4\ \text{ft/s}) + (21.6\ \text{kip})(8.86\ \text{ft/s}) + (-3.30\ \text{kip})(2.44\ \text{ft/s}) = 286\ \text{kip-ft/s} = 286{,}000\ \text{ft-lb/s}.$$

Converting this to horsepower,

$$286{,}000\ \text{ft-lb/s} = 286{,}000\ \text{ft-lb/s}\left(\frac{1\ \text{horsepower}}{550\ \text{ft-lb/s}}\right) = 521\ \text{horsepower}.$$

(a) $\mathbf{T}_p = 10.3\mathbf{i} + 3.73\mathbf{j} + 1.03\mathbf{k}$ (kip), $\mathbf{T}_n = -6.06\mathbf{i} + 17.9\mathbf{j} - 4.33\mathbf{k}$ (kip).
(b) 521 horsepower.

Problem 2.114 For the three-dimensional truss shown, determine the angle between members AB and AD.



Solution: The vector from A to B is

$$\mathbf{r}_{AB} = -4\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}\ \text{(ft)}.$$

The vector from A to D is

$$\begin{aligned}\mathbf{r}_{AD} &= (6\text{ ft} - 4\text{ ft})\mathbf{i} + (0 - 3\text{ ft})\mathbf{j} + (0 - 4\text{ ft})\mathbf{k}\\ &= 2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}\ \text{(ft)}.\end{aligned}$$

The magnitudes of these vectors are

$$\begin{aligned}|\mathbf{r}_{AB}| &= \sqrt{(-4\text{ ft})^2 + (-3\text{ ft})^2 + (-4\text{ ft})^2}\\ &= 6.40\text{ ft},\\ |\mathbf{r}_{AD}| &= \sqrt{(2\text{ ft})^2 + (-3\text{ ft})^2 + (-4\text{ ft})^2}\\ &= 5.39\text{ ft}.\end{aligned}$$

The value of their dot product is

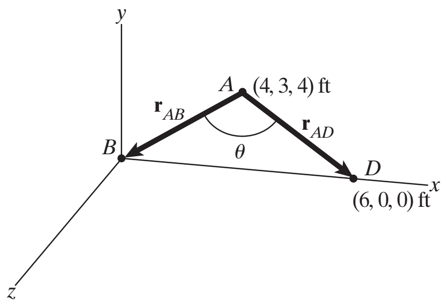
$$\begin{aligned}\mathbf{r}_{AB}\cdot\mathbf{r}_{AD} &= (-4\text{ ft})(2\text{ ft}) + (-3\text{ ft})(-3\text{ ft}) + (-4)(-4\text{ ft})\\ &= 17\text{ ft}^2.\end{aligned}$$

Using the definition of the dot product,

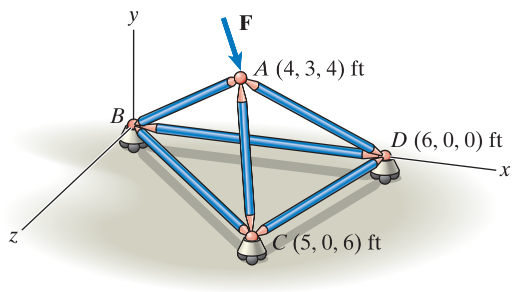
$$\begin{aligned}\cos\theta &= \frac{\mathbf{r}_{BA}\cdot\mathbf{r}_{BC}}{|\mathbf{r}_{BA}||\mathbf{r}_{BC}|}\\ &= \frac{17\text{ ft}^2}{(6.40\text{ ft})(5.39\text{ ft})}\\ &= 0.493.\end{aligned}$$

This yields

$\boxed{60.5^\circ.}$



Problem 2.115 For the three-dimensional truss shown, determine the angle between members AC and CD.



Solution: The vector from C to A is

$$\begin{aligned}\mathbf{r}_{CA} &= (4\text{ ft} - 5\text{ ft})\mathbf{i} + (3\text{ ft} - 0)\mathbf{j} + (4\text{ ft} - 6\text{ ft})\mathbf{k}\\ &= -\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\ \text{(ft)}.\end{aligned}$$

The vector from C to D is

$$\begin{aligned}\mathbf{r}_{CD} &= (6\text{ ft} - 5\text{ ft})\mathbf{i} + (0 - 0)\mathbf{j} + (0 - 6\text{ ft})\mathbf{k}\\ &= \mathbf{i} - 6\mathbf{k}\ \text{(ft)}.\end{aligned}$$

The magnitudes of these vectors are

$$\begin{aligned}|\mathbf{r}_{CA}| &= \sqrt{(-1\text{ ft})^2 + (3\text{ ft})^2 + (-2\text{ ft})^2}\\ &= 3.74\text{ ft},\\ |\mathbf{r}_{CD}| &= \sqrt{(1\text{ ft})^2 + (0)^2 + (-6\text{ ft})^2}\\ &= 6.08\text{ ft}.\end{aligned}$$

The value of their dot product is

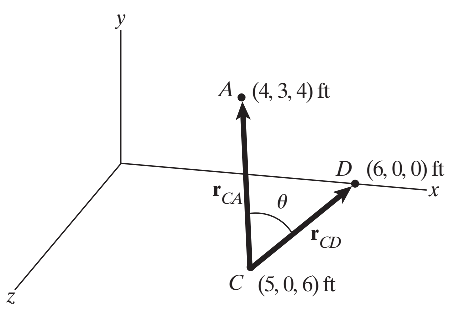
$$\begin{aligned}\mathbf{r}_{CA}\cdot\mathbf{r}_{CD} &= (-1\text{ ft})(1\text{ ft}) + (3\text{ ft})(0) + (-2)(-6\text{ ft})\\ &= 11\text{ ft}^2.\end{aligned}$$

Using the definition of the dot product,

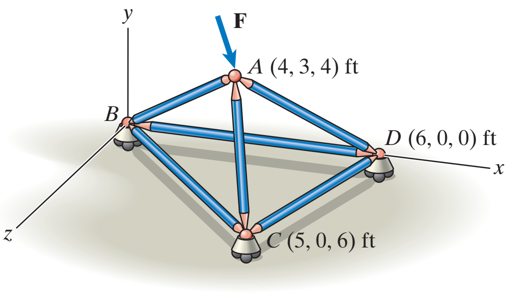
$$\begin{aligned}\cos\theta &= \frac{\mathbf{r}_{CA}\cdot\mathbf{r}_{CD}}{|\mathbf{r}_{CA}||\mathbf{r}_{CD}|}\\ &= \frac{11\text{ ft}^2}{(3.74\text{ ft})(6.08\text{ ft})}\\ &= 0.483.\end{aligned}$$

This yields

$\boxed{61.1^\circ.}$



Problem 2.116 The three-dimensional truss is subjected to a force $\mathbf{F} = 25\mathbf{i} - 100\mathbf{j} - 30\mathbf{k}$ (lb) at A. Determine the components of $\mathbf{F}$ parallel and normal (perpendicular) to member AD.



Solution: The vector from A to D is

$$\mathbf{r}_{AD} = (6\text{ ft} - 4\text{ ft})\mathbf{i} + (0 - 3\text{ ft})\mathbf{j} + (0 - 4\text{ ft})\mathbf{k}$$
$$= 2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}\text{ (ft)}.$$

Dividing this vector by its magnitude yields a unit vector that points from A toward D:

$$\mathbf{e}_{AD} = \frac{\mathbf{r}_{AD}}{|\mathbf{r}_{AD}|}$$
$$= \frac{2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}\text{ (ft)}}{\sqrt{(2\text{ ft})^2 + (-3\text{ ft})^2 + (-4\text{ ft})^2}}$$
$$= 0.371\mathbf{i} - 0.557\mathbf{j} - 0.743\mathbf{k}.$$

The component of $\mathbf{F}$ parallel to member AD is

$$\mathbf{F}_\text{p} = (\mathbf{e}_{AD} \cdot \mathbf{F})\mathbf{e}_{AD}.$$

The dot product of $\mathbf{e}_{AD}$ and $\mathbf{F}$ is

$$\mathbf{e}_{AD} \cdot \mathbf{F} = (0.371)(25\text{ lb}) + (-0.557)(-100\text{ lb}) + (-0.743)(-30\text{ lb})$$
$$= 87.3\text{ lb}.$$

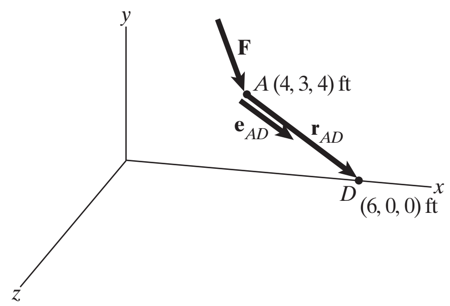
The parallel component is

$$\mathbf{F}_\text{p} = (\mathbf{e}_{AD} \cdot \mathbf{F})\mathbf{e}_{AD}$$
$$= (87.3\text{ lb})(0.371\mathbf{i} - 0.557\mathbf{j} - 0.743\mathbf{k})$$
$$= 32.4\mathbf{i} - 48.6\mathbf{j} - 64.8\mathbf{k}\text{ (lb)}.$$

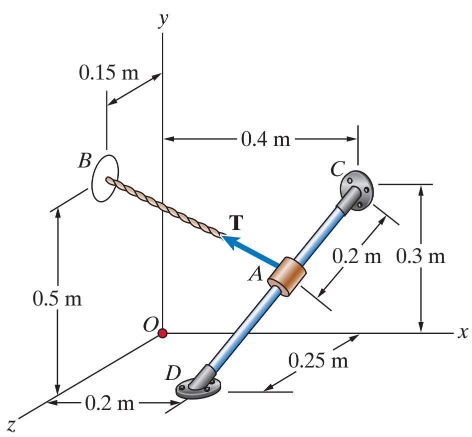
The component of $\mathbf{F}$ normal to member AD is

$$\mathbf{F}_\text{n} = \mathbf{F} - \mathbf{F}_\text{p}$$
$$= -7.41\mathbf{i} - 51.4\mathbf{j} + 34.8\mathbf{k}\text{ (lb)}.$$

$$\boxed{\mathbf{F}_\text{p} = 32.4\mathbf{i} - 48.6\mathbf{j} - 64.8\mathbf{k}\text{ (lb)},\ \mathbf{F}_\text{n} = -7.41\mathbf{i} - 51.4\mathbf{j} + 34.8\mathbf{k}\text{ (lb)}.}$$



Problem 2.117 The rope AB exerts a 50-N force $\mathbf{T}$ on collar A. Determine the vector component of $\mathbf{T}$ parallel to the bar CD.



Solution: We need to know the coordinates of point A. The vector from point D to point C is

$$\mathbf{r}_{DC} = (0.4\text{ m} - 0.2\text{ m})\mathbf{i} + (0.3\text{ m} - 0)\mathbf{j} + (0 - 0.25\text{ m})\mathbf{k}$$
$$= 0.2\mathbf{i} + 0.3\mathbf{j} - 0.25\mathbf{k}\text{ (m)}.$$

Its magnitude (the distance from D to C) is $|\mathbf{r}_{DC}| = 0.439$ m, so the distance from D to A is 0.239 m. The unit vector that points from D toward C is

$$\mathbf{e}_{DC} = \frac{\mathbf{r}_{DC}}{|\mathbf{r}_{DC}|} = 0.456\mathbf{i} + 0.684\mathbf{j} - 0.570\mathbf{k}.$$

The vector from the origin O to D is $\mathbf{r}_{OD} = 0.2\mathbf{i} + 0.25\mathbf{k}$ (m). Then the vector from the origin to A is

$$\mathbf{r}_{OA} = \mathbf{r}_{OD} + (0.239\text{ m})\mathbf{e}_{DC} = 0.309\mathbf{i} + 0.163\mathbf{j} + 0.114\mathbf{k}\text{ (m)}.$$

The vector from A to B is

$$\mathbf{r}_{AB} = (0 - 0.309\text{ m})\mathbf{i} + (0.5\text{ m} - 0.163\text{ m})\mathbf{j} + (0.15\text{ m} - 0.114\text{ m})\mathbf{k}$$
$$= -0.309\mathbf{i} + 0.337\mathbf{j} + 0.0360\mathbf{k}\text{ (m)}.$$

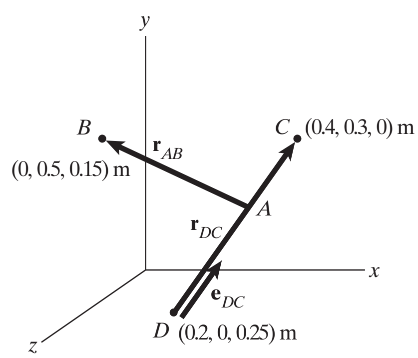
The force $\mathbf{T}$ in terms of its components is

$$\mathbf{T} = (50\text{ N})\frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = -33.7\mathbf{i} + 36.7\mathbf{j} + 3.93\mathbf{k}\text{ (N)}.$$

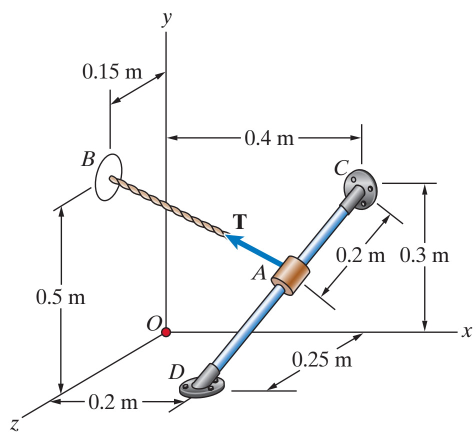
The component of $\mathbf{T}$ parallel to the bar is

$$\mathbf{T}_\text{p} = (\mathbf{e}_{DC} \cdot \mathbf{T})\mathbf{e}_{DC} = 3.43\mathbf{i} + 5.14\mathbf{j} - 4.29\mathbf{k}\text{ (N)}.$$

$$\boxed{\mathbf{T}_\text{p} = 3.43\mathbf{i} + 5.14\mathbf{j} - 4.29\mathbf{k}\text{ (N)}.}$$



Problem 2.118 In Problem 2.117, determine the vector component of **T** normal to the bar CD.



Solution: We need to know the coordinates of point A. The vector from point D to point C is

$$\mathbf{r}_{DC} = (0.4\text{ m} - 0.2\text{ m})\mathbf{i} + (0.3\text{ m} - 0)\mathbf{j} + (0 - 0.25\text{ m})\mathbf{k}$$
$$= 0.2\mathbf{i} + 0.3\mathbf{j} - 0.25\mathbf{k}\text{ (m)}.$$

Its magnitude (the distance from D to C) is $|\mathbf{r}_{DC}| = 0.439$ m, so the distance from D to A is 0.239 m. The unit vector that points from D toward C is

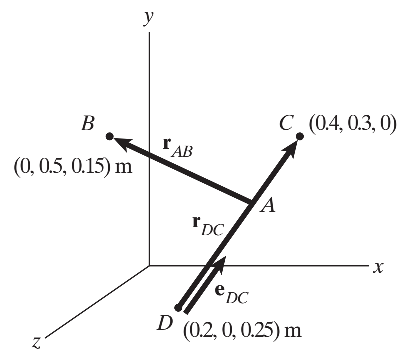
$$\mathbf{e}_{DC} = \frac{\mathbf{r}_{DC}}{|\mathbf{r}_{DC}|} = 0.456\mathbf{i} + 0.684\mathbf{j} - 0.570\mathbf{k}.$$

The vector from the origin O to D is $\mathbf{r}_{OD} = 0.2\mathbf{i} + 0.25\mathbf{k}$ (m). Then the vector from the origin to A is

$$\mathbf{r}_{OA} = \mathbf{r}_{OD} + (0.239\text{ m})\mathbf{e}_{DC} = 0.309\mathbf{i} + 0.163\mathbf{j} + 0.114\mathbf{k}\text{ (m)}.$$

The vector from A to B is

$$\mathbf{r}_{AB} = (0 - 0.309\text{ m})\mathbf{i} + (0.5\text{ m} - 0.163\text{ m})\mathbf{j} + (0.15\text{ m} - 0.114\text{ m})\mathbf{k}$$
$$= -0.309\mathbf{i} + 0.337\mathbf{j} + 0.0360\mathbf{k}\text{ (m)}.$$



The force **T** in terms of its components is

$$\mathbf{T} = (50\text{ N})\frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = -33.7\mathbf{i} + 36.7\mathbf{j} + 3.93\mathbf{k}\text{ (N)}.$$

The component of **T** parallel to the bar is

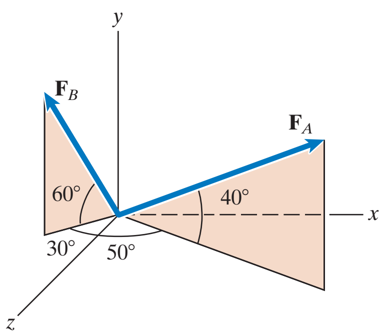
$$\mathbf{T}_{\text{p}} = (\mathbf{e}_{DC}\cdot\mathbf{T})\mathbf{e}_{DC} = 3.43\mathbf{i} + 5.14\mathbf{j} - 4.29\mathbf{k}\text{ (N)}.$$

Then the component normal to the bar is

$$\mathbf{T}_{\text{n}} = \mathbf{T} - \mathbf{T}_{\text{p}} = -37.1\mathbf{i} + 31.6\mathbf{j} + 8.22\mathbf{k}\text{ (N)}.$$

$$\boxed{\mathbf{T}_{\text{n}} = -37.1\mathbf{i} + 31.6\mathbf{j} + 8.22\mathbf{k}\text{ (N)}.}$$

Problem 2.119 The magnitudes of the two force vectors are $|\mathbf{F}_A| = 200$ lb and $|\mathbf{F}_B| = 160$ lb. What is the angle between the two vectors?



Solution: The components of $\mathbf{F}_A$ are

$$\mathbf{F}_A = (200\text{ lb})(\cos 40°\sin 50°\mathbf{i} + \sin 40°\mathbf{j} + \cos 40°\cos 50°\mathbf{k})$$
$$= 117\mathbf{i} + 129\mathbf{j} + 98.5\mathbf{k}\text{ (lb)}.$$

The components of $\mathbf{F}_B$ are

$$\mathbf{F}_B = (160\text{ lb})(-\cos 60°\sin 30°\mathbf{i} + \sin 60°\mathbf{j} + \cos 60°\cos 30°\mathbf{k})$$
$$= -40.0\mathbf{i} + 139\mathbf{j} + 69.3\mathbf{k}\text{ (lb)}.$$

Their dot product is

$$\mathbf{F}_A\cdot\mathbf{F}_B = (117\text{ lb})(-40.0\text{ lb}) + (129\text{ lb})(139\text{ lb}) + (98.5\text{ lb})(69.3\text{ lb})$$
$$= 19{,}900\text{ lb}^2.$$

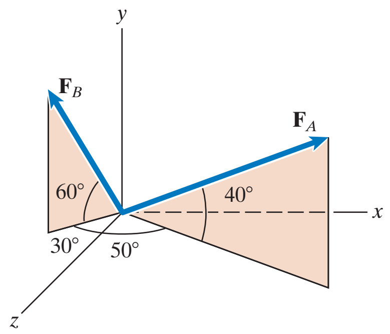
Using the definition of the dot product,

$$\cos\theta = \frac{\mathbf{F}_A\cdot\mathbf{F}_B}{|\mathbf{F}_A||\mathbf{F}_B|}$$
$$= \frac{19{,}900\text{ lb}^2}{(200\text{ lb})(160\text{ lb})}$$
$$= 0.623.$$

This yields

$\boxed{51.5°.}$

Problem 2.120 The magnitudes of the two force vectors are $|\mathbf{F}_A| = 200$ lb and $|\mathbf{F}_B| = 160$ lb. Determine the components of $\mathbf{F}_A$ parallel and normal to $\mathbf{F}_B$.



Solution: The components of $\mathbf{F}_A$ are

$$\mathbf{F}_A = (200 \text{ lb})(\cos 40^\circ \sin 50^\circ \mathbf{i} + \sin 40^\circ \mathbf{j} + \cos 40^\circ \cos 50^\circ \mathbf{k})$$
$$= 117\mathbf{i} + 129\mathbf{j} + 98.5\mathbf{k} \text{ (lb)}.$$

The components of $\mathbf{F}_B$ are

$$\mathbf{F}_B = (160 \text{ lb})(-\cos 60^\circ \sin 30^\circ \mathbf{i} + \sin 60^\circ \mathbf{j} + \cos 60^\circ \cos 30^\circ \mathbf{k})$$
$$= -40.0\mathbf{i} + 139\mathbf{j} + 69.3\mathbf{k} \text{ (lb)}.$$

We divide $\mathbf{F}_B$ by its magnitude to determine the components of a unit vector with the same direction:

$$\mathbf{e}_B = \frac{\mathbf{F}_B}{|\mathbf{F}_B|}$$
$$= \frac{-40.0\mathbf{i} + 139\mathbf{j} + 69.3\mathbf{k} \text{ (lb)}}{\sqrt{(-40.0 \text{ lb})^2 + (139 \text{ lb})^2 + (69.3 \text{ lb})^2}}$$
$$= -0.250\mathbf{i} + 0.866\mathbf{j} + 0.433\mathbf{k}.$$

The dot product of $\mathbf{e}_B$ with $\mathbf{F}_A$ is

$$\mathbf{e}_B \cdot \mathbf{F}_A = (-0.250)(117 \text{ lb}) + (0.866)(129 \text{ lb}) + (0.433)(98.5 \text{ lb})$$
$$= 125 \text{ lb}.$$

The component of $\mathbf{F}_A$ that is parallel to $\mathbf{F}_B$ is

$$\mathbf{F}_{Ap} = (\mathbf{e}_B \cdot \mathbf{F}_A)\mathbf{e}_B$$
$$= (125 \text{ lb})(-0.250\mathbf{i} + 0.866\mathbf{j} + 0.433\mathbf{k})$$
$$= -31.2\mathbf{i} + 108\mathbf{j} + 54.0\mathbf{k} \text{ (lb)}.$$

The component of $\mathbf{F}_A$ that is normal to $\mathbf{F}_B$ is

$$\mathbf{F}_{An} = \mathbf{F} - \mathbf{F}_{Ap}$$
$$= 149\mathbf{i} + 20.6\mathbf{j} + 44.5\mathbf{k} \text{ (lb)}.$$

$$\boxed{\mathbf{F}_{Ap} = -31.2\mathbf{i} + 108\mathbf{j} + 54.0\mathbf{k} \text{ (lb)}, \mathbf{F}_{An} = 149\mathbf{i} + 20.6\mathbf{j} + 44.5\mathbf{k} \text{ (lb)}.}$$

Problem 2.121 An astronaut in a maneuvering unit approaches the ISS. At the present instant, the station informs him that his position relative to the origin of the station's coordinate system is $\mathbf{r}_G = 50\mathbf{i} + 80\mathbf{j} + 180\mathbf{k}$ (m) and his velocity is $\mathbf{v} = -2.2\mathbf{j} - 3.6\mathbf{k}$ (m/s). The position of an airlock is $\mathbf{r}_A = -12\mathbf{i} + 20\mathbf{k}$ (m). Determine the angle between his velocity vector and the line from his position to the airlock's position.

Solution: The vector from the astronaut's position to the airlock is

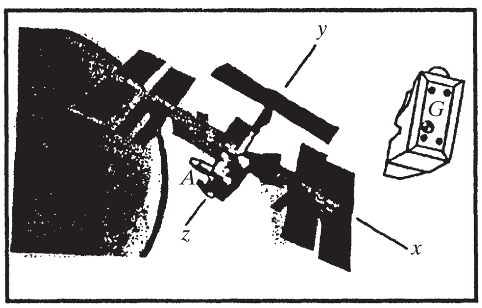
$$\mathbf{r} = \mathbf{r}_A - \mathbf{r}_G = -62\mathbf{i} - 80\mathbf{j} - 160\mathbf{k} \text{ (m)}.$$

Let θ denote the angle between his velocity vector and the line from his position to the airlock. Using the definition of the dot product,

$$\cos\theta = \frac{\mathbf{v} \cdot \mathbf{r}}{|\mathbf{v}||\mathbf{r}|} = 0.942.$$

We obtain

$$\boxed{\theta = 19.7^\circ.}$$



Problem 2.122 In Problem 2.121, determine the vector component of the astronaut's velocity parallel to the line from his position to the airlock's position.

Solution: The vector from the astronaut's position to the airlock is

$$\mathbf{r} = \mathbf{r}_A - \mathbf{r}_G = -62\mathbf{i} - 80\mathbf{j} - 160\mathbf{k} \text{ (m)}.$$

We divide this vector by its magnitude to obtain a unit vector that points from the astronaut's position toward the airlock:

$$\mathbf{e} = \frac{\mathbf{r}}{|\mathbf{r}|} = -0.328\mathbf{i} - 0.423\mathbf{j} - 0.845\mathbf{k}.$$

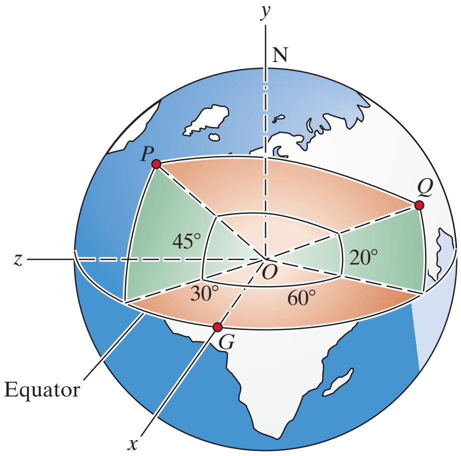
The component of $\mathbf{v}$ parallel to $\mathbf{e}$ is

$$\mathbf{v}_p = (\mathbf{e}\cdot\mathbf{v})\mathbf{e} = -1.30\mathbf{i} - 1.68\mathbf{j} - 3.36\mathbf{k} \text{ (m/s)}.$$

$$\boxed{\mathbf{v}_p = -1.30\mathbf{i} - 1.68\mathbf{j} - 3.36\mathbf{k} \text{ (m/s)}.}$$

Problem 2.123 Point P is at longitude 30°W and latitude 45°N on the Atlantic Ocean between Nova Scotia and France. Point Q is at longitude 60°E and latitude 20°N in the Arabian Sea. Use the dot product to determine the shortest distance along the surface of the earth from P to Q in terms of the radius of the earth R_E.

Strategy: Use the dot product to determine the angle between the lines OP and OQ; then use the definition of an angle in radians to determine the distance along the surface of the earth from P to Q.



Solution: The vector from O to P is

$$\mathbf{r}_{OP} = R_E(\cos 45^\circ \cos 30^\circ \mathbf{i} + \sin 45^\circ \mathbf{j} + \cos 45^\circ \sin 30^\circ \mathbf{k})$$
$$= R_E(0.612\mathbf{i} + 0.707\mathbf{j} + 0.354\mathbf{k}).$$

The vector from O to Q is

$$\mathbf{r}_{OQ} = R_E(\cos 20^\circ \cos 60^\circ \mathbf{i} + \sin 20^\circ \mathbf{j} - \cos 20^\circ \sin 60^\circ \mathbf{k})$$
$$= R_E(0.470\mathbf{i} + 0.342\mathbf{j} - 0.814\mathbf{k}).$$

Let θ denote the angle between these two vectors. Using the definition of the dot product, we have

$$\cos\theta = \frac{\mathbf{r}_{OP}\cdot\mathbf{r}_{OQ}}{|\mathbf{r}_{OP}||\mathbf{r}_{OQ}|} = 0.242.$$

Solving for the angle in radians, this gives $\theta = 1.33$. From the definition of an angle in radians, the desired distance is

$$\boxed{1.33R_E.}$$

Problem 2.124 Consider the vectors

$$\mathbf{U} = 12\mathbf{i} + 4\mathbf{j} - 8\mathbf{k},$$
$$\mathbf{V} = -6\mathbf{i} + 8\mathbf{j} + 14\mathbf{k}.$$

(a) Determine the components of the vector $\mathbf{W} = \mathbf{U} \times \mathbf{V}$.
(b) Evaluate the dot products $\mathbf{U} \cdot \mathbf{W}$ and $\mathbf{V} \cdot \mathbf{W}$. What is the explanation of these two results?

Solution:

(a) The vector $\mathbf{W}$ is

$$\begin{aligned}\mathbf{W} &= \mathbf{U} \times \mathbf{V} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 4 & -8 \\ -6 & 8 & 14 \end{vmatrix} \\ &= [(4)(14) - (-8)(8)]\mathbf{i} - [(12)(14) - (-8)(-6)]\mathbf{j} + [(12)(8) - (4)(-6)]\mathbf{k} \\ &= 120\mathbf{i} - 120\mathbf{j} + 120\mathbf{k}.\end{aligned}$$

(b) The dot products are

$$\begin{aligned}\mathbf{U} \cdot \mathbf{W} &= (12)(120) + (4)(-120) + (-8)(120) \\ &= 0, \\ \mathbf{V} \cdot \mathbf{W} &= (-6)(120) + (8)(-120) + (14)(120) \\ &= 0.\end{aligned}$$

(a) $\mathbf{W} = 120\mathbf{i} - 120\mathbf{j} + 120\mathbf{k}$. (b) $\mathbf{U} \cdot \mathbf{W} = 0$, $\mathbf{V} \cdot \mathbf{W} = 0$.
From the definition of the cross product, $\mathbf{W}$ is perpendicular to both $\mathbf{U}$ and $\mathbf{V}$.

Problem 2.125 Two vectors $\mathbf{U} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{V} = 2\mathbf{i} + 4\mathbf{j}$. (a) What is the cross product $\mathbf{U} \times \mathbf{V}$? (b) What is the cross product $\mathbf{V} \times \mathbf{U}$?

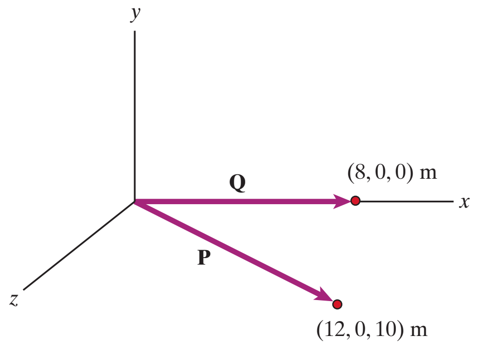
Solution: Use Eq. (2.34) and expand into 2 by 2 determinants.

$$\mathbf{U} \times \mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 0 \\ 2 & 4 & 0 \end{vmatrix} = \mathbf{i}((2)(0) - (4)(0)) - \mathbf{j}((3)(0) - (2)(0)) + \mathbf{k}((3)(4) - (2)(2)) = 8\mathbf{k}$$

$$\mathbf{V} \times \mathbf{U} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \mathbf{i}((4)(0) - (2)(0)) - \mathbf{j}((2)(0) - (3)(0)) + \mathbf{k}((2)(2) - (3)(4)) = -8\mathbf{k}$$

$\boxed{\text{(a) } \mathbf{U} \times \mathbf{V} = 8\mathbf{k}\text{. (b) } \mathbf{V} \times \mathbf{U} = -8\mathbf{k}.}$

Problem 2.126 Consider the position vectors **P** and **Q**. (a) Determine the cross product $\mathbf{P} \times \mathbf{Q}$ by using the definition, Eq. (2.28). (b) Determine the dot product $\mathbf{P} \cdot \mathbf{Q}$ by using Eq. (2.34).



Solution:

(a) The two vectors in terms of their components are

$$\mathbf{P} = 12\mathbf{i} + 10\mathbf{k} \text{ (m)},$$
$$\mathbf{Q} = 8\mathbf{i} \text{ (m)}.$$

Their magnitudes are

$$|\mathbf{P}| = \sqrt{(12 \text{ m})^2 + (10 \text{ m})^2} = 15.6 \text{ m},$$
$$|\mathbf{Q}| = 8 \text{ m}.$$

The angle between the two vectors is

$$\theta = \arctan\left(\frac{10 \text{ m}}{12 \text{ m}}\right) = 39.8^\circ.$$

From Eq. (2.28),

$$\mathbf{P} \times \mathbf{Q} = (15.6 \text{ m})(8 \text{ m})\sin 39.8^\circ \mathbf{e}$$
$$= (80 \text{ m}^2)\mathbf{e}.$$

The unit vector **e** is defined to be perpendicular to **P** and perpendicular to **Q**. Either **j** or $-\mathbf{j}$ satisfies those criteria. The remaining criterion states that **P**, **Q**, **e** form a right-handed system, which requires that $\mathbf{e} = \mathbf{j}$. Therefore,

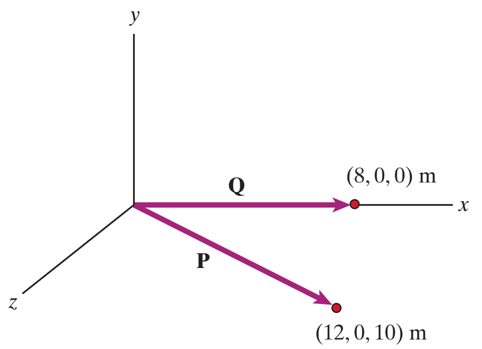
$$\mathbf{P} \times \mathbf{Q} = 80\mathbf{j} \text{ (m}^2).$$

(b) From Eq. (2.34),

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 \text{ m} & 0 & 10 \text{ m} \\ 8 \text{ m} & 0 & 0 \end{vmatrix}$$
$$= [(0)(0) - (10 \text{ m})(0)]\mathbf{i} - [(12 \text{ m})(0) - (10 \text{ m})(8 \text{ m})]\mathbf{j} + [(12 \text{ m})(0) - (0)(8 \text{ m})]\mathbf{k}$$
$$= 80\mathbf{j} \text{ (m}^2).$$

$\boxed{\text{(a), (b) } \mathbf{P} \times \mathbf{Q} = 80\mathbf{j} \text{ (m}^2).}$

Problem 2.127 Consider the position vectors **P** and **Q**. (a) Determine the cross product $\mathbf{Q} \times \mathbf{P}$ by using the definition, Eq. (2.28). (b) Determine the dot product $\mathbf{Q} \cdot \mathbf{P}$ by using Eq. (2.34).



Solution:

(a) The two vectors in terms of their components are

$$\mathbf{P} = 12\mathbf{i} + 10\mathbf{k} \text{ (m)},$$
$$\mathbf{Q} = 8\mathbf{i} \text{ (m)}.$$

Their magnitudes are

$$|\mathbf{P}| = \sqrt{(12 \text{ m})^2 + (10 \text{ m})^2} = 15.6 \text{ m},$$
$$|\mathbf{Q}| = 8 \text{ m}.$$

The angle between the two vectors is

$$\theta = \arctan\left(\frac{10 \text{ m}}{12 \text{ m}}\right) = 39.8^\circ.$$

From Eq. (2.28),

$$\mathbf{Q} \times \mathbf{P} = (8 \text{ m})(15.6 \text{ m})\sin 39.8^\circ \mathbf{e}$$
$$= (80 \text{ m}^2)\mathbf{e}.$$

The unit vector **e** is defined to be perpendicular to **Q** and perpendicular to **P**. Either **j** or $-\mathbf{j}$ satisfies those criteria. The remaining criterion states that **Q**, **P**, **e** form a right-handed system, which requires that $\mathbf{e} = -\mathbf{j}$. Therefore,

$$\mathbf{Q} \times \mathbf{P} = -80\mathbf{j} \text{ (m}^2).$$

(b) From Eq. (2.34),

$$\mathbf{Q} \times \mathbf{P} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 \text{ m} & 0 & 0 \\ 12 \text{ m} & 0 & 10 \text{ m} \end{vmatrix}$$
$$= [(0)(10 \text{ m}) - (0)(0)]\mathbf{i} - [(8 \text{ m})(10 \text{ m}) - (0)(12 \text{ m})]\mathbf{j} + [(8 \text{ m})(0) - (0)(12 \text{ m})]\mathbf{k}$$
$$= -80\mathbf{j} \text{ (m}^2).$$

$\boxed{\text{(a), (b) } \mathbf{Q} \times \mathbf{P} = -80\mathbf{j} \text{ (m}^2).}$

Problem 2.128 Suppose that the cross product of two vectors $\mathbf{U}$ and $\mathbf{V}$ is $\mathbf{U} \times \mathbf{V} = \mathbf{0}$. If $|\mathbf{U}| \neq 0$, what do you know about the vector $\mathbf{V}$?

Solution:

$\boxed{\text{Either } \mathbf{V} = 0 \text{ or } \mathbf{V} \parallel \mathbf{U}}$

Problem 2.129 The cross product of two vectors $\mathbf{U}$ and $\mathbf{V}$ is $\mathbf{U} \times \mathbf{V} = -30\mathbf{i} + 40\mathbf{k}$. The vector $\mathbf{V} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$. The vector $\mathbf{U} = 4\mathbf{i} + U_y\mathbf{j} + U_z\mathbf{k}$. Determine U_y and U_z.

Solution: From the given information we have

$$\mathbf{U} \times \mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & U_y & U_z \\ 4 & -2 & 3 \end{vmatrix}$$

$$= (3U_y + 2U_z)\mathbf{i} + (4U_z - 12)\mathbf{j} + (-8 - 4U_y)\mathbf{k}$$

$$\mathbf{U} \times \mathbf{V} = (-30i + 40\mathbf{k})$$

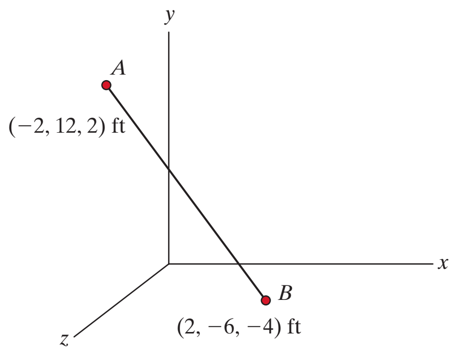
Equating the components we have

$$3U_y + 2U_z = -30, \quad 4U_z - 12 = 0, \quad -8 - 4U_y = 40.$$

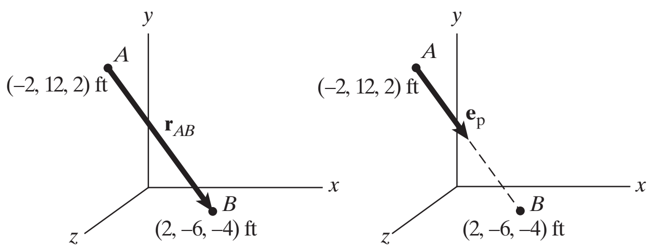
Solving any two of these three redundant equations gives

$$\boxed{U_y = -12, U_z = 3.}$$

Problem 2.130 (a) Determine the components of a unit vector $\mathbf{e}_p$ that is parallel to the line AB and points from A toward B. (b) Determine the components of a unit vector $\mathbf{e}_n$ that is normal (perpendicular) to the line AB. (Make sure your result is a unit vector.) Confirm that it is perpendicular to the line by showing that $\mathbf{e}_p \cdot \mathbf{e}_n = 0$.



Solution:



(a) The position vector from point A to point B is

$$\mathbf{r}_{AB} = (2 \text{ ft} - (-2 \text{ ft}))\mathbf{i} + (-6 \text{ ft} - 12 \text{ ft})\mathbf{j} + (-4 \text{ ft} - 2 \text{ ft})\mathbf{k}$$
$$= 4\mathbf{i} - 18\mathbf{j} - 6\mathbf{k} \text{ (ft)}.$$

Dividing this vector by its magnitude yields the unit vector $\mathbf{e}_p$:

$$\mathbf{e}_p = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|}$$
$$= \frac{4\mathbf{i} - 18\mathbf{j} - 6\mathbf{k} \text{ (ft)}}{\sqrt{(4 \text{ ft})^2 + (-18 \text{ ft})^2 + (-6 \text{ ft})^2}}$$
$$= 0.206\mathbf{i} - 0.928\mathbf{j} - 0.309\mathbf{k}.$$

(b) The cross product of $\mathbf{e}_p$ with any non-zero vector that is not parallel to the line will yield a vector that is perpendicular to the line, from the definition of the cross product. For example, the vector

$$\mathbf{e}_p \times \mathbf{i} = (0.206\mathbf{i} - 0.928\mathbf{j} - 0.309\mathbf{k}) \times \mathbf{i}$$
$$= 0.206(\mathbf{i} \times \mathbf{i}) - 0.928(\mathbf{j} \times \mathbf{i}) - 0.309(\mathbf{k} \times \mathbf{i})$$
$$= 0.928\mathbf{k} - 0.309\mathbf{j}$$

is perpendicular to $\mathbf{e}_p$. However, although it is the cross product of two unit vectors, it is not a unit vector:

$$|\mathbf{e}_p \times \mathbf{i}| = \sqrt{(0.928)^2 + (-0.309)^2}$$
$$= 0.978.$$

To obtain the desired unit vector, we must divide the vector $\mathbf{e}_p \times \mathbf{i}$ by its magnitude:

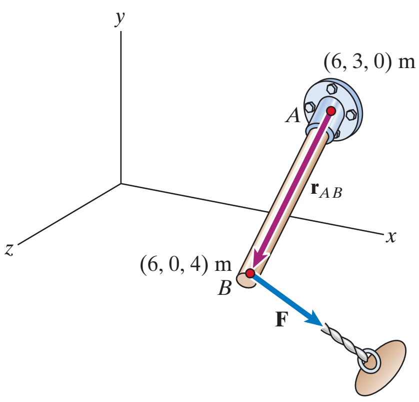
$$\mathbf{e}_n = \frac{\mathbf{e}_p \times \mathbf{i}}{|\mathbf{e}_p \times \mathbf{i}|}$$
$$= \frac{-0.309\mathbf{j} + 0.928\mathbf{k}}{0.978}$$
$$= -0.316\mathbf{j} + 0.949\mathbf{k}.$$

The dot product of this vector with $\mathbf{e}_p$ is zero, confirming that the two vectors are perpendicular:

$$\mathbf{e}_p \cdot \mathbf{e}_n = (0.206)(0) + (-0.928)(-0.316)$$
$$+ (-0.309)(0.949)$$
$$= 0.$$

$$\boxed{\text{(a) } \mathbf{e}_p = 0.206\mathbf{i} - 0.928\mathbf{j} - 0.309\mathbf{k}. \text{ (b) For example, } \mathbf{e}_n = -0.316\mathbf{j} + 0.949\mathbf{k}.}$$

Problem 2.131 The force $\mathbf{F} = 10\mathbf{i} - 4\mathbf{j}$ (N). Determine the cross product $\mathbf{r}_{AB} \times \mathbf{F}$.



Solution: The position vector is

$$\mathbf{r}_{AB} = (6 - 6)\mathbf{i} + (0 - 3)\mathbf{j} + (4 - 0)\mathbf{k} = 0\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$$

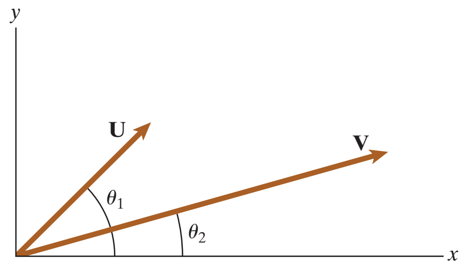
The cross product:

$$\mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -3 & 4 \\ 10 & -4 & 0 \end{vmatrix} = \mathbf{i}(16) - \mathbf{j}(-40) + \mathbf{k}(30)$$

$$= 16\mathbf{i} + 40\mathbf{j} + 30\mathbf{k} \text{ (N-m)}$$

$$\boxed{\mathbf{r}_{AB} \times \mathbf{F} = 16i + 40j + 30k \text{ (N-m).}}$$

Problem 2.132 By evaluating the cross product $\mathbf{U} \times \mathbf{V}$, prove the identity $\sin(\theta_1 - \theta_2) = \sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2$.



Solution: Assume that both **U** and **V** lie in the x-y plane. The strategy is to use the definition of the cross product (Eq. 2.28) and the Eq. (2.34), and equate the two. From Eq. (2.28) $\mathbf{U} \times \mathbf{V} = |\mathbf{U}||\mathbf{V}|\sin(\theta_1 - \theta_2)\mathbf{e}$. Since the positive z-axis is out of the paper, and $\mathbf{e}$ points into the paper, then $\mathbf{e} = -\mathbf{k}$. Take the dot product of both sides with $\mathbf{e}$, and note that $\mathbf{k} \cdot \mathbf{k} = 1$. Thus

$$\sin(\theta_1 - \theta_2) = -\left(\frac{(\mathbf{U} \times \mathbf{V}) \cdot \mathbf{k}}{|\mathbf{U}||\mathbf{V}|}\right)$$

The vectors are:

$\mathbf{U} = |\mathbf{U}|(\mathbf{i}\cos\theta_1 + \mathbf{j}\sin\theta_2)$, and $\mathbf{V} = |\mathbf{V}|(\mathbf{i}\cos\theta_2 + \mathbf{j}\sin\theta_2)$.

The cross product is

$$\mathbf{U} \times \mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ |\mathbf{U}|\cos\theta_1 & |\mathbf{U}|\sin\theta_1 & 0 \\ |\mathbf{V}|\cos\theta_2 & |\mathbf{V}|\sin\theta_2 & 0 \end{vmatrix}$$

$$= \mathbf{i}(0) - \mathbf{j}(0) + \mathbf{k}(|\mathbf{U}||\mathbf{V}|)(\cos\theta_1 \sin\theta_2 - \cos\theta_2 \sin\theta_1)$$

Substitute into the definition to obtain:

$\sin(\theta_1 - \theta_2) = \sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2$. Q.E.D.

Problem 2.133 In Example 2.15, what is the minimum distance from point B to the line OA?

Solution: Let θ be the angle between $\mathbf{r}_{OA}$ and $\mathbf{r}_{OB}$. Then the minimum distance is

$$d = |\mathbf{r}_{OB}|\sin\theta$$

Using the cross product, we have

$$|\mathbf{r}_{OA} \times \mathbf{r}_{OB}| = |\mathbf{r}_{OA}||\mathbf{r}_{OB}|\sin\theta = |\mathbf{r}_{OA}|d \Rightarrow d = \frac{|\mathbf{r}_{OA} \times \mathbf{r}_{OB}|}{|\mathbf{r}_{OA}|}$$

We have

$$\mathbf{r}_{OA} = (10\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})\text{ m}$$

$$\mathbf{r}_{OB} = (6\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})\text{ m}$$

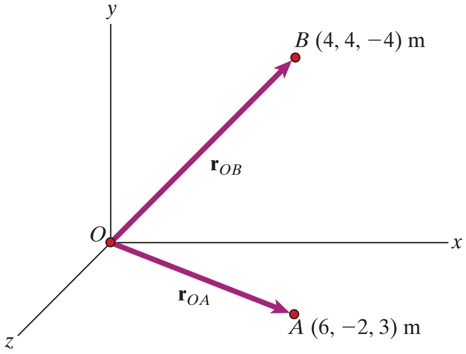
$$\mathbf{r}_{OA} \times \mathbf{r}_{OB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & -2 & 3 \\ 6 & 6 & -3 \end{vmatrix} = (-12\mathbf{i} + 48\mathbf{j} + 72\mathbf{k})\text{ m}^2$$

Thus

$$d = \frac{\sqrt{(-12\text{ m}^2) + (48\text{ m}^2)^2 + (72\text{ m}^2)^2}}{\sqrt{(10\text{ m})^2 + (-2\text{ m})^2 + (3\text{ m})^2}} = 8.22\text{ m}$$

$$\boxed{d = 8.22\text{ m}}$$

Problem 2.134 (a) What is the cross product $\mathbf{r}_{OA} \times \mathbf{r}_{OB}$? (b) Determine a unit vector $\mathbf{e}$ that is perpendicular to $\mathbf{r}_{OA}$ and $\mathbf{r}_{OB}$.



Solution: The two radius vectors are

$$\mathbf{r}_{OB} = 4\mathbf{i} + 4\mathbf{j} - 4\mathbf{k},\ \mathbf{r}_{OA} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$$

(a) The cross product is

$$\mathbf{r}_{OA} \times \mathbf{r}_{OB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -2 & 3 \\ 4 & 4 & -4 \end{vmatrix} = \mathbf{i}(8 - 12) - \mathbf{j}(-24 - 12) + \mathbf{k}(24 + 8)$$

$$= -4\mathbf{i} + 36\mathbf{j} + 32\mathbf{k}\ (\text{m}^2)$$

The magnitude is

$$|\mathbf{r}_{OA} \times \mathbf{r}_{OB}| = \sqrt{4^2 + 36^2 + 32^2} = 48.33\text{ m}^2$$

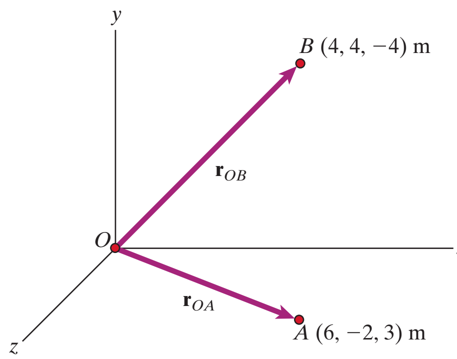
(b) The unit vector is

$$\mathbf{e} = \pm\left(\frac{\mathbf{r}_{OA} \times \mathbf{r}_{OB}}{|\mathbf{r}_{OA} \times \mathbf{r}_{OB}|}\right) = \pm(-0.0828\mathbf{i} + 0.7448\mathbf{j} + 0.6621\mathbf{k})$$

(Two vectors.)

$$\boxed{\begin{array}{l}\text{(a) } \mathbf{r}_{OA} \times \mathbf{r}_{OB} = -4\mathbf{i} + 36\mathbf{j} + 32\mathbf{k}\ (\text{m}^2). \\ \text{(b) } \pm(-0.0828\mathbf{i} + 0.7448\mathbf{j} + 0.662\mathbf{k}).\end{array}}$$

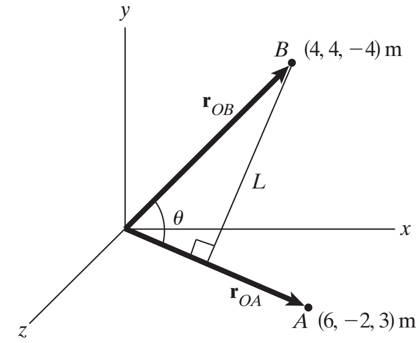
Problem 2.135 Use the cross product to determine the length of the shortest straight line from point B to the straight line that passes through points O and A.



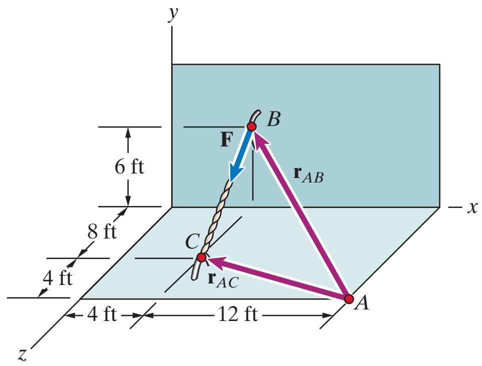
Solution: The magnitude of the cross product of $\mathbf{r}_{OA}$ and $\mathbf{r}_{OB}$ is $|\mathbf{r}_{OA}||\mathbf{r}_{OB}|\sin\theta$. Therefore the length L we seek is

$$L = |\mathbf{r}_{OB}|\sin\theta = \frac{|\mathbf{r}_{OA} \times \mathbf{r}_{OB}|}{|\mathbf{r}_{OA}|} = \frac{|-4\mathbf{i} + 36\mathbf{j} + 32\mathbf{k}|}{|6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}|} = 6.90 \text{ m}.$$

$\boxed{6.90 \text{ m.}}$



Problem 2.136 The cable BC exerts a 1000-lb force $\mathbf{F}$ on the hook at B. Determine $\mathbf{r}_{AB} \times \mathbf{F}$.



Solution: The vector $\mathbf{r}_{AB}$ is

$$\mathbf{r}_{AB} = (4 \text{ ft} - 16 \text{ ft})\mathbf{i} + (6 \text{ ft} - 0)\mathbf{j} + (0 - 12 \text{ ft})\mathbf{k}$$
$$= -12\mathbf{i} + 6\mathbf{j} - 12\mathbf{k} \text{ (ft)}.$$

We need to determine the components of the force. The vector $\mathbf{r}_{AB}$ is

$$\mathbf{r}_{BC} = (4 \text{ ft} - 4 \text{ ft})\mathbf{i} + (0 - 6 \text{ ft})\mathbf{j} + (8 \text{ ft} - 0)\mathbf{k}$$
$$= -6\mathbf{j} + 8\mathbf{k} \text{ (ft)}.$$

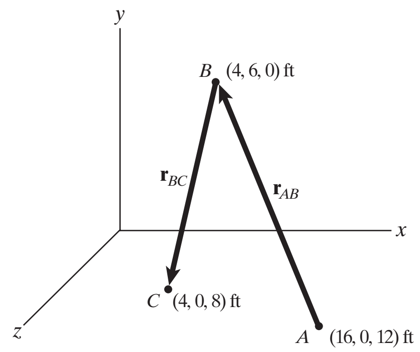
Then the force is

$$\mathbf{F} = (1000 \text{ lb})\frac{\mathbf{r}_{BC}}{|\mathbf{r}_{BC}|} = -600\mathbf{j} + 800\mathbf{k} \text{ (lb)}.$$

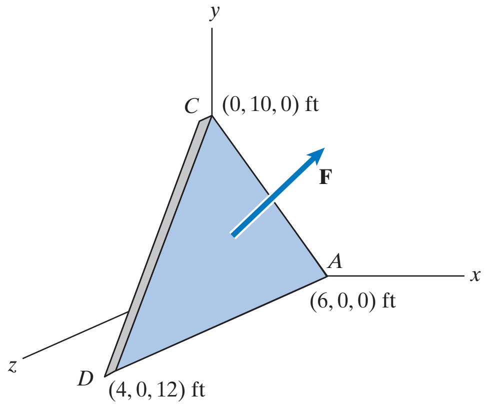
The desired cross product is

$$\mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -12 \text{ ft} & 6 \text{ ft} & -12 \text{ ft} \\ 0 & -600 \text{ lb} & 800 \text{ lb} \end{vmatrix}$$
$$= -2400\mathbf{i} + 9600\mathbf{j} + 7200\mathbf{k} \text{ (ft-lb)}.$$

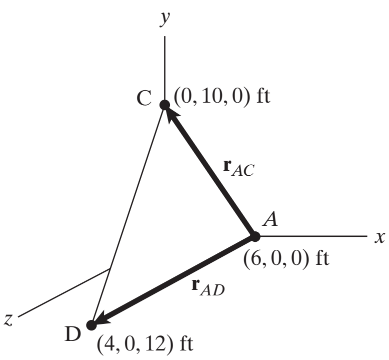
$$\boxed{\mathbf{r}_{AB} \times \mathbf{F} = -2400\mathbf{i} + 9600\mathbf{j} + 7200\mathbf{k} \text{ (ft-lb).}}$$



Problem 2.137* The force $\mathbf{F} = 200\mathbf{i} + 180\mathbf{j} + 100\mathbf{k}\,(\text{lb})$. Determine the vector component of $\mathbf{F}$ that is normal (perpendicular) to the flat plate ACD.



Solution:



The vector from A to C is

$$\mathbf{r}_{AC} = -6\mathbf{i} + 10\mathbf{j}\ (\text{ft}).$$

The vector from A to D is

$$\mathbf{r}_{AD} = -2\mathbf{i} + 12\mathbf{k}\ (\text{ft}).$$

The cross product $\mathbf{r}_{AC} \times \mathbf{r}_{AD}$ is perpendicular to the plate.

$$\mathbf{r}_{AC} \times \mathbf{r}_{AD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 10 & 0 \\ -2 & 0 & 12 \end{vmatrix}$$

$$= 120\mathbf{i} + 72\mathbf{j} + 20\mathbf{k}(\text{ft}^2).$$

Dividing this vector by its magnitude yields a unit vector that is perpendicular to the plate:

$$\mathbf{e} = \frac{\mathbf{r}_{AC} \times \mathbf{r}_{AD}}{|\mathbf{r}_{AC} \times \mathbf{r}_{AD}|}$$

$$= 0.849\mathbf{i} + 0.509\mathbf{j} + 0.141\mathbf{k}.$$

From Eq. (2.26), the vector component of $\mathbf{F}$ that is parallel to $\mathbf{e}$, and so is perpendicular to the plate, is

$$\mathbf{F}_{\text{n}} = (\mathbf{e} \cdot \mathbf{F})\mathbf{e}$$

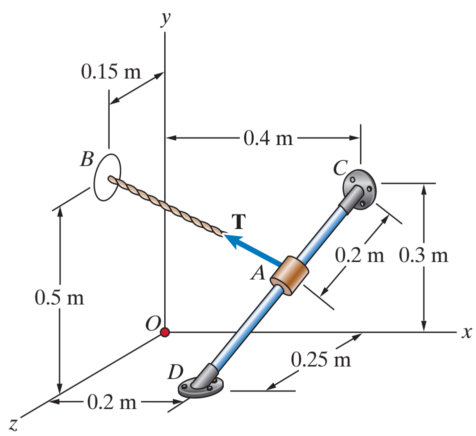
$$= [(0.849)(200\text{ lb}) + (0.509)(180\text{ lb})$$

$$+ (0.141)(100\text{ lb})](0.849\mathbf{i} + 0.509\mathbf{j} + 0.141\mathbf{k})$$

$$= 234\mathbf{i} + 140\mathbf{j} + 39\mathbf{k}(\text{lb}).$$

$$\boxed{\mathbf{F}_{\text{n}} = 234\mathbf{i} + 140\mathbf{j} + 39\mathbf{k}(\text{lb}).}$$

Problem 2.138 The rope AB exerts a 50-N force $\mathbf{T}$ on the collar at A. Let $\mathbf{r}_{CA}$ be the position vector from point C to point A. Determine the cross product $\mathbf{r}_{CA} \times \mathbf{T}$.



Solution: We define the appropriate vectors.

$$\mathbf{r}_{CD} = (-0.2\mathbf{i} - 0.3\mathbf{j} + 0.25\mathbf{k})\ \text{m}$$

$$\mathbf{r}_{CA} = (0.2\ \text{m})\frac{\mathbf{r}_{CD}}{|\mathbf{r}_{CD}|} = (-0.091\mathbf{i} - 0.137\mathbf{j} + 0.114\mathbf{k})\ \text{m}$$

$$\mathbf{r}_{OB} = (0.5\mathbf{j} + 0.15\mathbf{k})\ \text{m}$$

$$\mathbf{r}_{OC} = (0.4\mathbf{i} + 0.3\mathbf{j})\ \text{m}$$

$$\mathbf{r}_{AB} = \mathbf{r}_{OB} - (\mathbf{r}_{OC} + \mathbf{r}_{CA}) = (0.61\mathbf{i} - 1.22\mathbf{j} - 0.305\mathbf{k})\ \text{m}$$

$$\mathbf{T} = (50\ \text{N})\frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = (-33.7\mathbf{i} + 36.7\mathbf{j} + 3.93\mathbf{k})\ \text{N}$$

Now take the cross product

$$\mathbf{r}_{CA} \times \mathbf{T} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{l} \\ -0.091 & -0.137 & 0.114 \\ -33.7 & 36.7 & 3.93 \end{vmatrix} = (-4.72\mathbf{i} - 3.48\mathbf{j} + -7.96\mathbf{k})\ \text{N-m}$$

$$\boxed{\mathbf{r}_{CA} \times \mathbf{T} = (-47.2\mathbf{i} - 3.48\mathbf{j} + -7.96\mathbf{k})\ \text{N-m}}$$

Problem 2.139 In Example 2.16, suppose that the attachment point E is moved to the location (0.3, 0.3, 0) m and the magnitude of $\mathbf{T}$ increases to 600 N. What is the magnitude of the component of $\mathbf{T}$ perpendicular to the door?

Solution: We first develop the force $\mathbf{T}$.

$$\mathbf{r}_{CE} = (0.3\mathbf{i} + 0.1\mathbf{j})\text{ m}$$

$$\mathbf{T} = (600\text{ N})\frac{\mathbf{r}_{CE}}{|\mathbf{r}_{CE}|} = (569\mathbf{i} + 190\mathbf{j})\text{ N}$$

From Example 2.16 we know that the unit vector perpendicular to the door is

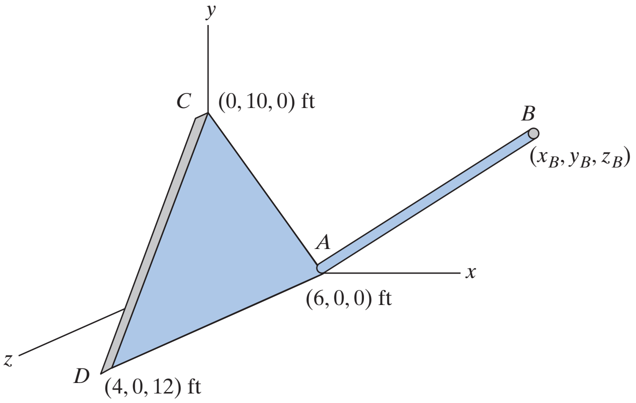
$$\mathbf{e} = (0.358\mathbf{i} + 0.894\mathbf{j} + 0.268\mathbf{k})$$

The magnitude of the force perpendicular to the door (parallel to $\mathbf{e}$) is then

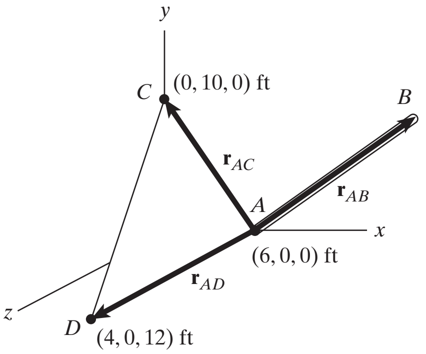
$$|\mathbf{T}_n| = \mathbf{T}\cdot\mathbf{e} = (569\text{ N})(0.358) + (190\text{ N})(0.894) = 373\text{ N}$$

$$\boxed{|\mathbf{T}_n| = 373\text{ N}}$$

Problem 2.140 The slender bar AB is 14 ft in length and is perpendicular to the flat plate ACD. (a) Determine the components of a unit vector $\mathbf{e}$ that is perpendicular to the plate ACD and points in the direction from A toward B. (b) Determine the coordinates (x_B, y_B, z_B) of point B.



Solution:



(a) The vector from A to C is

$$\mathbf{r}_{AC} = -6\mathbf{i} + 10\mathbf{j}\text{ (ft)}.$$

The vector from A to D is

$$\mathbf{r}_{AD} = -2\mathbf{i} + 12\mathbf{k}\text{ (ft)}.$$

The cross product $\mathbf{r}_{AC}\times\mathbf{r}_{AD}$ is perpendicular to the plate and points in the direction from A toward B.

$$\mathbf{r}_{AC}\times\mathbf{r}_{AD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 10 & 0 \\ -2 & 0 & 12 \end{vmatrix}$$

$$= 120\mathbf{i} + 72\mathbf{j} + 20\mathbf{k}\text{ (ft}^2\text{)}.$$

Dividing this vector by its magnitude yields the desired unit vector:

$$\mathbf{e} = \frac{\mathbf{r}_{AC}\times\mathbf{r}_{AD}}{|\mathbf{r}_{AC}\times\mathbf{r}_{AD}|}$$

$$= 0.849\mathbf{i} + 0.509\mathbf{j} + 0.141\mathbf{k}.$$

(b) The bar is 14 ft long, so we obtain the vector from A to B by multiplying $\mathbf{e}$ by 14 ft:

$$\mathbf{r}_{AB} = (14\text{ ft})\mathbf{e}$$

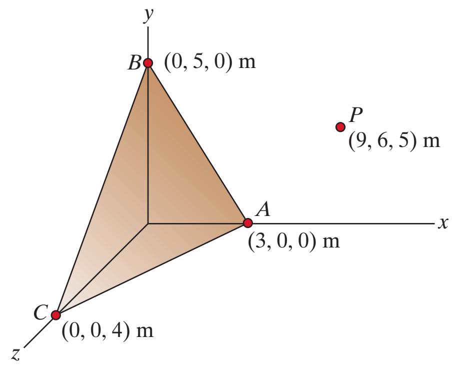
$$= 11.9\mathbf{i} + 7.13\mathbf{j} + 1.98\mathbf{k}\text{ (ft)}.$$

Adding the coordinates of point A to the components of this vector, we obtain the coordinates of point B:

$(x_B, y_B, z_B) = (17.9, 7.13, 1.98)$ ft.

$$\boxed{\begin{array}{l}\text{(a) } \mathbf{e} = 0.849\mathbf{i} + 0.509\mathbf{j} + 0.141\mathbf{k}. \\ \text{(b) } (x_B, y_B, z_B) = (17.9, 7.13, 1.98)\text{ ft.}\end{array}}$$

Problem 2.141* Determine the minimum distance from point P to the plane defined by the three points A, B, and C.



Solution: We'll begin by determining the components of a unit vector $\mathbf{e}$ that is normal to the plane and points from P toward the plane. According to the right hand rule, we obtain a vector that has that direction by evaluating the cross product $\mathbf{r}_{AC} \times \mathbf{r}_{AB}$. The components of these vectors are

$$\mathbf{r}_{AC} = -3\mathbf{i} + 4\mathbf{k} \text{ (m)},$$

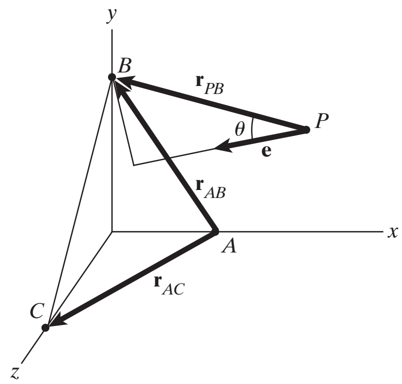
$$\mathbf{r}_{AB} = -3\mathbf{i} + 5\mathbf{j} \text{ (m)},$$

so we obtain

$$\mathbf{r}_{AC} \times \mathbf{r}_{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3\text{ m} & 0 & 4\text{ m} \\ -3\text{ m} & 5\text{ m} & 0 \end{vmatrix} = -20\mathbf{i} - 12\mathbf{j} - 15\mathbf{k} \text{ (m)}.$$

We obtain the desired unit vector by dividing this vector by its magnitude:

$$\mathbf{e} = \frac{\mathbf{r}_{AC} \times \mathbf{r}_{AB}}{|\mathbf{r}_{AC} \times \mathbf{r}_{AB}|} = -0.721\mathbf{i} - 0.433\mathbf{j} - 0.541\mathbf{k}.$$



The vector

$$\mathbf{r}_{PB} = -9\mathbf{i} - \mathbf{j} - 5\mathbf{k} \text{ (m)}.$$

The component of the vector $\mathbf{r}_{PB}$ in the direction of the vector $\mathbf{e}$ is normal to the plane. Its magnitude is the perpendicular distance we seek:

$$\mathbf{e} \cdot \mathbf{r}_{PB} = 9.63 \text{ m}.$$

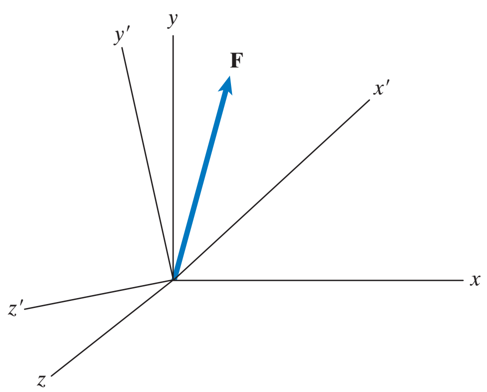
$\boxed{9.63 \text{ m.}}$

Problem 2.142 Two cartesian coordinate systems are shown. The unit vectors $\mathbf{i}'$ and $\mathbf{j}'$ of the $x'y'z'$ coordinate system are given in terms of their components in the xyz coordinate system by

$$\mathbf{i}' = 0.743\mathbf{i} + 0.557\mathbf{j} + 0.371\mathbf{k},$$

$$\mathbf{j}' = -0.408\mathbf{i} + 0.816\mathbf{j} - 0.408\mathbf{k}.$$

(a) Determine the unit vector $\mathbf{k}'$ of the $x'y'z'$ coordinate system in terms of its components in the xyz coordinate system. (b) The force $\mathbf{F} = 40\mathbf{i} + 80\mathbf{j} + 20\mathbf{k}$ (N). Use the dot product to determine the components of F in terms of the $x'y'z'$ coordinate system.



Solution:

(a) The vector $\mathbf{k}'$ is

$$\mathbf{k}' = \mathbf{i}' \times \mathbf{j}'$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.743 & 0.557 & 0.371 \\ -0.408 & 0.816 & -0.408 \end{vmatrix}$$

$$= -0.530\mathbf{i} + 0.152\mathbf{j} + 0.834\mathbf{k}.$$

(b) The x' component of $\mathbf{F}$ is

$$F_{x'} = \mathbf{F} \cdot \mathbf{i}'$$

$$= (40 \text{ N})(0.743) + (80 \text{ N})(0.557) + (20 \text{ N})(0.371)$$

$$= 81.7 \text{ N},$$

the y' component is

$$F_{y'} = \mathbf{F} \cdot \mathbf{j}'$$

$$= (40 \text{ N})(-0.408) + (80 \text{ N})(0.816) + (20 \text{ N})(-0.408)$$

$$= 40.8 \text{ N},$$

and the z' component is

$$F_{z'} = \mathbf{F} \cdot \mathbf{k}'$$

$$= (40 \text{ N})(-0.530) + (80 \text{ N})(0.152) + (20 \text{ N})(0.834)$$

$$= 7.61 \text{ N}.$$

Therefore $\mathbf{F} = 81.7\mathbf{i}' + 40.8\mathbf{j}' + 7.61\mathbf{k}'$ (N).

$\boxed{\begin{array}{l}\text{(a) } \mathbf{k}' = -0.530\mathbf{i} + 0.152\mathbf{j} + 0.834\mathbf{k}. \\ \text{(b) } \mathbf{F} = 81.7\mathbf{i}' + 40.8\mathbf{j}' + 7.61\mathbf{k}' \text{ (N).}\end{array}}$

Problem 2.143 For the vectors $\mathbf{U} = 6\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$, $\mathbf{V} = 2\mathbf{i} + 7\mathbf{j}$, and $\mathbf{W} = 3\mathbf{i} + 2\mathbf{k}$, evaluate the following mixed triple products:

(a) $\mathbf{U} \cdot (\mathbf{V} \times \mathbf{W})$;
(b) $\mathbf{W} \cdot (\mathbf{V} \times \mathbf{U})$;
(c) $\mathbf{V} \cdot (\mathbf{W} \times \mathbf{U})$.

Solution: Use Eq. (2.36).

(a) $$\mathbf{U} \cdot (\mathbf{V} \times \mathbf{W}) = \begin{vmatrix} 6 & 2 & -4 \\ 2 & 7 & 0 \\ 3 & 0 & 2 \end{vmatrix}$$

$$= 6(14) - 2(4) + (-4)(-21) = 160$$

(b) $$\mathbf{W} \cdot (\mathbf{V} \times \mathbf{U}) = \begin{vmatrix} 3 & 0 & 2 \\ 2 & 7 & 0 \\ 6 & 2 & -4 \end{vmatrix}$$

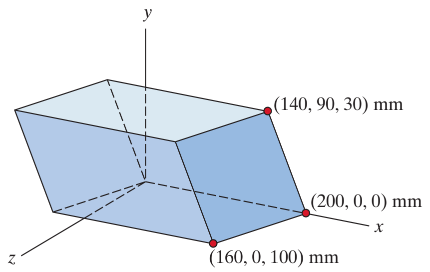
$$= 3(-28) - (0) + 2(4 - 42) = -160$$

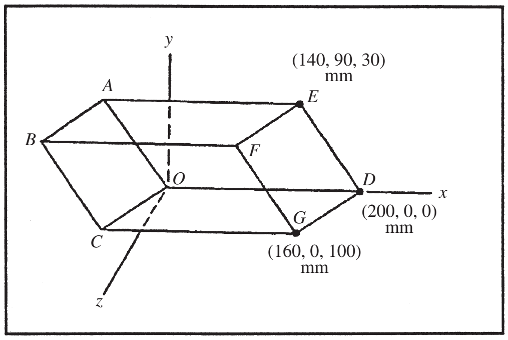
(c) $$\mathbf{V} \cdot (\mathbf{W} \times \mathbf{U}) = \begin{vmatrix} 2 & 7 & 0 \\ 3 & 0 & 2 \\ 6 & 2 & -4 \end{vmatrix}$$

$$= 2(-4) - 7(-12 - 12) + (0) = 160$$

$\boxed{\text{(a) } U \cdot (V \times W) = 160, \text{ (b) } W \cdot (V \times U) = -160, \text{ (c) } V \cdot (W \times U) = 160.}$

Problem 2.144 Use the mixed triple product to calculate the volume of the parallelepiped.





Solution: We are given the coordinates of point D. From the geometry, we need to locate points A and C. The key to doing this is to note that the length of side OD is 200 mm and that side OD is the x-axis. Sides OD, AE, and CG are parallel to the x-axis and the coordinates of the point pairs (O and D), (A and E), and (C and D) differ only by 200 mm in the x coordinate. Thus, the coordinates of point A are $(-60, 90, 30)$ mm and the coordinates of point C are $(-40, 0, 100)$ mm. Thus, the vectors $\mathbf{r}_{OA}$, $\mathbf{r}_{OD}$, and $\mathbf{r}_{OC}$ are $\mathbf{r}_{OD} = 200\mathbf{i}$ mm, $\mathbf{r}_{OA} = -60\mathbf{i} + 90\mathbf{j} + 30\mathbf{k}$ mm, and $\mathbf{r}_{OC} = -40\mathbf{i} + 0\mathbf{j} + 100\mathbf{k}$ mm. The mixed triple product of the three vectors is the volume of the parallelepiped. The volume is

$$\mathbf{r}_{OA} \cdot (\mathbf{r}_{OC} \times \mathbf{r}_{OD}) = \begin{vmatrix} -60 & 90 & 30 \\ -40 & 0 & 100 \\ 200 & 0 & 0 \end{vmatrix}$$

$$= -60(0) + 90(200)(100) + (30)(0) \text{ mm}^3$$

$$= 1{,}800{,}000 \text{ mm}^3$$

$\boxed{1.8\text{E6 mm}^2.}$

Problem 2.145 By using Eqs. (2.23) and (2.34), show that

$$\mathbf{U} \cdot (\mathbf{V} \times \mathbf{W}) = \begin{vmatrix} U_x & U_y & U_z \\ V_x & V_y & V_z \\ W_x & W_y & W_z \end{vmatrix}.$$

Solution: One strategy is to expand the determinant in terms of its components, take the dot product, and then collapse the expansion. Eq. (2.23) is an expansion of the dot product: Eq. (2.23): $\mathbf{U} \cdot \mathbf{V} = U_X V_X + U_Y V_Y + U_Z V_Z$. Eq. (2.34) is the determinant representation of the cross product:

$$\text{Eq. (2.34) } \mathbf{U} \times V = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ U_X & U_Y & U_Z \\ V_X & V_Y & V_Z \end{vmatrix}$$

For notational convenience, write $\mathbf{P} = (\mathbf{U} \times \mathbf{V})$. Expand the determinant about its first row:

$$\mathbf{P} = \mathbf{i}\begin{vmatrix} U_Y & U_Z \\ V_Y & V_Z \end{vmatrix} - \mathbf{j}\begin{vmatrix} U_X & U_Z \\ V_X & V_Z \end{vmatrix} + \mathbf{k}\begin{vmatrix} U_X & U_Z \\ V_X & V_Z \end{vmatrix}$$

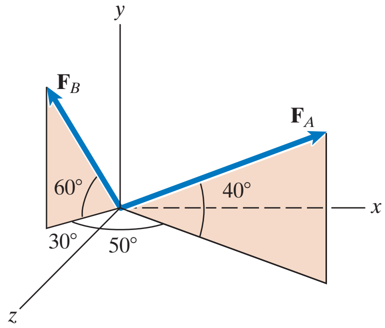
Since the two-by-two determinants are scalars, this can be written in the form: $\mathbf{P} = \mathbf{i}P_X + \mathbf{j}P_Y + \mathbf{k}P_Z$ where the scalars P_X, P_Y, and P_Z are the two-by-two determinants. Apply Eq. (2.23) to the dot product of a vector $\mathbf{Q}$ with $\mathbf{P}$. Thus $\mathbf{Q} \cdot \mathbf{P} = Q_X P_X + Q_Y P_Y + Q_Z P_Z$. Substitute P_X, P_Y, and P_Z into this dot product

$$\mathbf{Q} \cdot \mathbf{P} = Q_X\begin{vmatrix} U_Y & U_Z \\ V_Y & V_Z \end{vmatrix} - Q_Y\begin{vmatrix} U_X & U_Z \\ V_X & V_Z \end{vmatrix} + Q_Z\begin{vmatrix} U_X & U_Z \\ V_X & V_Z \end{vmatrix}$$

But this expression can be collapsed into a three-by-three determinant directly, thus:

$$\mathbf{Q} \cdot (\mathbf{U} \times \mathbf{V}) = \begin{vmatrix} Q_X & Q_Y & Q_Z \\ U_X & U_Y & U_Z \\ V_X & V_Y & V_Z \end{vmatrix}.$$ This completes the demonstration.

Problem 2.146 The magnitudes of the two force vectors are $|\mathbf{F}_A| = 200$ lb and $|\mathbf{F}_B| = 160$ lb. What is the magnitude of their cross product $\mathbf{F}_A \times \mathbf{F}_B$?



Solution: The components of $\mathbf{F}_A$ are

$$\mathbf{F}_A = (200\text{ lb})(\cos 40^\circ \sin 50^\circ \mathbf{i} + \sin 40^\circ \mathbf{j} + \cos 40^\circ \cos 50^\circ \mathbf{k})$$
$$= 117\mathbf{i} + 129\mathbf{j} + 98.5\mathbf{k}\text{ (lb)}.$$

The components of $\mathbf{F}_B$ are

$$\mathbf{F}_B = (160\text{ lb})(-\cos 60^\circ \sin 30^\circ \mathbf{i} + \sin 60^\circ \mathbf{j} + \cos 60^\circ \cos 30^\circ \mathbf{k})$$
$$= -40.0\mathbf{i} + 139\mathbf{j} + 69.3\mathbf{k}\text{ (lb)}.$$

Their cross product is

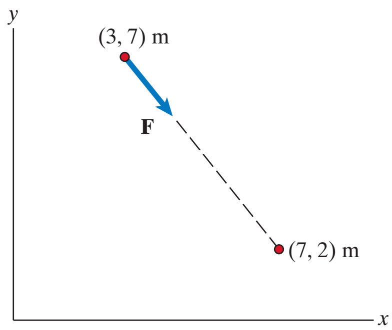
$$\mathbf{F}_A \times \mathbf{F}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 117\text{ lb} & 129\text{ lb} & 98.5\text{ lb} \\ -40.0\text{ lb} & 139\text{ lb} & 69.3\text{ lb} \end{vmatrix}$$
$$= -4740\mathbf{i} - 12{,}100\mathbf{j} + 21{,}400\mathbf{k}\text{ (lb}^2).$$

Its magnitude is

$$|\mathbf{F}_A \times \mathbf{F}_B| = \sqrt{(-4740\text{ lb}^2)^2 + (-12{,}100\text{ lb}^2)^2 + (21{,}400\text{ lb}^2)^2}$$
$$= 25{,}000\text{ lb}^2.$$

$$\boxed{|\mathbf{F}_A \times \mathbf{F}_B| = 25{,}000\text{ lb}^2.}$$

Problem 2.147 The magnitude of $\mathbf{F}$ is 8 kN. Express $\mathbf{F}$ in terms of scalar components.



Solution: The unit vector collinear with the force $\mathbf{F}$ is developed as follows: The collinear vector is $\mathbf{r} = (7 - 3)\mathbf{i} + (2 - 7)\mathbf{j} = 4\mathbf{i} - 5\mathbf{j}$

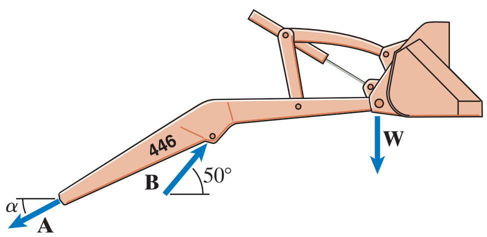
The magnitude: $|\mathbf{r}| = \sqrt{4^2 + 5^2} = 6.403$ m. The unit vector is

$\mathbf{e} = \dfrac{\mathbf{r}}{|\mathbf{r}|} = 0.6247\mathbf{i} - 0.7809\mathbf{j}$. The force vector is

$$\mathbf{F} = |\mathbf{F}|\mathbf{e} = 5.00\mathbf{i} - 6.25\mathbf{j}\text{ (kN)}.$$

$$\boxed{\mathbf{F} = 5.00\mathbf{i} - 6.25\mathbf{j}\text{ (kN)}.}$$

Problem 2.148 The magnitude of the vertical force $\mathbf{W}$ is 600 lb, and the magnitude of the force $\mathbf{B}$ is 1500 lb. Given that $\mathbf{A} + \mathbf{B} + \mathbf{W} = \mathbf{0}$, determine the magnitude of the force $\mathbf{A}$ and the angle α.



Solution: Let A, B, W denote the magnitudes of the forces. The forces in terms of their components are

$$\mathbf{A} = -A\cos\alpha\,\mathbf{i} - A\sin\alpha\,\mathbf{j},$$
$$\mathbf{B} = B\cos 50^\circ\mathbf{i} + B\sin 50^\circ\mathbf{j},$$
$$\mathbf{W} = -W\mathbf{j}.$$

The sum of the forces is zero:

$$\mathbf{A} + \mathbf{B} + \mathbf{W} = (-A\cos\alpha + B\cos 50^\circ)\mathbf{i} + (-A\sin\alpha + B\sin 50^\circ - W)\mathbf{j} = \mathbf{0}.$$

This requires that the sums of the $\mathbf{i}$ and $\mathbf{j}$ components each equal zero, giving two equations that we write as

$$A\cos\alpha = B\cos 50^\circ, \qquad (1)$$

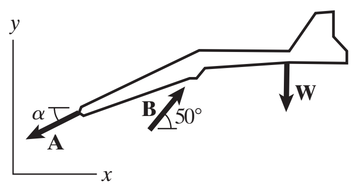
$$A\sin\alpha = B\sin 50^\circ - W. \qquad (2)$$

Dividing Eq. (2) by Eq. (1) gives

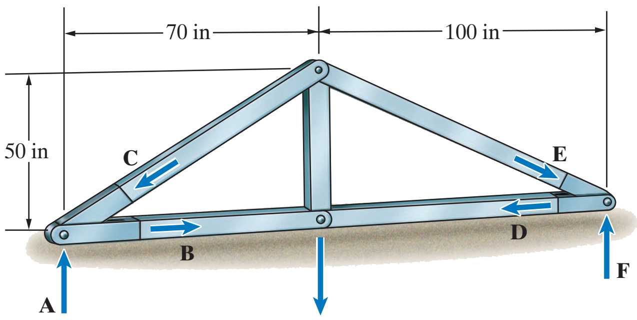
$$\tan\alpha = \frac{B\sin 50^\circ - W}{B\cos 50^\circ} = 0.569,$$

from which we obtain $\alpha = 29.7^\circ$. Then Eq. (1) gives $A = 1110$ lb.

$$\boxed{A = 1110\text{ lb}, \alpha = 29.7^\circ.}$$



Problem 2.149 The magnitude of the vertical force vector **A** is 200 lb. If $\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{0}$, what are the magnitudes of the force vectors **B** and **C**?

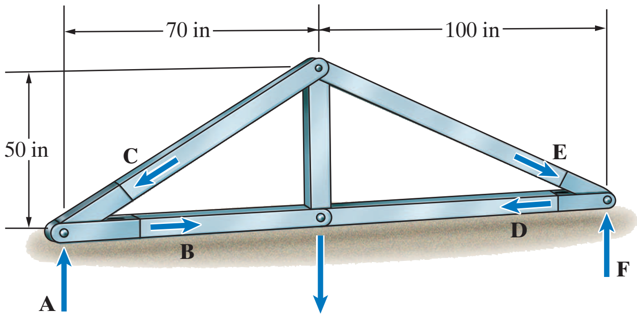


Solution: The strategy is to express the forces in terms of scalar components, and then solve the force balance equations for the unknowns. $\mathbf{C} = |\mathbf{C}|(-\mathbf{i}\cos\alpha - \mathbf{j}\sin\alpha)$, where

$$\tan\alpha = \frac{50}{70} = 0.7143, \text{ or } \alpha = 35.5°.$$

Thus $\mathbf{C} = |\mathbf{C}|(-0.8137\mathbf{i} - 0.5812\mathbf{j})$. Similarly, $\mathbf{B} = +|\mathbf{B}|\mathbf{i}$, and $\mathbf{A} = +200\mathbf{j}$. The force balance equation is $\mathbf{A} + \mathbf{B} + \mathbf{C} = 0$. Substituting, $(-0.8137|\mathbf{C}| + |\mathbf{B}|)\mathbf{i} = 0$, and $(-0.5812|\mathbf{C}| + 200)\mathbf{j} = 0$. Solving, $|\mathbf{C}| = 344.1$ lb, $|\mathbf{B}| = 280$ lb

Problem 2.150 The magnitude of the horizontal force vector **D** is 280 lb. If $\mathbf{D} + \mathbf{E} + \mathbf{F} = \mathbf{0}$, what are the magnitudes of the force vectors **E** and **F**?



Solution: The strategy is to express the force vectors in terms of scalar components, and then solve the force balance equation for the unknowns. The force vectors are:

$\mathbf{E} = |\mathbf{E}|(\mathbf{i}\cos\beta - \mathbf{j}\sin\beta)$, where $\tan\beta = \frac{50}{100} = 0.5$, or $\beta = 26.6°$.

Thus

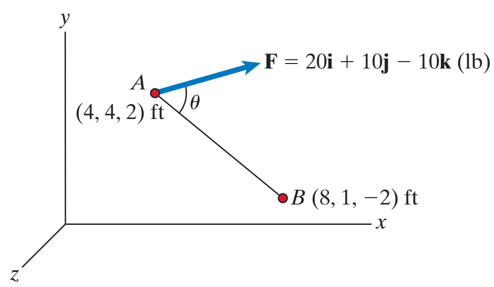
$$\mathbf{E} = |\mathbf{E}|(0.8944\mathbf{i} - 0.4472\mathbf{j})$$
$$\mathbf{D} = -280\mathbf{i}, \text{ and } \mathbf{F} = |\mathbf{F}|\mathbf{j}.$$

The force balance equation is $\mathbf{D} + \mathbf{E} + \mathbf{F} = 0$. Substitute and resolve into two equations:

$(0.8944|\mathbf{E}| - 280)\mathbf{i} = 0$, and $(-0.4472|\mathbf{E}| + |\mathbf{F}|)\mathbf{j} = 0$.

Solve: $|\mathbf{E}| = 313.1$ lb, $|\mathbf{F}| = 140$ lb

Problem 2.151 What are the direction cosines of **F**?



Solution: Use the definition of the direction cosines and the ensuing discussion.

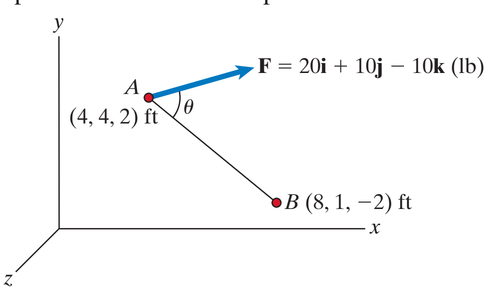
The magnitude of **F**: $|\mathbf{F}| = \sqrt{20^2 + 10^2 + 10^2} = 24.5$.

The direction cosines are $\cos\theta_x = \frac{F_x}{|\mathbf{F}|} = \frac{20}{24.5} = 0.8165$,

$$\cos\theta_y = \frac{F_y}{|\mathbf{F}|} = \frac{10}{24.5} = 0.4082$$
$$\cos\theta_z = \frac{F_z}{|\mathbf{F}|} = \frac{-10}{24.5} = -0.4082$$

$\boxed{\cos\theta_x = 0.817,\ \cos\theta_y = 0.408,\ \cos\theta_z = -0.408.}$

Problem 2.152 Determine the components of a unit vector parallel to line AB that points from A toward B.



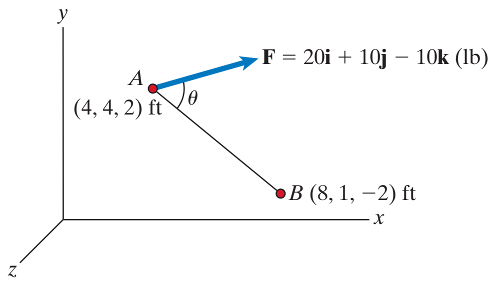
Solution: Use the definition of the unit vector, we get

The position vectors are: $\mathbf{r}_A = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$, $\mathbf{r}_B = 8\mathbf{i} + 1\mathbf{j} - 2\mathbf{k}$. The vector from A to B is $\mathbf{r}_{AB} = (8-4)\mathbf{i} + (1-4)\mathbf{j} + (-2-2)\mathbf{k} = 4\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$. The magnitude: $|\mathbf{r}_{AB}| = \sqrt{4^2 + 3^2 + 4^2} = 6.4$. The unit vector is

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = \frac{4}{6.4}\mathbf{i} - \frac{3}{6.4}\mathbf{j} - \frac{4}{6.4}\mathbf{k} = 0.6247\mathbf{i} - 0.4685\mathbf{j} - 0.6247\mathbf{k}$$

$\boxed{\mathbf{e}_{AB} = 0.625\mathbf{i} - 0.469\mathbf{j} - 0.625\mathbf{k}.}$

Problem 2.153 What is the angle θ between the line AB and the force $\mathbf{F}$?



Solution: The vector from A to B is

$$\mathbf{r}_{AB} = (8-4)\mathbf{i} + (1-4)\mathbf{j} + (-2-2)\mathbf{k}\ (\text{ft}) = 4\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}\ (\text{ft}).$$

The dot product of this vector with $\mathbf{F}$ is

$$\mathbf{r}_{AB}\cdot\mathbf{F} = (4)(20) + (-3)(10) + (-4)(-10)\ \text{ft-lb} = 90\ \text{ft-lb}.$$

From the definition of the dot product

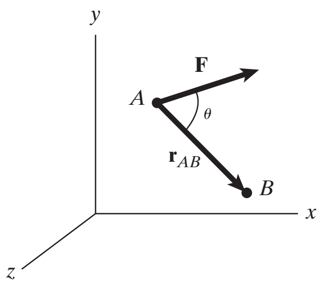
$\mathbf{r}_{AB}\cdot\mathbf{F} = |\mathbf{r}_{AB}||\mathbf{F}|\cos\theta,$

We obtain

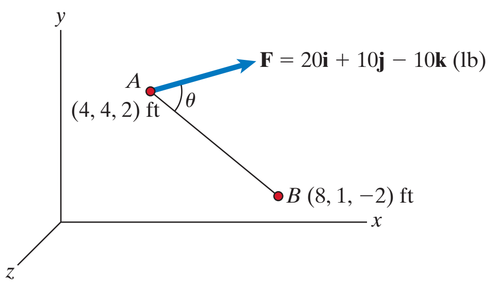
$$\cos\theta = \frac{\mathbf{r}_{AB}\cdot\mathbf{F}}{|\mathbf{r}_{AB}||\mathbf{F}|} = \frac{90\ \text{ft-lb}}{\sqrt{(4\ \text{ft})^2 + (-3\ \text{ft})^2 + (-4\ \text{ft})^2}\sqrt{(20\ \text{lb})^2 + (10\ \text{lb})^2 + (-10\ \text{lb})^2}} = 0.574.$$

This gives $\theta = 55.0°$.

$\boxed{\theta = 55.0°.}$



Problem 2.154 Determine the vector component of $\mathbf{F}$ that is parallel to the line AB.



Solution: The vector from A to B is $\mathbf{r}_{AB} = 4\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$ (ft). A unit vector $\mathbf{e}$ that points from A toward B is

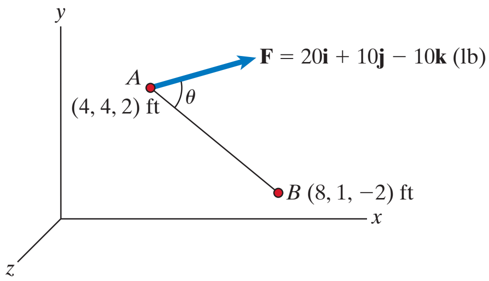
$$\mathbf{e} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = 0.625\mathbf{i} - 0.469\mathbf{j} - 0.625\mathbf{k}.$$

The component of $\mathbf{F}$ that is parallel to the line from A to B is

$$\mathbf{F}_p = (\mathbf{e}\cdot\mathbf{F})\mathbf{e} = 8.78\mathbf{i} - 6.59\mathbf{j} - 8.78\mathbf{k}\ (\text{lb}).$$

$\boxed{\mathbf{F}_p = 8.78\mathbf{i} - 6.59\mathbf{j} - 8.78\mathbf{k}\ (\text{lb}).}$

Problem 2.155 Determine the vector component of $\mathbf{F}$ that is normal to the line AB.



Solution: The vector from A to B is $\mathbf{r}_{AB} = 4\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$ (ft). A unit vector $\mathbf{e}$ that points from A toward B is

$$\mathbf{e} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = 0.625\mathbf{i} - 0.469\mathbf{j} - 0.625\mathbf{k}.$$

The component of $\mathbf{F}$ that is parallel to the line from A to B is

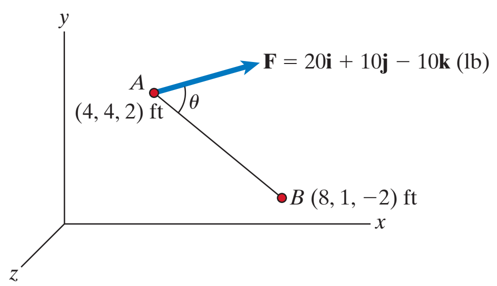
$$\mathbf{F}_p = (\mathbf{e}\cdot\mathbf{F})\mathbf{e} = 8.78\mathbf{i} - 6.59\mathbf{j} - 8.78\mathbf{k}\ (\text{lb}).$$

Then the component of $\mathbf{F}$ that is normal to the line from A to B is

$$\mathbf{F}_n = \mathbf{F} - \mathbf{F}_p = 11.2\mathbf{i} + 16.6\mathbf{j} - 1.22\mathbf{k}\ (\text{lb}).$$

$\boxed{\mathbf{F}_n = 11.2\mathbf{i} + 16.6\mathbf{j} - 1.22\mathbf{k}\ (\text{lb}).}$

Problem 2.156 Determine the vector $\mathbf{r}_{BA}\times\mathbf{F}$, where $\mathbf{r}_{BA}$ is the position vector from B to A.

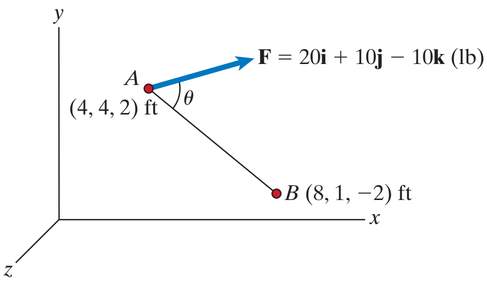


Solution: The vector from B to A is $\mathbf{r}_{BA} = -4\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ (ft). The cross product is

$$\mathbf{r}_{BA}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 3 & 4 \\ 20 & 10 & -10 \end{vmatrix} = (-30-40)\mathbf{i} - (40-80)\mathbf{j} + (-40-60) = -70\mathbf{i} + 40\mathbf{j} - 100\mathbf{k}\ (\text{ft-lb})$$

$\boxed{\mathbf{r}_{BA}\times\mathbf{F} = -70\mathbf{i} + 40\mathbf{j} - 100\mathbf{k}\ (\text{ft-lb}).}$

Problem 2.157 (a) Write the position vector $\mathbf{r}_{AB}$ from point A to point B in terms of components. (b) A vector $\mathbf{R}$ has magnitude $|\mathbf{R}| = 200$ lb and is parallel to the line from A to B. Write $\mathbf{R}$ in terms of components.



Solution:

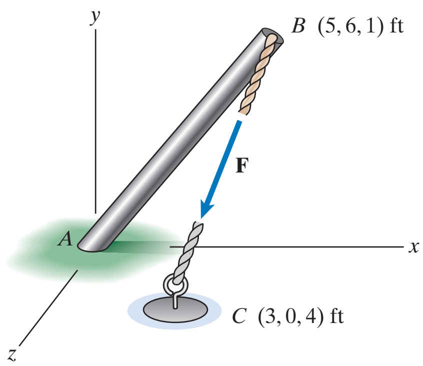
(a) $\mathbf{r}_{AB} = (8\text{ ft} - 4\text{ ft})\mathbf{i} + (1\text{ ft} - 4\text{ ft})\mathbf{j} + (-2\text{ ft} - 2\text{ ft})\mathbf{k}$

$= 4\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$ (ft).

(b) $\mathbf{R} = (200\text{ lb})\dfrac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = 125\mathbf{i} - 93.7\mathbf{j} - 125\mathbf{k}$ (lb).

$\boxed{\text{(a) } \mathbf{r}_{AB} = 4\mathbf{i} - 3\mathbf{j} - 4\mathbf{k} \text{ (ft). (b) } \mathbf{R} = 125\mathbf{i} - 93.7\mathbf{j} - 125\mathbf{k} \text{ (lb).}}$

Problem 2.158 The rope exerts a force of magnitude $|\mathbf{F}| = 200$ lb on the top of the pole at B. (a) Determine the vector $\mathbf{r}_{AB} \times \mathbf{F}$, where $\mathbf{r}_{AB}$ is the position vector from A to B. (b) Determine the vector $\mathbf{r}_{AC} \times \mathbf{F}$, where $\mathbf{r}_{AC}$ is the position vector from A to C.



Solution: The strategy is to define the unit vector pointing from B to A, express the force in terms of this unit vector, and take the cross product of the position vectors with this force. The position vectors

$\mathbf{r}_{AB} = 5\mathbf{i} + 6\mathbf{j} + 1\mathbf{k}$, $\mathbf{r}_{AC} = 3\mathbf{i} + 0\mathbf{j} + 4\mathbf{k}$,

$\mathbf{r}_{BC} = (3 - 5)\mathbf{i} + (0 - 6)\mathbf{j} + (4 - 1)\mathbf{k} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$.

The magnitude $|\mathbf{r}_{BC}| = \sqrt{2^2 + 6^2 + 3^2} = 7$. The unit vector is

$$\mathbf{e}_{BC} = \frac{\mathbf{r}_{BC}}{|\mathbf{r}_{BC}|} = -0.2857\mathbf{i} - 0.8571\mathbf{j} + 0.4286\mathbf{k}.$$

The force vector is

$$\mathbf{F} = |\mathbf{F}|\mathbf{e}_{BC} = 200\mathbf{e}_{BC} = -57.14\mathbf{i} - 171.42\mathbf{j} + 85.72\mathbf{k}.$$

The cross products:

$$\mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 6 & 1 \\ -57.14 & -171.42 & 85.72 \end{vmatrix}$$

$$= 685.74\mathbf{i} - 485.74\mathbf{j} - 514.26\mathbf{k}$$

$$= 685.7\mathbf{i} - 485.7\mathbf{j} - 514.3\mathbf{k} \text{ (ft-lb)}$$

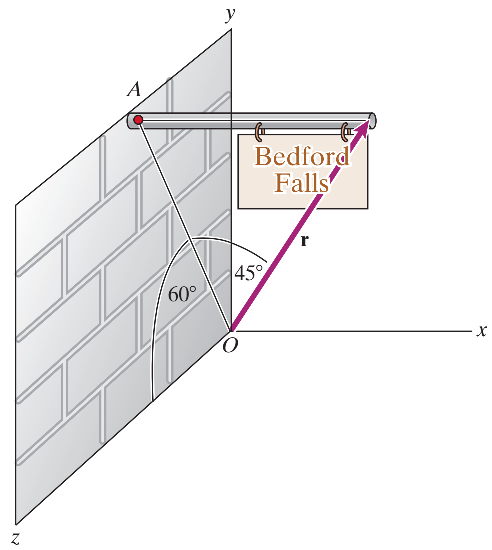
$$\mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 4 \\ -57.14 & -171.42 & 85.72 \end{vmatrix}$$

$$= 685.68\mathbf{i} - 485.72\mathbf{j} - 514.26\mathbf{k}$$

$$= 685.7\mathbf{i} - 485.7\mathbf{j} - 514.3\mathbf{k} \text{ (ft-lb)}$$

$\boxed{\text{(a),(b) } 686\mathbf{i} - 486\mathbf{j} - 514\mathbf{k} \text{ (ft-lb).}}$

Problem 2.159 The pole supporting the sign is parallel to the x-axis and is 6 ft long. Point A is contained in the y–z plane. (a) Express the vector $\mathbf{r}$ in terms of components. (b) What are the direction cosines of $\mathbf{r}$?



Solution: The vector $\mathbf{r}$ is

$$\mathbf{r} = |\mathbf{r}|(\sin 45^\circ \mathbf{i} + \cos 45^\circ \sin 60^\circ \mathbf{j} + \cos 45^\circ \cos 60^\circ \mathbf{k})$$

The length of the pole is the x component of $\mathbf{r}$. Therefore

$$|\mathbf{r}|\sin 45^\circ = 6 \text{ ft} \Rightarrow |\mathbf{r}| = \frac{6 \text{ ft}}{\sin 45^\circ} = 8.49 \text{ ft}$$

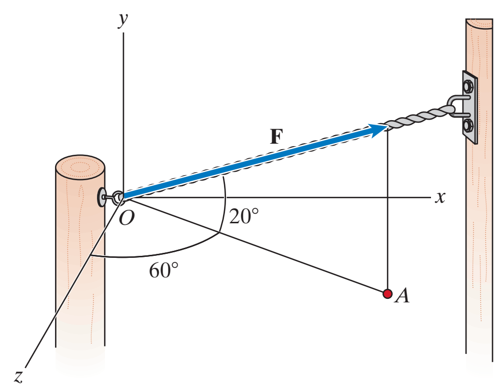
(a) $\boxed{\mathbf{r} = (6.00\mathbf{i} + 5.20\mathbf{j} + 3.00\mathbf{k}) \text{ ft}}$

(b) The direction cosines are

$$\cos\theta_x = \frac{r_x}{|\mathbf{r}|} = 0.707, \; \cos\theta_y = \frac{r_y}{|\mathbf{r}|} = 0.612, \; \cos\theta_z = \frac{r_z}{|\mathbf{r}|} = 0.354$$

$$\boxed{\cos\theta_x = 0.707, \; \cos\theta_y = 0.612, \; \cos\theta_z = 0.354}$$

Problem 2.160 The z component of the force $\mathbf{F}$ is 80 lb. (a) Express $\mathbf{F}$ in terms of components. (b) What are the angles θ_x, θ_y, and θ_z between $\mathbf{F}$ and the positive coordinate axes?



Solution: We can write the force as

$$\mathbf{F} = |\mathbf{F}|(\cos 20^\circ \sin 60^\circ \mathbf{i} + \sin 20^\circ \mathbf{j} + \cos 20^\circ \cos 60^\circ \mathbf{k})$$

We know that the z component is 80 lb. Therefore

$$|\mathbf{F}|\cos 20^\circ \cos 60^\circ = 80 \text{ lb} \Rightarrow |\mathbf{F}| = 170 \text{ lb}$$

(a) $\boxed{\mathbf{F} = (139\mathbf{i} + 58.2\mathbf{j} + 80\mathbf{k}) \text{ lb}}$

(b) The direction cosines can be found:

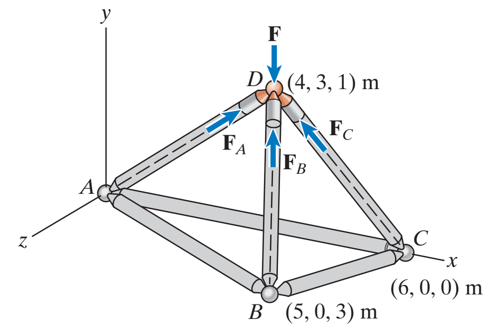
$$\theta_x = \cos^{-1}\left(\frac{139}{170}\right) = 35.5^\circ$$

$$\theta_y = \cos^{-1}\left(\frac{58.2}{170}\right) = 70.0^\circ$$

$$\theta_z = \cos^{-1}\left(\frac{80}{170}\right) = 62.0^\circ$$

$$\boxed{\theta_x = 35.5^\circ, \theta_y = 70.0^\circ, \theta_z = 62.0^\circ}$$

Problem 2.161 The magnitude of the force vector $\mathbf{F}_B$ is 2 kN. Express it in terms of components.



Solution: The strategy is to determine the unit vector collinear with $\mathbf{F}_B$ and then express the force in terms of this unit vector.

The radius vector collinear with $\mathbf{F}_B$ is

$\mathbf{r}_{BD} = (4 - 5)\mathbf{i} + (3 - 0)\mathbf{j} + (1 - 3)\mathbf{k}$ or $\mathbf{r}_{BD} = -1\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$.

The magnitude is

$$|\mathbf{r}_{BD}| = \sqrt{1^2 + 3^2 + 2^2} = 3.74.$$

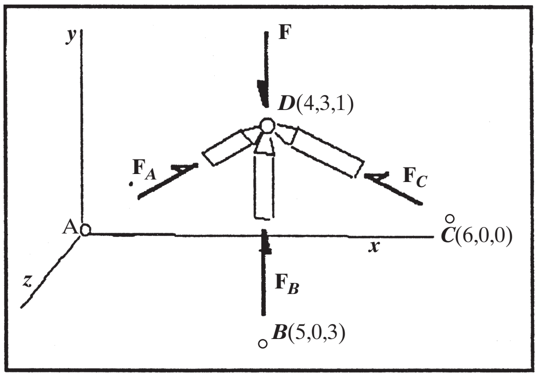
The unit vector is

$$\mathbf{e}_{BD} = \frac{\mathbf{r}_{BD}}{|\mathbf{r}_{BD}|} = -0.2673\mathbf{i} + 0.8018\mathbf{j} - 0.5345\mathbf{k}$$

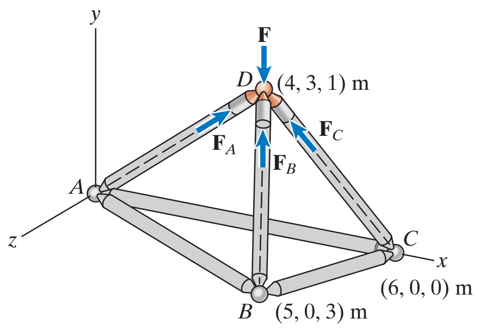
The force is

$$\mathbf{F}_B = |\mathbf{F}_B|\mathbf{e}_{BD} = 2\mathbf{e}_{BD} \text{ (kN) } \mathbf{F}_B = -0.5345\mathbf{i} + 1.6036\mathbf{j} - 1.0693\mathbf{k}$$
$$= -0.53\mathbf{i} + 1.60\mathbf{j} - 1.07\mathbf{k} \text{ (kN)}$$

$$\boxed{\mathbf{F}_B = -0.53\mathbf{i} + 1.60\mathbf{j} - 1.07\mathbf{k} \text{ (kN).}}$$



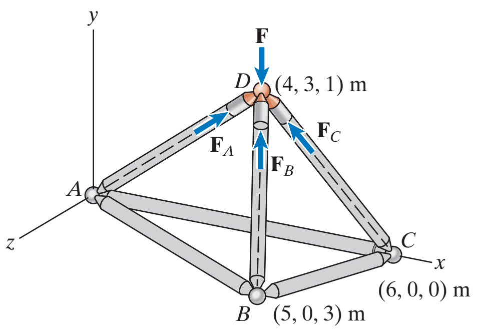
Problem 2.162 The magnitude of the vertical force vector $\mathbf{F}$ is 6 kN. Determine the vector components of $\mathbf{F}$ parallel and normal to the line from B to D.



Solution: The projection of the force $\mathbf{F}$ onto the line from B to D is $\mathbf{F}_P = (\mathbf{F} \cdot \mathbf{e}_{BD})\mathbf{e}_{BD}$. The vertical force has the component $\mathbf{F} = -6\mathbf{j}$ (kN). From Problem 2.139, the unit vector pointing from B to D is $\mathbf{e}_{BD} = -0.2673\mathbf{i} + 0.8018\mathbf{j} - 0.5345\mathbf{k}$. The dot product is $\mathbf{F} \cdot \mathbf{e}_{BD} = -4.813$. Thus the component parallel to the line BD is $\mathbf{F}_P = -4.813\mathbf{e}_{BD} = +1.29\mathbf{i} - 3.86\mathbf{j} + 2.57\mathbf{k}$ (kN). The component perpendicular to the line is: $\mathbf{F}_N = \mathbf{F} - \mathbf{F}_P$. Thus $\mathbf{F}_N = -1.29\mathbf{i} - 2.14\mathbf{j} - 2.57\mathbf{k}$ (kN)

$$\boxed{\mathbf{F}_P = 1.29\mathbf{i} - 3.86\mathbf{j} + 2.57\mathbf{k} \text{ (kN)}, \mathbf{F}_N = -1.29\mathbf{i} - 2.14\mathbf{j} - 2.57\mathbf{k} \text{ (kN).}}$$

Problem 2.163 The magnitude of the vertical force vector $\mathbf{F}$ is 6 kN. Given that $\mathbf{F} + \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{0}$, what are the magnitudes of $\mathbf{F}_A$, $\mathbf{F}_B$, and $\mathbf{F}_C$?



Solution:
The strategy is to expand the forces into scalar components, and then use the force balance equation to solve for the unknowns. The unit vectors are used to expand the forces into scalar components. The position vectors, magnitudes, and unit vectors are:

$\mathbf{r}_{AD} = 4\mathbf{i} + 3\mathbf{j} + 1\mathbf{k}$, $|\mathbf{r}_{AD}| = \sqrt{26} = 5.1$,

$\mathbf{e}_{AD} = 0.7845\mathbf{i} + 0.5883\mathbf{j} + 0.1961\mathbf{k}$.

$\mathbf{r}_{BD} = -1\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, $|\mathbf{r}_{BD}| = \sqrt{14} = 3.74$,

$\mathbf{e}_{BD} = -0.2673\mathbf{i} + 0.8018\mathbf{j} - 0.5345\mathbf{k}$.

$\mathbf{r}_{CD} = -2\mathbf{i} + 3\mathbf{j} + 1\mathbf{k}$, $|\mathbf{r}_{CD}| = \sqrt{14} = 3.74$,

$\mathbf{e}_{CD} = -0.5345\mathbf{i} + 0.8018\mathbf{j} + 0.2673\mathbf{k}$

The forces are:

$\mathbf{F}_A = |\mathbf{F}_A|\mathbf{e}_{AD}$, $\mathbf{F}_B = |\mathbf{F}_B|\mathbf{e}_{BD}$, $\mathbf{F}_C = |\mathbf{F}_C|\mathbf{e}_{CD}$, $\mathbf{F} = -6\mathbf{j}$ (kN).

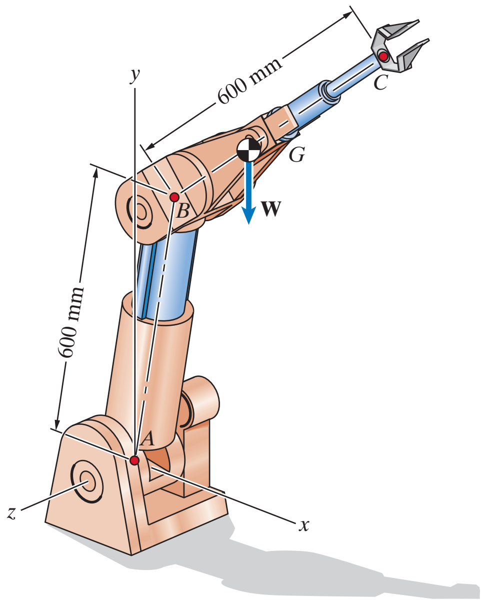
Substituting into the force balance equation

$$\mathbf{F} + \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = 0,$$
$$(0.7843|\mathbf{F}_A| - 0.2674|\mathbf{F}_B| - 0.5348|\mathbf{F}_C|)\mathbf{i} = 0$$
$$(0.5882|\mathbf{F}_A| + 0.8021|\mathbf{F}_B| + 0.8021|\mathbf{F}_C| - 6)\mathbf{j}$$
$$= 0(0.1961|\mathbf{F}_A| - 0.5348|\mathbf{F}_B| + 0.2674|\mathbf{F}_C|)\mathbf{k} = 0$$

These simple simultaneous equations can be solved a standard method (e.g., Gauss elimination) or, conveniently, by using a commercial package, such as TK Solver®, Mathcad®, or other. An HP-28S hand held calculator was used here: $|\mathbf{F}_A| = 2.83$ (kN), $|\mathbf{F}_B| = 2.49$ (kN), $|\mathbf{F}_C| = 2.91$ (kN)

$$\boxed{|\mathbf{F}_A| = 2.83 \text{ (kN)}, |\mathbf{F}_B| = 2.49 \text{ (kN)}, |\mathbf{F}_C| = 2.91 \text{ (kN).}}$$

Problem 2.164 The magnitude of the vertical force **W** is 160 N. The direction cosines of the position vector from A to B are $\cos\theta_x = 0.500$, $\cos\theta_y = 0.866$, and $\cos\theta_z = 0$, and the direction cosines of the position vector from B to C are $\cos\theta_x = 0.707$, $\cos\theta_y = 0.619$, and $\cos\theta_z = -0.342$. Point G is the midpoint of the line from B to C. Determine the vector $\mathbf{r}_{AG} \times \mathbf{W}$, where $\mathbf{r}_{AG}$ is the position vector from A to G.



Solution: The distance from B to G is 0.3 m. Using the given direction cosines, the vector from A to B is

$$\mathbf{r}_{AB} = (0.6 \text{ m})(0.5\mathbf{i} + 0.866\mathbf{j}),$$

and the vector from B to G is

$$\mathbf{r}_{BG} = (0.3 \text{ m})(0.707\mathbf{i} + 0.619\mathbf{j} - 0.342\mathbf{k}).$$

Then the vector from A to G is

$$\mathbf{r}_{AG} = \mathbf{r}_{AB} + \mathbf{r}_{BG} = 0.512\mathbf{i} + 0.705\mathbf{j} - 0.103\mathbf{k} \text{ (m)}.$$

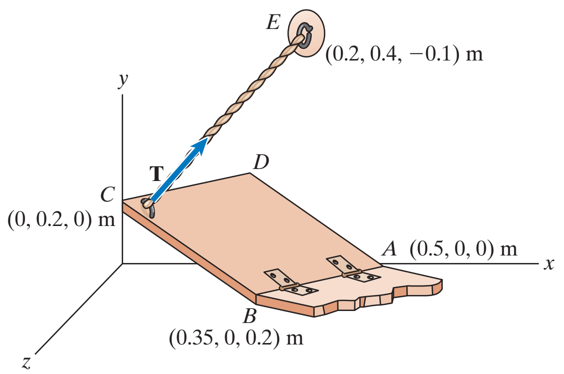
The weight $\mathbf{W} = -160\mathbf{j}$ (N), so the desired cross product is

$$\mathbf{r}_{AG} \times \mathbf{W} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.512 \text{ m} & 0.705 \text{ m} & -0.341 \text{ m} \\ 0 & -160 \text{ N} & 0 \end{vmatrix}$$

$$= -16.4\mathbf{i} - 81.9\mathbf{k} \text{ (N-m)}.$$

$$\boxed{\mathbf{r}_{AG} \times \mathbf{W} = -16.4\mathbf{i} - 81.9\mathbf{k} \text{ (N-m)}.}$$

Problem 2.165 The rope CE exerts a 500-N force **T** on the hinged door. (a) Express **T** in terms of components. (b) Determine the vector component of **T** parallel to the line from point A to point B.



Solution: We have

$$\mathbf{r}_{CE} = (0.2\mathbf{i} + 0.2\mathbf{j} - 0.1\mathbf{k}) \text{ m}$$

$$\mathbf{T} = (500 \text{ N})\frac{\mathbf{r}_{CE}}{|\mathbf{r}_{CE}|} = (333\mathbf{i} + 333\mathbf{j} - 167\mathbf{k}) \text{ N}$$

(a) $\mathbf{T} = (333\mathbf{i} + 333\mathbf{j} - 167\mathbf{k})$ N

(b) We define the unit vector in the direction of AB and then use this vector to find the component parallel to AB.

$$\mathbf{r}_{AB} = (-0.15\mathbf{i} + 0.2\mathbf{k}) \text{ m}$$

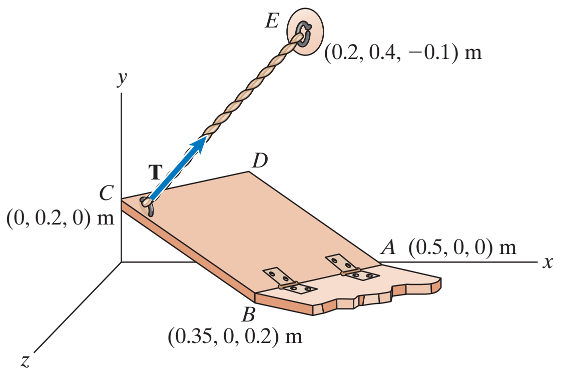
$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = (-0.6\mathbf{i} + 0.8\mathbf{k})$$

$$\mathbf{T}_p = (\mathbf{e}_{AB} \cdot \mathbf{T})\mathbf{e}_{AB} = ([-0.6][333 \text{ N}] + [0.8][-167 \text{ N}])(-0.6\mathbf{i} + 0.8\mathbf{k})$$

$$\mathbf{T}_p = (200\mathbf{i} - 267\mathbf{k}) \text{ N}$$

$$\boxed{\text{(a) } \mathbf{T} = 333\mathbf{i} + 333\mathbf{j} - 167\mathbf{k} \text{ (N)}. \text{ (b) } \mathbf{T}_p = 200\mathbf{i} - 267\mathbf{k} \text{ (N)}.}$$

Problem 2.166 In Problem 2.165, let $\mathbf{r}_{BC}$ be the position vector from point B to point C. Determine the cross product $\mathbf{r}_{BC} \times \mathbf{T}$.



Solution: From Problem 2.165 we know that

$\mathbf{T} = (333\mathbf{i} + 333\mathbf{j} - 167\mathbf{k})$ N

The vector $\mathbf{r}_{BC}$ is

$\mathbf{r}_{BC} = (-035\mathbf{i} + 0.2\mathbf{j} - 0.2\mathbf{k})$ m

The cross product is

$$\mathbf{r}_{BC} \times \mathbf{T} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.35 & 0.2 & -0.2 \\ 333 & 333 & -137 \end{vmatrix} = (33.3\mathbf{i} - 125\mathbf{j} - 183\mathbf{k}) \text{ Nm}$$

$$\boxed{\mathbf{r}_{BC} \times \mathbf{T} = 33.3\mathbf{i} - 125\mathbf{j} - 183\mathbf{k} \text{ (N-m).}}$$