TEXTBOOK TESTBANKS

Solutions Manual for Elementary and Intermediate Algebra 8th Bittinger

SOLUTIONS MANUAL SAMPLE PREVIEW

PUBLISHER
Pearson

COPYRIGHT
2024

RESOURCE TYPE
Solutions Manual

ISBN
9780137977451

INSTRUCTOR'S SOLUTIONS MANUAL

CHRISTI VERITY

ELEMENTARY AND INTERMEDIATE ALGEBRA: CONCEPTS AND APPLICATIONS

EIGHTH EDITION

Bittinger/Ellenbogen/Johnson

Pearson

Pearson

ISBN-13: 978-0-13-797737-6

CONTENTS

Just-in-Time Review

1 Prime Factorizations

1. $33 = 3 \cdot 11$
The prime factorization is $3 \cdot 11$.

2. $121 = 11 \cdot 11$
The prime factorization is $11 \cdot 11$.

3. $18 = 2 \cdot 9 = 2 \cdot 3 \cdot 3$
The prime factorization is $2 \cdot 3 \cdot 3$.

4. $56 = 2 \cdot 28 = 2 \cdot 2 \cdot 14 = 2 \cdot 2 \cdot 2 \cdot 7$
The prime factorization is $2 \cdot 2 \cdot 2 \cdot 7$.

5. $120 = 2 \cdot 60 = 2 \cdot 2 \cdot 30 = 2 \cdot 2 \cdot 2 \cdot 15 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$
The prime factorization is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$.

6. $90 = 2 \cdot 45 = 2 \cdot 3 \cdot 15 = 2 \cdot 3 \cdot 3 \cdot 5$
The prime factorization is $2 \cdot 3 \cdot 3 \cdot 5$.

7. $210 = 2 \cdot 105 = 2 \cdot 3 \cdot 35 = 2 \cdot 3 \cdot 5 \cdot 7$
The prime factorization is $2 \cdot 3 \cdot 5 \cdot 7$.

8. $91 = 7 \cdot 13$
The prime factorization is $7 \cdot 13$.

2 Greatest Common Factor

1. $36 = 2 \cdot 2 \cdot \quad 3 \cdot 3$
$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$
The GCF of 36 and 48 is $2 \cdot 2 \cdot 3$, or 12.

2. $13 = \quad 13$
$52 = 2 \cdot 2 \cdot 13$
The GCF of 13 and 52 is 13.

3. $27 = \quad 3 \cdot 3 \cdot 3$
$40 = 2 \cdot 2 \cdot 2 \cdot \quad 5$
The GCF of 27 and 40 is 1.

4. $54 = 2 \cdot \quad 3 \cdot 3 \cdot 3$
$180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot \quad 5$
The GCF of 54 and 180 is $2 \cdot 3 \cdot 3$, or 18.

5. $18 = 2 \cdot 3 \cdot 3$
$66 = 2 \cdot 3 \cdot \quad 11$
The GCF of 18 and 66 is $2 \cdot 3$, or 6.

6. $30 = 2 \cdot 3 \cdot \quad 5$
$135 = \quad 3 \cdot 3 \cdot 3 \cdot 5$
The GCF of 30 and 135 is $3 \cdot 5$, or 15.

7. $40 = 2 \cdot 2 \cdot 2 \cdot 5$
$220 = 2 \cdot 2 \cdot \quad 5 \cdot 11$
The GCF of 40 and 220 is $2 \cdot 2 \cdot 5$, or 20.

8. $14 = 2 \cdot \quad 7$
$42 = 2 \cdot 3 \cdot 7$
The GCF of 14 and 42 is $2 \cdot 7$, or 14.

9. $15 = \quad 3 \cdot 5$
$40 = 2 \cdot 2 \cdot 2 \cdot \quad 5$
$60 = 2 \cdot 2 \cdot \quad 3 \cdot 5$
The GCF of 15, 40, and 60 is 5.

10. $70 = 2 \cdot \quad 5 \cdot \quad 7$
$105 = \quad 3 \cdot 5 \cdot \quad 7$
$350 = 2 \cdot \quad 5 \cdot 5 \cdot 7$
The GCF of 70, 105, and 350 is $5 \cdot 7$, or 35.

3 Least Common Multiple

1. $24 = 2 \cdot 2 \cdot 2 \cdot 3$
$27 = 3 \cdot 3 \cdot 3$
The LCM of 24 and 27 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$, or 216.

2. $3 = 3$
$15 = 3 \cdot 5$
The LCM of 3 and 15 is $3 \cdot 5$, or 15.

3. $50 = 2 \cdot 5 \cdot 5$
$60 = 2 \cdot 2 \cdot 3 \cdot 5$
The LCM of 50 and 60 is $2 \cdot 2 \cdot 3 \cdot 5 \cdot 5$, or 300.

4. $13 = 13$
$23 = 23$
The LCM of 13 and 23 is $13 \cdot 23$, or 299.

5. $45 = 3 \cdot 3 \cdot 5$
$72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$
The LCM of 45 and 72 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$, or 360.

6. $30 = 2 \cdot 3 \cdot 5$
$36 = 2 \cdot 2 \cdot 3 \cdot 3$
The LCM of 30 and 36 is $2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$, or 180.

7. $12 = 2 \cdot 2 \cdot 3$
$28 = 2 \cdot 2 \cdot 7$
The LCM of 12 and 28 is $2 \cdot 2 \cdot 3 \cdot 7$, or 84.

8. $8 = 2 \cdot 2 \cdot 2$
$16 = 2 \cdot 2 \cdot 2 \cdot 2$
$22 = 2 \cdot 11$
The LCM of 8, 16, and 22 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 11$, or 176.

9. $5 = 5$
$12 = 2 \cdot 2 \cdot 3$
$15 = 3 \cdot 5$
The LCM of 5, 12, and 15 is $2 \cdot 2 \cdot 3 \cdot 5$, or 60.

10. $24 = 2 \cdot 2 \cdot 2 \cdot 3$
$35 = 5 \cdot 7$
$45 = 3 \cdot 3 \cdot 5$
The LCM of 24, 35, and 45 is
$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$, or 2520.

4 Equivalent Expressions and Fraction Notation

1. $\frac{7}{8}\cdot 1=\frac{7}{8}\cdot\frac{3}{3}=\frac{21}{24}$

2. $\frac{5}{6}\cdot 1=\frac{5}{6}\cdot\frac{8}{8}=\frac{40}{48}$

3. $\frac{5}{4}\cdot 1=\frac{5}{4}\cdot\frac{4}{4}=\frac{20}{16}$

4. $\frac{2}{9}\cdot 1=\frac{2}{9}\cdot\frac{6}{6}=\frac{12}{54}$

5. $\frac{3}{11}\cdot 1=\frac{3}{11}\cdot\frac{7}{7}=\frac{21}{77}$

6. $\frac{13}{16}\cdot 1=\frac{13}{16}\cdot\frac{5}{5}=\frac{65}{80}$

5 Mixed Numerals

1. $5\frac{2}{3}=\frac{17}{3}\quad 3\cdot 5=15,\;\; 15+2=17$

2. $9\frac{1}{10}=\frac{91}{10}\quad 10\cdot 9=90,\;\; 90+1=91$

3. $30\frac{4}{5}=\frac{154}{5}\quad 5\cdot 30=150,\;\; 150+4=154$

4. $1\frac{5}{8}=\frac{13}{8}\quad 8\cdot 1=8,\;\; 8+5=13$

5. $66\frac{2}{3}=\frac{200}{3}\quad 3\cdot 66=198,\;\; 198+2=200$

6. $\frac{18}{5}\to \begin{array}{r} 3 \\ 5\overline{)18} \\ \underline{15} \\ 3 \end{array} \to \frac{18}{5}=3\frac{3}{5}$

7. $\frac{29}{6}\to \begin{array}{r} 4 \\ 6\overline{)29} \\ \underline{24} \\ 5 \end{array} \to \frac{29}{6}=4\frac{5}{6}$

8. $\frac{57}{10}\to \begin{array}{r} 5 \\ 10\overline{)57} \\ \underline{50} \\ 7 \end{array} \to \frac{57}{10}=5\frac{7}{10}$

9. $\frac{40}{3}\to \begin{array}{r} 13 \\ 3\overline{)40} \\ \underline{39} \\ 1 \end{array} \to \frac{40}{3}=13\frac{1}{3}$

10. $\frac{757}{100}\to \begin{array}{r} 7 \\ 100\overline{)757} \\ \underline{700} \\ 57 \end{array} \to \frac{757}{100}=7\frac{57}{100}$

6 Converting from Decimal Notation to Fraction Notation

1. $5.3=\frac{53}{10}$ 1 place, 1 zero

2. $0.67=\frac{67}{100}$ 2 places, 2 zeros

3. $4.0008=\frac{40{,}008}{10{,}000}$ 4 places, 4 zeros

4. $1122.3=\frac{11{,}223}{10}$ 1 place, 1 zero

5. $14.703=\frac{14{,}703}{1000}$ 3 places, 3 zeros

6. $0.9=\frac{9}{10}$ 1 place, 1 zero

7. $183.42=\frac{18{,}342}{100}$ 2 places, 2 zeros

8. $0.006=\frac{6}{1000}$ 3 places, 3 zeros

7 Adding and Subtracting Decimal Notation

1. $\begin{array}{r} {\scriptstyle 1\;\;1} \\ 415.78 \\ +\;\; 29.16 \\ \hline 444.94 \end{array}$

2. $\begin{array}{r} {\scriptstyle 2} \\ 35.000 \\ 7.214 \\ +\;\; 128.630 \\ \hline 170.844 \end{array}$

3. $\begin{array}{r} {\scriptstyle 2\,2\;\;1} \\ 17.95 \\ 16.99 \\ +\;\; 28.85 \\ \hline 63.79 \end{array}$

4. $\begin{array}{r} {\scriptstyle 1} \\ 0.600 \\ 2000.430 \\ +\;\; 7.213 \\ \hline 2008.243 \end{array}$

5. $\begin{array}{r} {\scriptstyle 7\;\;10\,10\,10} \\ 7\,\cancel{8}.\cancel{1}\,\cancel{1}\,\cancel{0} \\ -\;\; 4\,5.8\,7\,6 \\ \hline 3\,2.2\,3\,4 \end{array}$

6.
$$\begin{array}{r} 3\overset{7}{\cancel{8}}.\overset{16}{\overset{\cancel{6}}{\cancel{7}}}\,\overset{9}{\overset{\cancel{10}}{\cancel{0}}}\,\overset{10}{\cancel{0}} \\ -\ 11.865 \\ \hline 26.835 \end{array}$$

7.
$$\begin{array}{r} 2.\overset{15}{\cancel{6}}\,\overset{10}{\cancel{0}} \\ -\ 1.08 \\ \hline 1.52 \end{array}$$

8.
$$\begin{array}{r} \overset{2}{\cancel{3}}.\overset{9}{\cancel{0}}\,\overset{9}{\cancel{0}}\,\overset{9}{\cancel{0}}\,\overset{10}{\cancel{0}} \\ -\ 1.0807 \\ \hline 1.9193 \end{array}$$

8 Multiplying and Dividing Decimal Notation

1.
$$\begin{array}{rl} 7.34 & \text{2 decimal places} \\ \times\quad 1.8 & \text{1 decimal place} \\ \hline 5872 & \\ 7340 & \\ \hline 13.212 & \text{3 decimal places} \end{array}$$

2.
$$\begin{array}{rl} 0.86 & \text{2 decimal places} \\ \times\quad 0.93 & \text{2 decimal places} \\ \hline 258 & \\ 7740 & \\ \hline 0.7998 & \text{4 decimal places} \end{array}$$

3.
$$\begin{array}{rl} 0.0024 & \text{4 decimal places} \\ \times\quad 0.015 & \text{3 decimal places} \\ \hline 120 & \\ 240 & \\ \hline 0.000360 & \text{7 decimal places} \end{array}$$

4.
$$\begin{array}{rl} 0.457 & \text{3 decimal places} \\ \times\quad 3.08 & \text{2 decimal places} \\ \hline 3656 & \\ 0000 & \\ 137100 & \\ \hline 1.40756 & \text{5 decimal places} \end{array}$$

5. $7.8\overline{)72.54} \to$
$$\begin{array}{r} 9.3 \\ 78\overline{)725.4} \\ \underline{702} \\ 234 \\ \underline{234} \\ 0 \end{array}$$

6.
$$\begin{array}{r} 2.3 \\ 72\overline{)165.6} \\ \underline{144} \\ 216 \\ \underline{216} \\ 0 \end{array}$$

7. $1.05\overline{)693} \to$
$$\begin{array}{r} 660 \\ 105\overline{)69300} \\ \underline{630} \\ 630 \\ \underline{630} \\ 0 \end{array}$$

8. $0.47\overline{)0.1222} \to$
$$\begin{array}{r} 0.26 \\ 47\overline{)12.22} \\ \underline{94} \\ 282 \\ \underline{282} \\ 0 \end{array}$$

9 Converting from Fraction Notation to Decimal Notation

1. $\frac{11}{32} \to$
$$\begin{array}{r} 0.34375 \\ 32\overline{)11.00000} \\ \underline{96} \\ 140 \\ \underline{128} \\ 120 \\ \underline{96} \\ 240 \\ \underline{224} \\ 160 \\ \underline{160} \\ 0 \end{array}$$

2. $\frac{7}{8} \to$
$$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ \underline{64} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

3. $\frac{13}{11} \to$
$$\begin{array}{r} 1.1818 \\ 11\overline{)13.000} \\ \underline{11} \\ 20 \\ \underline{11} \\ 90 \\ \underline{88} \\ 20 \\ \underline{11} \\ 9 \end{array}$$
$\to \frac{13}{11} = 1.\overline{18}$

4. $\frac{17}{12} \rightarrow \begin{array}{r} 1.416 \\ 12\overline{)17.000} \\ \underline{12} \\ 50 \\ \underline{48} \\ 20 \\ \underline{12} \\ 80 \\ \underline{72} \\ 8 \end{array} \rightarrow \frac{17}{12} = 1.41\overline{6}$

5. $\frac{5}{9} \rightarrow \begin{array}{r} 0.55 \\ 9\overline{)5.00} \\ \underline{45} \\ 50 \\ \underline{45} \\ 5 \end{array} \rightarrow \frac{5}{9} = 0.\overline{5}$

6. $\frac{5}{6} \rightarrow \begin{array}{r} 0.83 \\ 6\overline{)5.00} \\ \underline{48} \\ 20 \\ \underline{18} \\ 2 \end{array} \rightarrow \frac{5}{6} = 0.8\overline{3}$

7. $\frac{19}{9} \rightarrow \begin{array}{r} 2.1 \\ 9\overline{)19.0} \\ \underline{18} \\ 10 \\ \underline{9} \\ 1 \end{array} \rightarrow \frac{19}{9} = 2.\overline{1}$

8. $\frac{9}{11} \rightarrow \begin{array}{r} 0.8181 \\ 11\overline{)9.0000} \\ \underline{88} \\ 20 \\ \underline{11} \\ 90 \\ \underline{88} \\ 20 \\ \underline{11} \\ 9 \end{array} \rightarrow \frac{9}{11} = 0.\overline{81}$

10 Rounding with Decimal Notation

1. 745.06534
hundredth: 745.07
tenth: 745.1
one: 745
ten: 750
hundred: 700

2. 6780.50568
hundredth: 6780.51
tenth: 6780.5
one: 6781
ten: 6780
hundred: 6800

3. \$17.988
cent: \$17.99
dollar: \$18

4. \$20.492
cent: \$20.49
dollar: \$20

5. $\frac{5}{12} \approx 0.416\overline{6}$
ten-thousandth: 0.4167
thousandth: 0.417
hundredth: 0.42
tenth: 0.4
one: 0

6. $\frac{1000}{81} \approx 12.34567901$
ten-thousandth: 12.3457
thousandth: 12.346
hundredth: 12.35
tenth: 12.3
one: 12

11 Converting between Percent Notation and Decimal Notation

1. 63% = 0.63

2. 94.1% = 0.941

3. 240% = 2.4

4. 0.81% = 0.0081

5. 2.3% = 0.023

6. 100% = 1

7. 0.76 = 76%

8. 5 = 500%

9. 0.093 = 9.3%

10. 0.0047 = 0.47%

11. 0.675 = 67.5%

12. 1.34 = 134%

12 Converting between Percent Notation and Fraction Notation

1. $60\% = 60 \cdot \frac{1}{100} = \frac{60}{100}$

2. $28.9\% = 28.9 \cdot \frac{1}{100} = \frac{28.9}{100} \cdot \frac{10}{10} = \frac{289}{1000}$

3. $110\% = 110 \cdot \frac{1}{100} = \frac{110}{100}$

4. $0.042\% = 0.042 \cdot \frac{1}{100} = \frac{0.042}{100} \cdot \frac{1000}{1000} = \frac{42}{100{,}000}$

5. $320\% = 320 \cdot \frac{1}{100} = \frac{320}{100}$

6. $3.47\% = 3.47 \cdot \frac{1}{100} = \frac{3.47}{100} \cdot \frac{100}{100} = \frac{347}{10{,}000}$

7. $\frac{7}{10} \to \begin{array}{r} 0.7 \\ 10\overline{)7.0} \\ \underline{70} \\ 0 \end{array} \to \frac{7}{10} = 0.7$

$0.70 = 70\%$

8. $\frac{14}{25} \to \begin{array}{r} 0.56 \\ 25\overline{)14.00} \\ \underline{125} \\ 150 \\ \underline{150} \\ 0 \end{array} \to \frac{14}{25} = 0.56$

$0.56 = 56\%$

9. $\frac{3.17}{100} \to \begin{array}{r} 0.0317 \\ 100\overline{)3.1700} \\ \underline{300} \\ 170 \\ \underline{100} \\ 700 \\ \underline{700} \\ 0 \end{array} \to \frac{3.17}{100} = 0.0317$

$0.0317 = 3.17\%$

10. $\frac{17}{50} \to \begin{array}{r} 0.34 \\ 50\overline{)17.00} \\ \underline{150} \\ 200 \\ \underline{200} \\ 0 \end{array} \to \frac{17}{50} = 0.34$

$0.34 = 34\%$

11. $\frac{3}{8} \to \begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{24} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array} \to \frac{3}{8} = 0.375$

$0.375 = 37.5\%, \text{ or } 37\frac{1}{2}\%$

12. $\frac{1}{6} \to \begin{array}{r} 0.166 \\ 6\overline{)1.000} \\ \underline{6} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array} \to \frac{1}{6} = 0.16\overline{6}$

$0.16\overline{6} = 16.\overline{6}\%, \text{ or } 16\frac{2}{3}\%$

13 Order of Operations

1. $9 + 2 \times 8 = 9 + 16 = 25$

2. $$\begin{aligned} 39 - 4 \times 2 + 2 &= 39 - 8 + 2 \\ &= 31 + 2 \\ &= 33 \end{aligned}$$

3. $$\begin{aligned} 32 - 8 \div 4 - 2 &= 32 - 2 - 2 \\ &= 30 - 2 \\ &= 28 \end{aligned}$$

4. $3 \cdot 2^3 = 3 \cdot 8 = 24$

5. $$\begin{aligned} 6 \times 8 - 10(15 - 12) &= 6 \times 8 - 10(3) \\ &= 48 - 30 \\ &= 18 \end{aligned}$$

6. $$\begin{aligned} 20 + 4^3 \div 8 - 4 &= 20 + 64 \div 8 - 4 \\ &= 20 + 8 - 4 \\ &= 28 - 4 \\ &= 24 \end{aligned}$$

7. $$\begin{aligned} 400 \times 0.64 \div 3.2 &= 256 \div 3.2 \\ &= 80 \end{aligned}$$

8. $$\begin{aligned} 14 - 2 \times 6 + 7 &= 14 - 12 + 7 \\ &= 2 + 7 \\ &= 9 \end{aligned}$$

9. $$\begin{aligned} 12 \div 3(9 - 5) &= 12 \div 3(4) \\ &= 4(4) \\ &= 16 \end{aligned}$$

10. $$\begin{aligned} \frac{3}{4} - \left(\frac{1}{2}\right)^2 &= \frac{3}{4} - \frac{1}{4} \\ &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$

11. $1000 \div \frac{1}{10} \cdot \frac{4}{5} = 10{,}000 \cdot \frac{4}{5} = 8000$

12. $1000 \div 100 \div 10 = 10 \div 10 = 1$

13. $$\begin{aligned} 8[11 - (2 + 6)] &= 8[11 - (8)] \\ &= 8[3] \\ &= 24 \end{aligned}$$

14. $$\begin{aligned} \frac{1}{10}\left[(5^3 - 5) + 30\right] &= \frac{1}{10}[(125 - 5) + 30] \\ &= \frac{1}{10}[120 + 30] \\ &= \frac{1}{10}[150] \\ &= 15 \end{aligned}$$

14 Unit Conversion

1. $3.5\text{ ft} = 3.5 \cdot 1\text{ ft} = 3.5 \cdot 12\text{ in.} = 42\text{ in.}$

2. $1\frac{1}{5}\text{ mi} = 1\frac{1}{5} \cdot 1\text{ mi} = 1\frac{1}{5} \cdot 5280\text{ ft} = 6336\text{ ft}$

3. $4.06\text{ km} = 4.06 \cdot 1\text{ km} = 4.06 \cdot 1000\text{ m} = 4060\text{ m}$

4. $15\text{ in.} = 15 \cdot 1\text{ in.} = 15 \cdot 2.54\text{ cm} = 38.1\text{ cm}$

5. $2000\text{ cm} = \frac{2000\ \cancel{\text{cm}}}{1} \cdot \frac{1\text{ m}}{100\ \cancel{\text{cm}}} = 20\text{ m}$

6. $1760\text{ ft} = \frac{1760\ \cancel{\text{ft}}}{1} \cdot \frac{1\text{ mi}}{5280\ \cancel{\text{ft}}} = \frac{1760}{3 \cdot 1760}\text{ mi} = \frac{1}{3}\text{ mi}$

7. $13{,}200\text{ in.} = \frac{13{,}200\ \cancel{\text{in}}}{1} \cdot \frac{1\ \cancel{\text{ft}}}{12\ \cancel{\text{in}}} \cdot \frac{1\text{ mi}}{5280\ \cancel{\text{ft}}} = \frac{5}{24}\text{ mi}$

8. $3\text{ km} \approx \frac{3\text{ km}}{1} \cdot \frac{1\text{ mi}}{1.609\text{ km}} \approx 1.86\text{ mi}$

9. $5\frac{1}{3}\text{ yd} = 5\frac{1}{3} \cdot 1\text{ yd} = 5\frac{1}{3} \cdot 3\text{ ft} = 16\text{ ft}$

10. $66\text{ in.} = \frac{66\text{ in.}}{1} \cdot \frac{1\text{ ft}}{12\text{ in.}} = 5\frac{1}{2}\text{ ft}$

11. $2\frac{1}{2}\text{ km} = 2\frac{1}{2} \cdot 1\text{ km} = 2\frac{1}{2} \cdot 1000\text{ m} = 2500\text{ m}$

12. $6.1\text{ km} = \frac{6.1\text{ km}}{1} \cdot \frac{1000\text{ m}}{1\text{ km}} \cdot \frac{100\text{ cm}}{1\text{ m}} = 610{,}000\text{ cm}$

13. $8\text{ cm} \approx \frac{8\text{ cm}}{1} \cdot \frac{1\text{ in.}}{2.54\text{ cm}} \approx 3.15\text{ in.}$

14. $6\text{ km} \approx \frac{6\text{ km}}{1} \cdot \frac{1\text{ mi}}{1.609\text{ km}} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \approx 19{,}689\text{ ft}$

***ZOO end**

Chapter 1

Introduction to Algebraic Expressions

Exercise Set 1.1

1. In the expression 4 + x, the number 4 is a *constant*.

2. In the expression 4 + x, the symbol + indicates the *operation* of addition.

3. To *evaluate* an algebraic expression, we substitute a number for each variable and carry out the operations.

4. An *equation* contains an equal sign.

5. $10n-1$ does not contain an equals sign, so it is an expression.

6. $3x=21$ contains an equals sign, so it is an equation.

7. $2x-5=9$ contains an equals sign, so it is an equation.

8. $5(x+2)$ does not contain an equals sign, so it is an expression.

9. $45=a-1$ contains an equals sign, so it is an equation.

10. $4a-5b$ does not contain an equals sign, so it is an expression.

11. $2x-3y=8$ contains an equals sign, so it is an equation.

12. $r(t+7)+5$ does not contain an equals sign, so it is an expression.

13. Substitute 9 for a and multiply.
$$5a=5\cdot 9=45$$

14. $11\cdot 7=77$

15. Substitute 4 for r and subtract.
$$12-4=8$$

16. $t+8=2+8=10$

17. $\dfrac{a}{b}=\dfrac{45}{9}=5$

18. $\dfrac{14+13}{3}=\dfrac{27}{3}=9$

19. $\dfrac{x+y}{4}=\dfrac{2+14}{4}=\dfrac{16}{4}=4$

20. $\dfrac{54}{9}=6$

21. $\dfrac{p-q}{7}=\dfrac{55-20}{7}=\dfrac{35}{7}=5$

22. $\dfrac{9m}{q}=\dfrac{9\cdot 6}{18}=\dfrac{54}{18}=3$

23. $\dfrac{5z}{y}=\dfrac{5\cdot 9}{15}=\dfrac{45}{15}=3$

24. $\dfrac{20-8}{2}=\dfrac{12}{2}=6$

25. $$\begin{aligned} bh &= (6\text{ ft})(4\text{ ft}) \\ &= (6)(4)(\text{ft})(\text{ft}) \\ &= 24\text{ ft}^2,\text{ or 24 square feet} \end{aligned}$$

26. $\dfrac{27{,}000}{1125}=24\text{ hr}$

27. $$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(5\text{ cm})(6\text{ cm}) \\ &= \frac{1}{2}(5)(6)(\text{cm})(\text{cm}) \\ &= \frac{5}{2}\cdot 6\text{ cm}^2 \\ &= 15\text{ cm}^2,\text{ or 15 square centimeters} \end{aligned}$$

28. (a) $3(30\text{ sec})=90\text{ sec}$;

    (b) $3(90\text{ sec})=270\text{ sec}$;

    (c) $3(2\text{ min})=6\text{ min}$

29. $$\begin{aligned} A &= bh \\ &= (67\text{ ft})(12\text{ ft}) \\ &= (67)(12)(\text{ft})(\text{ft}) \\ &= 804\text{ ft}^2,\text{ or 804 square feet} \end{aligned}$$

30. $\dfrac{h}{a}=\dfrac{12}{18}\approx 0.667$

31. Let z represent Zane's age. Then we have $z+5$, or $5+z$.

32. $4a$, or $a\cdot 4$

33. $6b$, or $b\cdot 6$

34. Let h represent Isabel's height. Then we have $h+7$, or $7+h$.

35. $c-9$

36. $d-4$

37. $6+q$, or $q+6$

38. $11+z$, or $z+11$

39. $p-t$

40. $n - m$

41. $y - x$

42. Let a represent Xander's age. Then we have $a - 2$.

43. $x \div w$, or $\frac{x}{w}$

44. Let s and t represent the numbers. Then we have $s \div t$, or $\frac{s}{t}$.

45. Let l and h represent the box's length and height, respectively. Then we have $l + h$, or $h + l$.

46. $d + f$, or $f + d$

47. $9 \cdot 2m$, or $2m \cdot 9$

48. Let a represent Amaya's speed and w represent the wind speed. Then we have $a - 2w$.

49. Let y represent "some number." Then we have $\frac{1}{4}y - 13$, or $\frac{y}{4} - 13$.

50. Let n represent the number; $10n - 4$

51. Let a and b represent the two numbers. Then we have $5(a - b)$.

52. Let x and y represent the numbers. Then we have $\frac{1}{3}(x + y)$, or $\frac{x + y}{3}$.

53. Let w represent the number of women attending. Then we have 64% of w, or $0.64w$.

54. Let y represent "a number." Then we have 38% of y, or $0.38y$.

55. Let x represent the number.
Translating:

$$\begin{array}{ccccc} \underbrace{\text{What number}} & \underbrace{\text{added to}} & 73 & \text{is} & 201? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ x & + & 73 & = & 201 \end{array}$$

$x + 73 = 201$

56. Let x represent the number.
$7x = 1596$

57. Let x represent the number.

$$\begin{array}{lccccc} \text{Rewording:} & 42 & \text{times} & \underbrace{\text{what number}} & \text{is} & 2352? \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{Translating:} & 42 & \cdot & x & = & 2352 \end{array}$$

$42x = 2352$

58. Let x represent the number.
$x + 345 = 987$

59. Let s represent the number of unoccupied squares.

$$\begin{array}{lccccc} \text{Rewording:} & \underbrace{\text{The number of unoccupied squares}} & \underbrace{\text{added to}} & 19 & \text{is} & 64. \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{Translating:} & s & + & 19 & = & 64 \end{array}$$

$s + 19 = 64$

60. Let h represent the number of hours the carpenter worked.
$\$55h = \$5720$

61. Let w represent the total amount of waste generated, in millions of tons.

$$\begin{array}{lccccc} \text{Rewording:} & \underbrace{32.1\%} & \text{of} & \underbrace{\text{the total amount of waste}} & \text{is} & \underbrace{\text{94 million tons.}} \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{Translating:} & 32.1\% & \cdot & w & = & 94 \end{array}$$

$32.1\% \cdot w = 94$, or $0.321w = 94$

62. Let t represent the length of the average commute in South Dakota, in minutes.
$t + 16.3 = 33.3$

63. We look for a pattern in the data. We try subtracting.

$8 - 3 = 5$ $\quad 11 - 6 = 5$
$9 - 4 = 5$ $\quad 12 - 7 = 5$
$10 - 5 = 5$ $\quad 13 - 8 = 5$

The amount is the same, 5, for each pair of numbers. Let a represent the age of the child and f represent the number of grams of dietary fiber.
We reword and translate as follows:

$$\begin{array}{lccccc} \text{Rewording:} & \underbrace{\text{dietary fiber}} & \text{is} & \underbrace{\text{child's age}} & \text{added to} & \underbrace{5} \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{Translating:} & f & = & a & + & 5 \end{array}$$

$f = a + 5$

64. Let c represent the cost of tuition and h represent the hours of classes.
$c = 150h$

65. We look for a pattern in the data. We try dividing.

$24{,}000 \div 2 = 12{,}000$ $\quad 120{,}000 \div 10 = 12{,}000$
$60{,}000 \div 5 = 12{,}000$ $\quad 180{,}000 \div 15 = 12{,}000$

The amount is the same, 12,000, for each pair of numbers. Let n represent the number of insects eaten daily and b represent the number of birds.
We reword and translate as follows:

$$\begin{array}{ccccc} \underbrace{\text{number of insects eaten}} & \text{is} & \underbrace{12{,}000} & \text{times} & \underbrace{\text{number of birds}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ n & = & 12{,}000 & + & b \end{array}$$

$n = 12{,}000b$

66. Let r represent the amount received and s represent the amount spent.

$r = s - 3$

67. We look for a pattern in the data. We try subtracting.

$27.75 - 23.75 = 4$
$28.60 - 24.60 = 4$
$41.60 - 37.60 = 4$

The amount is the same, 4, for each pair of numbers. Let n represent the nonmachinable cost and m represent the machinable cost.
We reword and translate as follows:

$$\underbrace{\text{nonmachinable cost}} \quad \text{is} \quad \underbrace{\text{machinable cost}} \quad \text{added to} \quad \underbrace{4}$$
$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$
$$n \qquad = \qquad m \qquad + \qquad 4$$

$n = m + 4$

68. Let w represent the depth of water and s represent the depth of snow.

$w = s \div 10$

69. The sum of two numbers m and n is $m+n$, and twice the sum is $2(m+n)$. Choice (f) is the correct answer.

70. Five less than a number x is $x-5$. If this expression is equal to 12, we have the equation $x-5=12$. Choice (h) is the correct answer.

71. Twelve more than a number t is $t+12$. If this expression is equal to 5, we have the equation $t+12=5$. Choice (d) is the correct answer.

72. The product of two numbers a and b is $a \cdot b$. Half of this product is $\frac{1}{2} \cdot a \cdot b$. Choice (c) is the correct answer.

73. The sum of a number t and 5 is $t+5$, and 3 times the sum is $3(t+5)$. Choice (g) is the correct answer.

74. The sum of two numbers x and y is $x+y$, and twice this sum is $2(x+y)$. If this expression is equal to 48, we have the equation $2(x+y)=48$. Choice (b) is the correct answer.

75. The product of two numbers a and b is ab, and 1 less than this product is $ab-1$. If this expression is equal to 48, we have the equation $ab-1=48$. Choice (e) is the correct answer.

76. The quotient of two numbers x and y is $\frac{x}{y}$, and 6 more than this quotient is $\frac{x}{y}+6$. Choice (a) is the correct answer.

77. *Writing Exercise*. A *variable* is a letter that is used to stand for any number chosen from a set of numbers. An *algebraic expression* is an expression that consists of variables, constants, operation signs, and/or grouping symbols. A *variable expression* is an algebraic expression that contains a variable. An *equation* is a number sentence with the verb =. The symbol = is used to indicate that the algebraic expressions on either side of the symbol represent the same number.

78. *Writing Exercise*. To evaluate an algebraic expression means to find the value of the expression when its variables are given values.

79. *Writing Exercise*. No; for a square with side s, the area is given by $A = s \cdot s$. The area of a square with side $2s$ is given by $(2s)(2s) = 4 \cdot s \cdot s = \ 4A \neq 2A$.

80. *Writing Exercise*. Answers may vary.
Skylar was born in 2006. Find Skylar's age in 2014.

81. Area of sign: $A = \frac{1}{2}(3\text{ ft})(2.5\text{ ft}) = 3.75\text{ ft}^2$

Cost of sign: $\$120(3.75) = \$450$

82. The shaded area is the area of a rectangle with dimensions 20 cm by 10 cm less the area of a triangle with base 20 cm – 4 cm – 5 cm, or 11 cm, and height 7.5 cm. We perform the computation

$$(20\text{ cm})(10\text{ cm}) - \frac{1}{2}(11\text{ cm})(7.5)$$
$$= 200\text{ cm}^2 - 41.25\text{ cm}^2$$
$$= 158.75\text{ cm}^2,\text{ or }158.75\text{ square centimeters}$$

83. When x is twice y, then y is one-half x, so $y = \frac{12}{2} = 6$.

$$\frac{x-y}{3} = \frac{12-6}{3} = \frac{6}{3} = 2$$

84. $x = 6,\ y = 2x = 2 \cdot 6 = 12$

$$\frac{x+y}{2} = \frac{6+12}{2} = \frac{18}{2} = 9$$

85. When a is twice b, then b is one-half a, so $b = \frac{16}{2} = 8$.

$$\frac{a+b}{4} = \frac{16+8}{4} = \frac{24}{4} = 6$$

86. When a is three times b, then b is one-third a, so $b = \frac{18}{3} = 6$.

$$\frac{a-b}{3} = \frac{18-6}{3} = \frac{12}{3} = 4$$

87. The next whole number is one more than $w+3$:

$w+3+1 = w+4$

88. The preceding odd number is 2 less than $d+2$:

$d+2-2 = d$

89. $l+w+l+w$, or $2l+2w$

90. $s+s+s+s$, or $4s$

91. If t is Pari's race time, then Dion's race time is $t+3$ and Ellie's race time is

$t+3+5 = t+8.$

92. $a+2+7$, or $a+9$

93. *Writing Exercise*. Yes; the area of a triangle with base b and height h is given by $A=\frac{1}{2}bh$. The area of a triangle with base b and height $2h$ is given by $\frac{1}{2}b(2h)=2\left(\frac{1}{2}bh\right)=2A$.

Exercise Set 1.2

1. *Equivalent* expressions represent the same number.
2. Changing the order of multiplication does not affect the answer. This is an example of a *commutative* law.
3. The result of addition is called a *sum*.
4. The numbers in a product are called *factors*.
5. Commutative
6. Associative
7. Distributive
8. Commutative
9. Commutative
10. Distributive
11. $t+11$ Changing the order
12. $2+a$
13. $8x+4$
14. $c+ab$
15. $3y+9x$
16. $7b+3a$
17. $5(1+a)$
18. $9(5+x)$
19. $x\cdot 7$ Changing the order
20. yx
21. ts
22. $m\cdot 13$
23. $5+ba$
24. $x+y\cdot 3$
25. $(a+1)5$
26. $(x+5)9$
27. $x+(8+y)$
28. $5+(m+r)$
29. $(u+v)+7$
30. $(x+2)+y$
31. $ab+(c+d)$
32. $m+(np+r)$
33. $10(xy)$
34. $4(uv)$
35. $(2a)b$
36. $(9\cdot 7)r$
37. $(3\cdot 2)(a+b)$
38. $(5x)(2+y)$
39. $s+(t+6)=(s+t)+6$
    $=(t+6)+s$
40. $7+(v+w)=(v+w)+7=v+(w+7)$
    $7+(v+w)=(7+v)+w=(v+7)+w$
    Answers may vary.
41. $(17a)b=17(ab)$ Using the associative law
    $(17a)b=b(17a)$ Using the commutative law
    $=b(a17)$ Using the commutative law again
    Answers may vary.
42. $x(3y)=(3y)x=3(yx)$
    $x(3y)=(x\cdot 3)y=(3x)y$
43. $(1+x)+2=(x+1)+2$ Commutative law
    $=x+(1+2)$ Associative law
    $=x+3$ Simplifying
44. $(2a)4=4(2a)$ Commutative law
    $=(4\cdot 2)a$ Associative law
    $=8a$ Simplifying
45. $(m\cdot 3)7=m(3\cdot 7)$ Associative law
    $=m\cdot 21$ Simplifying
    $=21m$ Commutative law
46. $4+(9+x)$
    $=(4+9)+x$ Associative law
    $=13+x$ Simplifying
    $=x+13$ Commutative law
47. $x+xyz+1$
    The term are separated by plus signs. They are x, xyz, and 1.
48. 9, $17a$, abc
49. $2a+\frac{a}{3b}+5b$
    The terms are separated by plus signs. They are $2a$, $\frac{a}{3b}$, and $5b$.

50. $3xy, 20, \frac{4a}{b}$

51. $4x, 4y$

52. $14, 2y$

53. $5n = 5 \cdot n$
The factors are 5 and n.

54. $uv = u \cdot v$
The factors are u and v.

55. $3(x+y) = 3 \cdot (x+y)$
The factors are 3 and $(x+y)$.

56. $(a+b)12 = (a+b) \cdot 12$
The factors are $(a+b)$ and 12.

57. The factors are 7, a, and b.

58. The factors are m, n, and 2.

59. $(a-b)(x-y) = (a-b) \cdot (x-y)$
The factors are $(a-b)$ and $(x-y)$.

60. $(3-a)(b+c) = (3-a) \cdot (b+c)$
The factors are $(3-a)$ and $(b+c)$.

61. $2(x+15) = 2 \cdot x + 2 \cdot 15 = 2x + 30$

62. $3x + 15$

63. $4(1+a) = 4 \cdot 1 + 4 \cdot a = 4 + 4a$

64. $7 + 7y$

65. $10(9x+6) = 10 \cdot 9x + 10 \cdot 6 = 90x + 60$

66. $54m + 63$

67. $5(r+2+3t) = 5 \cdot r + 5 \cdot 2 + 5 \cdot 3t = 5r + 10 + 15t$

68. $20x + 32 + 12p$

69. $(a+b)2 = a(2) + b(2) = 2a + 2b$

70. $7x + 14$

71. $(x+y+2)5 = x(5) + y(5) + 2(5) = 5x + 5y + 10$

72. $12 + 6a + 6b$

73. $2 \cdot a + 2 \cdot b$ Using the distributive law
$= 2(a+b)$ The common factor is 2.
Check: $2(a+b) = 2 \cdot a + 2 \cdot b = 2a + 2b$

74. $5y + 5z = 5(y+z)$
Check: $5(y+z) = 5y + 5z$

75. $7 + 7y = 7 \cdot 1 + 7 \cdot y$ The common factor is 7.
$= 7(1+y)$ Using the distributive law
Check: $7(1+y) = 7 \cdot 1 + 7 \cdot y = 7 + 7y$

76. $13 + 13x = 13(1+x)$
Check: $13(1+x) = 13 + 13x$

77. $32x + 2 = 2 \cdot 16x + 2 \cdot 1 = 2(16x+1)$
Check: $2(16x+1) = 2 \cdot 16x + 2 \cdot 1 = 32x + 2$

78. $20a + 5 = 5(4a+1)$
Check: $5(4a+1) = 20a + 5$

79. $5x + 10 + 15y = 5 \cdot x + 5 \cdot 2 + 5 \cdot 3y = 5(x+2+3y)$
Check: $5(x+2+3y) = 5 \cdot x + 5 \cdot 2 + 5 \cdot 3y$
$= 5x + 10 + 15y$

80. $3 + 27b + 6c = 3(1+9b+2c)$
Check: $3(1+9b+2c) = 3 + 27b + 6c$

81. $7a + 35b = 7 \cdot a + 7 \cdot 5b = 7(a+5b)$
Check: $7(a+5b) = 7 \cdot a + 7 \cdot 5b = 7a + 35b$

82. $3x + 24y = 3(x+8y)$
Check: $3(x+8y) = 3x + 24y$

83. $44x + 11y + 22z = 11 \cdot 4x + 11 \cdot y + 11 \cdot 2z$
$= 11(4x + y + 2z)$
Check: $11(4x+y+2z) = 11 \cdot 4x + 11 \cdot y + 11 \cdot 2z$
$= 44x + 11y + 22z$

84. $14a + 56b + 7 = 7(2a + 8b + 1)$
Check: $7(2a+8b+1) = 14a + 56b + 7$

85. $3(2+x)$
$= 3(x+2)$ Commutative law of addition
$= 3 \cdot x + 3 \cdot 2$ Distributive law
$= 3x + 6$ Multiplying

86. $(y+4)5$
$= 5(y+2)$ Commutative law of multiplication
$= 5 \cdot y + 5 \cdot 4$ Distributive law
$= 5y + 20$ Multiplying

87. $7(2x+3y)$
$= 7(2x) + 7(3y)$ Distributive law
$= (7 \cdot 2)x + (7 \cdot 3)y$ Associative law of multiplication
$= 14x + 21y$ Multiplying

88. $(4a+2)8$
$= 8(4a+2)$ Commutative law of multiplication
$= 8(4a) + 8(2)$ Distributive law
$= (8 \cdot 4)a + 8(2)$ Associative law of multiplication
$= 32a + 16$ Multiplying

89. *Writing Exercise*. No; in general, when subtracting, the result depends on the order in which the operation is performed.

90. *Writing Exercise*. No; in general, when dividing, the result depends on the grouping.

91. *Writing Exercise*. Terms are added; factors are multiplied. Examples will vary.

92. *Writing Exercise.*
$2(3x+4y)$
$= 2(3x)+2(4y)$ Distributive law
$= (2\cdot 3)x+(2\cdot 4)y$ Associative law
$= 6x+8y$

93. $[2(x+1)]+3x$
$=[2\cdot x+2\cdot 1)]+3x$ Distributive law
$=[2x+2]+3x$ Multiplying
$=2x+[2+3x]$ Associative law of addition
$=2x+[3x+2]$ Commutative law of addition
$=[2x+3x]+2$ Associative law of addition
$=[(2+3)x]+2$ Distributive law
$=[5x]+2$ Adding
$=5x+2$

94. $12a+4(b+5)$
$=(4\cdot 3)a+4(b+5)$ Writing 12 as a product
$=4(3a)+4(b+5)$ Associative law of multiplication
$=4(3a)+4(b)+4(5)$ Distributive law
$=4(3a+b+5)$ Distributive law

95. The expressions are equivalent by the distributive law.
$8+4(a+b)=8+4a+4b=4(2+a+b)$

96. The expressions are not equivalent.
Let $a=2$ and $b=3$. Then we have:
$5(2\cdot 3)=5\cdot 6=30,$ but
$5\cdot 2\cdot 5\cdot 3=10\cdot 5\cdot 3=50\cdot 3=150.$

97. The expressions are not equivalent.
Let $m=1$. Then we have:
$7\div 3\cdot 1=\frac{7}{3}\cdot 1=\frac{7}{3},$ but
$1\cdot 3\div 7=3\div 7=\frac{3}{7}.$

98. The expressions are equivalent by the commutative law of multiplication and the distributive law.
$(rt+st)5=5(rt+st)=5\cdot t(r+s)=5t(r+s)$

99. The expressions are not equivalent.
Let $x=1$ and $y=0$. Then we have:
$30\cdot 0+1\cdot 15=0+15=15,$ but
$5[2(1+3\cdot 0)]=5[2(1)]=5\cdot 2=10.$

100. The expressions are equivalent by the commutative and associative laws of multiplication and the distributive law.
$[c(2+3b)]5=5[c(2+3b)]$
$=5c(2+3b)$
$=10c+15bc$

101. *Writing Exercise.* $3(2+x)=3(2+0)=3\cdot 2=6$;
$6+x=6+0=6$
The result indicates that $3(2+x)$ and $6+x$ are equivalent when $x=0$. (By the distributive law, we know they are not equivalent for all values of x.)

102. *Writing Exercise.*
$15x+40=5(3x+8)$
$15\cdot 4+40=60+40=100$
$5(3\cdot 4+8)=5(12+8)=5\cdot 20=100$
Although the expressions $15x+40$ and $5(3x+8)$ are equivalent when $x=4$, this result does not guarantee that the factorization is correct. (See Exercise 101.)

103. Answers may vary.
a. Let x represent the number of overtime hours worked in one week.
Asad: $20(1.5x+40)$
Bella: $20\cdot 40+20(1.5)x$
Cody: $30x+800$
b. We simplify each expression.
Asad: $20(1.5x+40)=20(1.5x)+20(40)$
$=30x+800$
Bella: $20\cdot 40+20(1.5)x=800+30x$
Cody: $30x+800$
All expressions are equivalent to $30x+800$.

Exercise Set 1.3

1. The top number in a fraction is called the *numerator*.

2. A *prime* number has exactly two different factors.

3. To divide two fractions, multiply by the *reciprocal* of the divisor.

4. We need a common denominator in order to *add* fractions.

5. Since $35=5\cdot 7$, choice (b) is correct.

6. Since $60=3\cdot 20$, choice (c) is correct.

7. Since 65 is an odd number and has more than two different factors, choice (d) is correct.

8. The only even prime number is 2, so choice (a) is correct.

9. 9 is composite because it has more than two different factors. They are 1, 3, and 9.

10. 15 is composite because it has more than two different factors. They are 1, 3, 5, and 15.

11. 41 is prime because it has only two different factors, 41 and 1.

12. 49 is composite because it has more than two different factors. They are 1, 7, and 49.

13. 77 is composite because it has more than two different factors. They are 1, 7, 11, and 77.

14. 37 is prime because it has only two different factors, 37 and 1.

15. 2 is prime because it has only two different factors, 2 and 1.

16. 1 is not prime because it does not have two different factors. It is not composite because it does not have more than two different factors. Thus 1 is neither prime nor composite.

17. The terms "prime" and "composite" apply only to natural numbers. Since 0 is not a natural number, it is neither prime nor composite.

18. 16 is composite because it has more than two factors. They are 1, 2, 4, 8, and 16.

19. Factorizations:
$1\cdot 50,\ 2\cdot 25,\ 5\cdot 10$
List all of the factors of 50:
1, 2, 5, 10, 25, 50

20. Factorizations: $1\cdot 70$, $2\cdot 35$, $5\cdot 14$, $7\cdot 10$
Factors: 1, 2, 5, 7, 10, 14, 35, 70

21. Factorizations:
$1\cdot 42$, $2\cdot 21$, $3\cdot 14$, $6\cdot 7$
List all of the factors of 42:
1, 2, 3, 6, 7, 14, 21, 42

22. Factorizations:
$1\cdot 60$, $2\cdot 30$, $3\cdot 20$, $4\cdot 15$, $5\cdot 12$, $6\cdot 10$
Factors: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

23. $39 = 3\cdot 13$

24. $2\cdot 17$

25. We begin factoring 30 in any way that we can and continue factoring until each factor is prime.
$30 = 2\cdot 15 = 2\cdot 3\cdot 5$

26. $2\cdot 7\cdot 7$

27. We begin by factoring 27 in any way that we can and continue factoring until each factor is prime.
$27 = 3\cdot 9 = 3\cdot 3\cdot 3$

28. $2\cdot 3\cdot 3\cdot 3$

29. We begin by factoring 150 in any way that we can and continue factoring until each factor is prime.
$150 = 2\cdot 75 = 2\cdot 3\cdot 25 = 2\cdot 3\cdot 5\cdot 5$

30. $2\cdot 2\cdot 2\cdot 7$

31. 31 has exactly two different factors, 31 and 1. Thus, 31 is prime.

32. $180 = 2\cdot 2\cdot 3\cdot 3\cdot 5$

33. $210 = 2\cdot 105 = 2\cdot 3\cdot 35 = 2\cdot 3\cdot 5\cdot 7$

34. 79 is prime.

35. $115 = 5\cdot 23$

36. $11\cdot 13$

37. $\frac{21}{35} = \frac{7\cdot 3}{7\cdot 5}$ Factoring numerator and denominator
$= \frac{7}{7}\cdot\frac{3}{5}$ Rewriting as a product of two fractions
$= 1\cdot\frac{3}{5}$ $\frac{7}{7} = 1$
$= \frac{3}{5}$ Using the identity property of 1

38. $\frac{20}{26} = \frac{\cancel{2}\cdot 10}{\cancel{2}\cdot 13} = \frac{10}{13}$

39. $\frac{16}{56} = \frac{2\cdot 8}{7\cdot 8} = \frac{2}{7}\cdot\frac{8}{8} = \frac{2}{7}\cdot 1 = \frac{2}{7}$

40. $\frac{72}{27} = \frac{8\cdot\cancel{9}}{3\cdot\cancel{9}} = \frac{8}{3}$

41. $\frac{12}{48} = \frac{1\cdot 12}{4\cdot 12}$ Factoring and using the identity property of 1 to to write 12 as $1\cdot 12$
$= \frac{1}{4}\cdot\frac{12}{12}$
$= \frac{1}{4}\cdot 1 = \frac{1}{4}$

42. $\frac{18}{84} = \frac{\cancel{6}\cdot 3}{\cancel{6}\cdot 14} = \frac{3}{14}$

43. $\frac{52}{13} = \frac{13\cdot 4}{13\cdot 1} = 1\cdot\frac{4}{1} = 4$

44. $\frac{132}{11} = \frac{12\cdot\cancel{11}}{1\cdot\cancel{11}} = 12$

45. $\frac{19}{76} = \frac{1\cdot 19}{4\cdot 19}$ Factoring and using the identity property of 1 to write 19 as $1\cdot 19$
$= \frac{1}{4}\cdot\frac{\cancel{19}}{\cancel{19}}$ Removing a factor equal to 1: $\frac{19}{19} = 1$
$= \frac{1}{4}$

46. $\frac{17}{51} = \frac{1\cdot\cancel{17}}{3\cdot\cancel{17}} = \frac{1}{3}$

47. $\frac{150}{25} = \frac{6\cdot 25}{1\cdot 25}$ Factoring and using the identity property of 1 to write 25 as $1\cdot 25$
$= \frac{6}{1}\cdot\frac{\cancel{25}}{\cancel{25}}$ Removing a factor equal to 1: $\frac{25}{25} = 1$
$= \frac{6}{1}$
$= 6$ Simplifying

48. $\frac{180}{36} = \frac{5\cdot\cancel{36}}{1\cdot\cancel{36}} = 5$

49. $\frac{42}{50}=\frac{2\cdot 21}{2\cdot 25}$ Factoring the numerator and the denominator

$=\frac{\cancel{2}\cdot 21}{\cancel{2}\cdot 25}$ Removing a factor equal to 1: $\frac{2}{2}=1$

$=\frac{21}{25}$

50. $\frac{75}{80}=\frac{\cancel{5}\cdot 15}{\cancel{5}\cdot 16}=\frac{15}{16}$

51. $\frac{120}{82}=\frac{2\cdot 60}{2\cdot 41}$ Factoring

$=\frac{\cancel{2}\cdot 60}{\cancel{2}\cdot 41}$ Removing a factor equal to 1: $\frac{2}{2}=1$

$=\frac{60}{41}$

52. $\frac{75}{45}=\frac{5\cdot\cancel{15}}{3\cdot\cancel{15}}=\frac{5}{3}$

53. $\frac{210}{98}=\frac{2\cdot 7\cdot 15}{2\cdot 7\cdot 7}$ Factoring

$=\frac{\cancel{2}\cdot\cancel{7}\cdot 15}{\cancel{2}\cdot\cancel{7}\cdot 7}$ Removing a factor equal to1: $\frac{2\cdot 7}{2\cdot 7}=1$

$=\frac{15}{7}$

54. $\frac{140}{350}=\frac{\cancel{2}\cdot 2\cdot\cancel{5}\cdot\cancel{7}}{\cancel{2}\cdot\cancel{5}\cdot 5\cdot\cancel{7}}=\frac{2}{5}$

55. $\frac{1}{2}\cdot\frac{3}{5}=\frac{1\cdot 3}{2\cdot 5}$ Multiplying numerators and denominators

$=\frac{3}{10}$

56. $\frac{11}{10}\cdot\frac{8}{5}=\frac{11\cdot 8}{10\cdot 5}=\frac{11\cdot\cancel{2}\cdot 4}{\cancel{2}\cdot 5\cdot 5}=\frac{44}{25}$

57. $\frac{9}{2}\cdot\frac{4}{3}=\frac{9\cdot 4}{2\cdot 3}=\frac{3\cdot\cancel{3}\cdot\cancel{2}\cdot 2}{\cancel{2}\cdot\cancel{3}}=6$

58. By the commutative law of multiplication, $11\cdot 12$ and $12\cdot 11$ are equivalent. Thus, the product of the numerators will be equivalent to the product of the denominators, so the result is a number divided by itself, or 1.

$\frac{11}{12}\cdot\frac{12}{11}=\frac{11\cdot 12}{12\cdot 11}=1$

59. $\frac{1}{8}+\frac{3}{8}=\frac{1+3}{8}$ Adding numerators; keeping the common denominator

$=\frac{4}{8}$

$=\frac{1\cdot\cancel{4}}{2\cdot\cancel{4}}=\frac{1}{2}$ Simplifying

60. $\frac{1}{10}+\frac{7}{10}=\frac{8}{10}=\frac{2\cdot 4}{2\cdot 5}=\frac{4}{5}$

61. $\frac{4}{9}+\frac{13}{18}=\frac{4}{9}\cdot\frac{2}{2}+\frac{13}{18}$ Using 18 as the common denominator

$=\frac{8}{18}+\frac{13}{18}$

$=\frac{21}{18}$

$=\frac{7\cdot\cancel{3}}{6\cdot\cancel{3}}=\frac{7}{6}$ Simplifying

62. $\frac{4}{5}+\frac{8}{15}=\frac{12}{15}+\frac{8}{15}=\frac{20}{15}=\frac{4}{3}$

63. $\frac{3}{a}\cdot\frac{b}{7}=\frac{3b}{7a}$ Multiplying numerators and denominators

64. $\frac{x}{5}\cdot\frac{y}{z}=\frac{xy}{5z}$

65. $\frac{4}{n}+\frac{6}{n}=\frac{10}{n}$ Adding numerators; keeping the common denominator

66. $\frac{9}{x}-\frac{5}{x}=\frac{4}{x}$

67. $\frac{3}{10}+\frac{8}{15}=\frac{3}{10}\cdot\frac{3}{3}+\frac{8}{15}\cdot\frac{2}{2}$ Using 30 as the common denominator

$=\frac{9}{30}+\frac{16}{30}$

$=\frac{25}{30}$

$=\frac{5\cdot\cancel{5}}{6\cdot\cancel{5}}=\frac{5}{6}$ Simplifying

68. $\frac{7}{8}+\frac{5}{12}=\frac{21}{24}+\frac{10}{24}=\frac{31}{24}$

69. $\frac{11}{7}-\frac{4}{7}=\frac{7}{7}=1$

70. $\frac{12}{5}-\frac{2}{5}=\frac{10}{5}=2$

71. $\frac{13}{18}-\frac{4}{9}=\frac{13}{18}-\frac{4}{9}\cdot\frac{2}{2}$ Using 18 as the common denominator

$=\frac{13}{18}-\frac{8}{18}$

$=\frac{5}{18}$

72. $\frac{13}{15}-\frac{11}{45}=\frac{39}{45}-\frac{11}{45}=\frac{28}{45}$

73. $\frac{11}{30}-\frac{2}{9}=\frac{11}{30}\cdot\frac{3}{3}-\frac{2}{9}\cdot\frac{10}{10}$ Using 90 as the common denominator

$=\frac{33}{90}-\frac{20}{90}$

$=\frac{13}{90}$

74. $\frac{5}{14}-\frac{5}{21}=\frac{15}{42}-\frac{10}{42}=\frac{5}{42}$

75. $\frac{7}{6} \div \frac{3}{5} = \frac{7}{6} \cdot \frac{5}{3}$ Multiplying by the reciprocal of the divisor

$= \frac{35}{18}$

76. $\frac{7}{5} \div \frac{10}{3} = \frac{7}{5} \cdot \frac{3}{10} = \frac{21}{50}$

77. $12 \div \frac{4}{9} = \frac{12}{1} \cdot \frac{9}{4} = \frac{\cancel{4} \cdot 3 \cdot 9}{1 \cdot \cancel{4}} = 27$

78. $\frac{9}{4} \div 9 = \frac{9}{4} \cdot \frac{1}{9} = \frac{\cancel{9} \cdot 1}{4 \cdot \cancel{9}} = \frac{1}{4}$

79. Note that we have a number divided by itself. Thus, the result is 1. We can also do this exercise as follows:

$$\frac{7}{13} \div \frac{7}{13} = \frac{7}{13} \cdot \frac{13}{7} = \frac{7 \cdot 13}{7 \cdot 13} = 1$$

80. $\frac{1}{10} \div \frac{1}{5} = \frac{1}{10} \cdot \frac{5}{1} = \frac{5}{10} = \frac{1}{2}$

81. $\dfrac{\frac{2}{7}}{\frac{5}{3}} = \frac{2}{7} \div \frac{5}{3} = \frac{2}{7} \cdot \frac{3}{5} = \frac{2 \cdot 3}{7 \cdot 5} = \frac{6}{35}$

82. $\dfrac{\frac{3}{8}}{\frac{1}{5}} = \frac{3}{8} \cdot \frac{5}{1} = \frac{15}{8}$

83. $\dfrac{9}{\frac{1}{2}} = 9 \div \frac{1}{2} = \frac{9}{1} \cdot \frac{2}{1} = \frac{9 \cdot 2}{1 \cdot 1} = 18$

84. $\dfrac{\frac{3}{7}}{6} = \frac{3}{7} \cdot \frac{1}{6} = \frac{1}{14}$

85. *Writing Exercise*. If the fractions have the same denominator and the numerators and/or denominators are very large numbers, it would probably be easier to compute the sum of the fractions than their product.

86. *Writing Exercise*. If the fractions have different denominators, it would probably be easier to compute the product of the fractions than their sum.

87. *Writing Exercise*. Bryce is canceling incorrectly. The number 2 is not a common factor of both terms in the numerator, so it cannot be canceled. For example, let $x = 1$. Then $(2+1)/8 = 3/8$ but $(1+1)/4 = 2/4 = 1/2$. The expressions are not equivalent.

89.

Product	56	63	36	72	140	96	168
Factor	7	7	2	36	14	8	8
Factor	8	9	18	2	10	12	21
Sum	15	16	20	38	24	20	29

90. We need to find the smallest number that has both 6 and 8 as factors. Starting with 6 we list some numbers with a factor of 6, and starting with 8 we also list some numbers with a factor of 8. Then we find the first number that is on both lists.

6, 12, 18, 24, 30, 36, …
8, 16, 24, 32, 40, 48, …

Since 24 is the smallest number that is on both lists, the carton should be 24 in. long.

91. $\frac{16 \cdot 9 \cdot 4}{15 \cdot 8 \cdot 12} = \frac{\cancel{4} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{2} \cdot 2}{\cancel{3} \cdot 5 \cdot \cancel{2} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{4}} = \frac{2}{5}$

92. $\frac{9 \cdot 8xy}{2xy \cdot 36} = \frac{\cancel{9} \cdot \cancel{2} \cdot \cancel{4} \cdot \cancel{x} \cdot \cancel{y} \cdot 1}{\cancel{2} \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{4} \cdot \cancel{9}} = 1$

93. $\frac{45pqrs}{9prst} = \frac{5 \cdot \cancel{9} \cdot \cancel{p} \cdot q \cdot \cancel{r} \cdot \cancel{s}}{\cancel{9} \cdot \cancel{p} \cdot \cancel{r} \cdot \cancel{s} \cdot t} = \frac{5q}{t}$

94. $\frac{247}{323} = \frac{13 \cdot \cancel{19}}{17 \cdot \cancel{19}} = \frac{13}{17}$

95. $\frac{15 \cdot 4xy \cdot 9}{6 \cdot 25x \cdot 15y} = \frac{\cancel{15} \cdot \cancel{2} \cdot 2 \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{3} \cdot 3}{\cancel{2} \cdot \cancel{3} \cdot 25 \cdot \cancel{x} \cdot \cancel{15} \cdot \cancel{y}} = \frac{6}{25}$

96. $\frac{10x \cdot 12 \cdot 25y}{2z \cdot 30x \cdot 20y} = \frac{\cancel{10} \cdot \cancel{x} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{5} \cdot 5 \cdot \cancel{y}}{\cancel{2} \cdot z \cdot \cancel{3} \cdot \cancel{10} \cdot \cancel{x} \cdot \cancel{2} \cdot 2 \cdot \cancel{5} \cdot \cancel{y}} = \frac{5}{2z}$

97. $\dfrac{\frac{27ab}{15mn}}{\frac{18bc}{25np}} = \frac{27ab}{15mn} \div \frac{18bc}{25np} = \frac{27ab}{15mn} \cdot \frac{25np}{18bc}$

$= \frac{27ab \cdot 25np}{15mn \cdot 18bc} = \frac{\cancel{3} \cdot \cancel{9} \cdot a \cdot \cancel{b} \cdot \cancel{5} \cdot 5 \cdot \cancel{n} \cdot p}{\cancel{3} \cdot \cancel{5} \cdot m \cdot \cancel{n} \cdot 2 \cdot \cancel{9} \cdot \cancel{b} \cdot c}$

$= \frac{5ap}{2cm}$

98. $\dfrac{\frac{45xyz}{24ab}}{\frac{30xz}{32ac}} = \frac{45xyz}{24ab} \cdot \frac{32ac}{30xz}$

$= \frac{\cancel{3} \cdot \cancel{15} \cdot \cancel{x} \cdot y \cdot \cancel{z} \cdot \cancel{2} \cdot 2 \cdot \cancel{8} \cdot \cancel{a} \cdot c}{\cancel{3} \cdot \cancel{8} \cdot \cancel{a} \cdot b \cdot \cancel{2} \cdot \cancel{15} \cdot \cancel{x} \cdot \cancel{z}} = \frac{2cy}{b}$

99. $\dfrac{5\frac{3}{4}rs}{4\frac{1}{2}st} = \dfrac{\frac{23}{4}rs}{\frac{9}{2}st} = \dfrac{\frac{23rs}{4}}{\frac{9st}{2}} = \frac{23rs}{4} \div \frac{9st}{2}$

$= \frac{23rs}{4} \cdot \frac{2}{9st} = \frac{23rs \cdot 2}{4 \cdot 9st}$

$= \frac{23 \cdot r \cdot \cancel{s} \cdot \cancel{2}}{\cancel{2} \cdot 2 \cdot 9 \cdot \cancel{s} \cdot t} = \frac{23r}{18t}$

100. $\dfrac{3\frac{5}{7}mn}{2\frac{4}{5}np} = \dfrac{\frac{26mn}{7}}{\frac{14np}{5}} = \frac{26mn}{7} \cdot \frac{5}{14np}$

$= \frac{\cancel{2} \cdot 13 \cdot m \cdot \cancel{n} \cdot 5}{7 \cdot \cancel{2} \cdot 7 \cdot \cancel{n} \cdot p} = \frac{65m}{49p}$

101. $A = lw = \left(\frac{4}{5}\text{ m}\right)\left(\frac{7}{9}\text{ m}\right)$
$= \left(\frac{4}{5}\right)\left(\frac{7}{9}\right)(\text{m})(\text{m})$
$= \frac{28}{45}\text{m}^2$, or $\frac{28}{45}$ square meters

102. $A = \frac{1}{2}bh = \frac{1}{2}\left(\frac{10}{7}\text{ m}\right)\left(\frac{5}{4}\text{m}\right)$
$= \frac{1}{2}\left(\frac{10}{7}\right)\left(\frac{5}{4}\right)(\text{m})(\text{m})$
$= \frac{1\cdot 10\cdot 5}{2\cdot 7\cdot 4}\text{ m}^2$
$= \frac{1\cdot \cancel{2}\cdot 5\cdot 5}{\cancel{2}\cdot 7\cdot 4}\text{ m}^2$
$= \frac{25}{28}\text{ m}^2$, or $\frac{25}{28}$ square meters

103. $P = 4s = 4\left(3\frac{5}{9}\text{ m}\right) = 4\cdot\frac{32}{9}\text{ m} = \frac{128}{9}\text{ m}$,
or $14\frac{2}{9}\text{m}$

104. $P = 2l + 2w = 2\left(\frac{4}{5}\text{ m}\right) + 2\left(\frac{7}{9}\text{ m}\right)$
$= \frac{8}{5}\text{ m} + \frac{14}{9}\text{ m}$
$= \left(\frac{8}{5} + \frac{14}{9}\right)\text{m}$
$= \left(\frac{8}{5}\cdot\frac{9}{9} + \frac{14}{9}\cdot\frac{5}{5}\right)\text{m}$
$= \left(\frac{72}{45} + \frac{70}{45}\right)\text{m}$
$= \frac{142}{45}\text{ m, or } 3\frac{7}{45}\text{ m}$

105. There are 12 edges, each with length $2\frac{3}{10}$ cm. We multiply to find the total length of the edges.
$12\cdot 2\frac{3}{10}\text{ cm} = 12\cdot\frac{23}{10}\text{ cm}$
$= \frac{12\cdot 23}{10}\text{ cm}$
$= \frac{\cancel{2}\cdot 6\cdot 23}{\cancel{2}\cdot 5}\text{ cm}$
$= \frac{138}{5}\text{ cm, or } 27\frac{3}{5}\text{ cm}$

Exercise Set 1.4

1. Since $\frac{3}{20} = 0.15$, we can write $\frac{3}{20}$ as a *terminating* decimal.

2. If a number is an *integer*, it is either a whole number or the opposite of a whole number.

3. 0 is the only *whole number* that is not a natural number.

4. A number like $\sqrt{5}$, which cannot be written precisely in fraction notation or decimal notation, is an example of an *irrational number*.

5. The *opposite* of 1 is –1.

6. When two numbers are opposites, they have the same *absolute value*.

7. $-n$ is the opposite of n.

8. $|x|$

9. $-10 < x$

10. $6 \ge y$

11. The real number –9500 corresponds to borrowing \$9500. The real number 5000 corresponds to the award of \$5000.

12. –150; 65

13. The real number 100 corresponds to 100°F. The real number –80 corresponds to 80°F below zero.

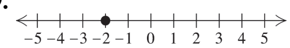
14. –1312; 29,035

15. The real number –2997 corresponds to a 2997-point fall. The real number 2112.98 corresponds to a 2112.98-point gain.

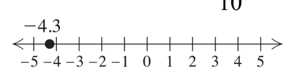
16. The real number 10,000 corresponds to a \$10,000 grant, and the real number –4500 corresponds to \$4500 spent.

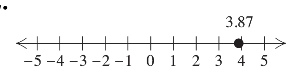
17. The real number 8 corresponds to an 8-yd gain, and the real number –5 corresponds to a 5-yd loss.

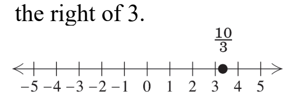
18. 8, –3

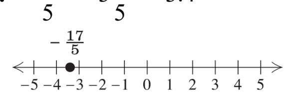
19.


20.

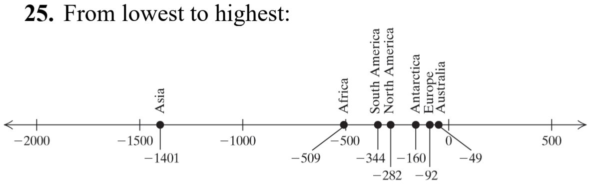

21. The graph of –4.3 is $\frac{3}{10}$ of a unit to the left of –4.


22.


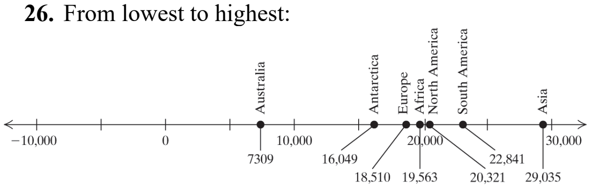
23. Since $\frac{10}{3} = 3\frac{1}{3}$, its graph is $\frac{1}{3}$ of a unit to the right of 3.


24. $-\frac{17}{5} = -3\frac{2}{5} = -3.4$


25. From lowest to highest:



26. From lowest to highest:



27. $\frac{7}{8}$ means $7 \div 8$, so we divide.

$$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ \underline{6\,4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

We have $\frac{7}{8} = 0.875.$

28.
$$\begin{array}{r} 0.125 \\ 8\overline{)1.000} \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$\frac{1}{8} = 0.125,$ so $-\frac{1}{8} = -0.125.$

29. We first find decimal notation for $\frac{3}{4}$. Since $\frac{3}{4}$ means $3 \div 4$, we divide.

$$\begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ \underline{2\,8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Thus, $\frac{3}{4} = 0.75,$ so $-\frac{3}{4} = -0.75.$

30.
$$\begin{array}{r} 1.833\ldots \\ 6\overline{)11.000} \\ \underline{6} \\ 50 \\ \underline{48} \\ 20 \\ \underline{18} \\ 20 \end{array}$$

$\frac{11}{6} = 1.8\overline{3}.$

31. $\frac{7}{6}$ means $7 \div 6$, so we divide.

$$\begin{array}{r} 1.166\ldots \\ 6\overline{)7.000} \\ \underline{6} \\ 10 \\ \underline{6} \\ 40 \\ \underline{36} \\ 40 \end{array}$$

Thus $\frac{7}{6} = 1.1\overline{6},$ so $-\frac{7}{6} = -1.1\overline{6}.$

32.
$$\begin{array}{r} 0.4166\ldots \\ 12\overline{)5.0000} \\ \underline{4\,8} \\ 20 \\ \underline{12} \\ 80 \\ \underline{72} \\ 80 \\ \underline{72} \\ 8 \end{array}$$

$\frac{5}{12} = 0.41\overline{6},$ so $-\frac{5}{12} = -0.41\overline{6}.$

33. $\frac{2}{3}$ means $2 \div 3$, so we divide.

$$\begin{array}{r} 0.666\ldots \\ 3\overline{)2.000} \\ \underline{1\,8} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

We have $\frac{2}{3} = 0.\overline{6}.$

34.
$$\begin{array}{r} 0.25 \\ 4\overline{)1.00} \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

$\frac{1}{4} = 0.25.$

35. We first find decimal notation for $\frac{1}{2}$. Since $\frac{1}{2}$ means $1 \div 2$, we divide.

$$\begin{array}{r} 0.5 \\ 2\overline{)1.0} \\ \underline{1\,0} \\ 0 \end{array}$$

Thus, $\frac{1}{2} = 0.5,$ so $-\frac{1}{2} = -0.5.$

36.
$$\begin{array}{r} 0.111\ldots \\ 9\overline{)1.000} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

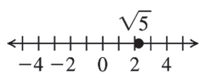
$\frac{1}{9} = 0.\overline{1}$, so $-\frac{1}{9} = -0.\overline{1}$.

37. Since the denominator is 100, we know that $\frac{13}{100} = 0.13$. We could also divide 13 by 100 to find this result.

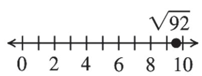
38.
$$\begin{array}{r} 0.45 \\ 20\overline{)9.00} \\ \underline{8\,0} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

$\frac{9}{20} = 0.45$, so $-\frac{9}{20} = -0.45$.

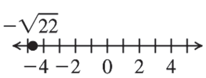
39.



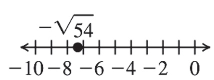
40.



41.



42.



43. Since 5 is to the right of 0, we have $5 > 0$.

44. $8 > -8$

45. Since –9 is to the left of 9, we have $-9 < 9$.

46. $0 > -7$

47. Since –8 is to the left of –5, we have $-8 < -5$.

48. $-4 < -3$

49. Since –5 is to the right of –11, we have $-5 > -11$.

50. $-3 > -4$

51. Since -12.5 is to the left of –10.2, we have $-12.5 < -10.2$.

52. $-10.3 > -14.5$

53. We convert to decimal notation.
$\frac{5}{12} = 0.41\overline{6}$ and $\frac{11}{25} = 0.44$. Thus, $\frac{5}{12} < \frac{11}{25}$.

54. $-\frac{14}{17} \approx -0.82353$ and $-\frac{27}{35} \approx -0.77143$.
Thus, $-\frac{14}{17} < -\frac{27}{35}$.

55. $-2 > x$ has the same meaning as $x < -2$.

56. $9 < a$

57. $10 \le y$ has the same meaning as $y \ge 10$.

58. $t \le -12$

59. $-83,\ -4.7,\ 0,\ \frac{5}{9},\ 2.\overline{16},\ 62$

60. 62

61. –83, 0, 62

62. $\pi, \sqrt{17}$

63. All are real numbers.

64. 0, 62

65. $|-58| = 58$ since –58 is 58 units from 0.

66. 47

67. $|-12.2| = 12.2$ since –12.2 is 12.2 units from 0.

68. 4.3

69. $|\sqrt{2}| = \sqrt{2}$ since $\sqrt{2}$ is $\sqrt{2}$ units from 0.

70. 456

71. $\left|-\frac{9}{7}\right| = \frac{9}{7}$ since $-\frac{9}{7}$ is $\frac{9}{7}$ units from 0.

72. $\sqrt{3}$

73. $|0| = 0$ since 0 is 0 units from itself.

74. $\frac{3}{4}$

75. $|x| = |-8| = 8$

76. $|a| = |-5| = 5$

77. *Writing Exercise*. Yes; every integer can be written as $n/1$, a quotient of the form a/b where $b \ne 0$.

78. *Writing Exercise*. No; for instance, –2 is an integer and –2 is not a natural number.

79. *Writing Exercise*. No; $|0| = 0$ which is neither positive nor negative.

80. *Writing Exercise*. There are infinitely many rational numbers between 0 and 1. Consider only rational numbers of the form $\frac{1}{n}$, where n is an integer greater than 1. There are infinitely many integers greater than 1, so there are infinitely many numbers $\frac{1}{n}$, all between 0 and 1. (These numbers are a subset of the rational numbers between 0 and 1.)

81. *Writing Exercise*. No; every positive number is nonnegative, but zero is nonnegative and zero is not positive.

82. List the numbers as they occur on the number line, from left to right: –17, –12, 5, 13

83. List the numbers as they occur on the number line, from left to right: –23, –17, 0, 4

84. $-\frac{2}{3}, \frac{1}{2}, -\frac{3}{4}, -\frac{5}{6}, \frac{3}{8}, \frac{1}{6}$ can be written in decimal notation as $-0.66\overline{6}, 0.5, -0.75, -0.83\overline{3}, 0.375,$ $0.16\overline{6},$ respectively. Listing from least to greatest (in fractional form), we have

$$-\frac{5}{6}, -\frac{3}{4}, -\frac{2}{3}, \frac{1}{6}, \frac{3}{8}, \frac{1}{2}.$$

85. Converting to decimal notation, we can write $\frac{4}{5}, \frac{4}{3}, \frac{4}{8}, \frac{4}{6}, \frac{4}{9}, \frac{4}{2}, -\frac{4}{3}$ as $0.8,\ 1.3\overline{3},\ 0.5,\ 0.6\overline{6},$ $0.4\overline{4},\ 2, -1.3\overline{3},$ respectively. List the numbers (in fractional form) as they occur on the number line, from left to right:

$$-\frac{4}{3}, \frac{4}{9}, \frac{4}{8}, \frac{4}{6}, \frac{4}{5}, \frac{4}{3}, \frac{4}{2}$$

86. $|-5| = 5$ and $|-2| = 2,$ so $|-5| > |-2|.$

87. $|4| = 4$ and $|-7| = 7,$ so $|4| < |-7|.$

88. $|-8| = 8$ and $|8| = 8,$ so $|-8| = |8|.$

89. $|23| = 23$ and $|-23| = 23,$ so $|23| = |-23|.$

90. $|x| = 19$

x represents a number whose distance from 0 is 19. Thus, $x = 19$ or $x = -19.$

91. x represents an integer whose distance from 0 is less than 3 units. Thus, $x = -2, -1, 0, 1, 2.$

92. $2 < |x| < 5$

x represents an integer whose distance from 0 is greater than 2 and also less than 5. Thus, $x = -4, -3, 3, 4.$

93. $0.1\overline{1} = \frac{0.3\overline{3}}{3} = \frac{\frac{1}{3}}{3} = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

94. $0.9\overline{9} = 3(0.3\overline{3}) = 3 \cdot \frac{1}{3} = \frac{3}{3}$

95. $5.5\overline{5} = 50(0.1\overline{1}) = 50 \cdot \frac{1}{9} = \frac{50}{9}$

(See Exercise 93.)

96. $7.7\overline{7} = 70(0.1\overline{1}) = 70 \cdot \frac{1}{9} = \frac{70}{9}$

(See Exercise 94.)

97. $a < 0$

98. Nonpositive numbers include zero. Thus, $x \le 0.$

99. $|x| \le 10$

100. Distance from zero can go in the positive or negative direction. Thus, $|t| \ge 20.$

101. *Writing Exercise*. The number entered by hand is an approximation of $\sqrt{2}$ while the value that is squared immediately after being calculated is actually regarded by the calculator as $\sqrt{2}.$

102. *Writing Exercise*. The statement $\sqrt{a^2} = |a|$ for any real number a is true. If a is nonnegative, then $\sqrt{a^2} = a.$ If a is negative, $\sqrt{a^2} = -a$ since $\sqrt{a^2}$ must be nonnegative. Thus, for a nonnegative number, the result is the number and, for a negative number, the result is the opposite of the number. This describes the absolute value of a number.

Mid-Chapter Review

1. $\frac{x-y}{3} = \frac{22-10}{3}$ Substituting

$= \frac{12}{3}$ Subtracting

$= 4$ Dividing

2. $14x + 7 = 7 \cdot 2x + 7 \cdot 1 = 7(2x+1)$

3. $x + y = 3 + 12$ Substituting

$= 15$

4. $\frac{2a}{5} = \frac{2 \cdot 10}{5} = 4$

5. $d - 10$

6. Let h represent the number of hours worked; $8h$

7. Let s represent the number of students originally enrolled; $s - 5 = 27$

8. No; $13t = 13 \cdot 8 = 104 \ne 94$

9. $10x + 7$

10. $(3a)b$

11. $4(2x + 8) = 8x + 32$

12. $3(2m + 5n + 10) = 6m + 15n + 30$

13. $18x + 15 = 3(6x) + 3(5) = 3(6x + 5)$

14. $9c + 12d + 3 = 3(3c + 4d + 1)$

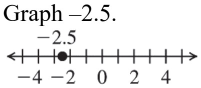
15. $84 = 2 \cdot 2 \cdot 3 \cdot 7$

16. $\dfrac{135}{315} = \dfrac{\cancel{3} \cdot \cancel{3} \cdot 3 \cdot \cancel{5}}{\cancel{3} \cdot \cancel{3} \cdot \cancel{5} \cdot 7} = \dfrac{3}{7}$

17. $\dfrac{11}{12} - \dfrac{3}{8} = \dfrac{22}{24} - \dfrac{9}{24} = \dfrac{13}{24}$

18. $\dfrac{8}{15} \div \dfrac{6}{11} = \dfrac{8}{15} \cdot \dfrac{11}{6} = \dfrac{\cancel{2} \cdot 2 \cdot 2 \cdot 11}{3 \cdot 5 \cdot \cancel{2} \cdot 3} = \dfrac{44}{45}$

19. Graph –2.5.



20.
$$\begin{array}{r} 0.15 \\ 20\overline{)3.00} \\ \underline{2\,0} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

$\dfrac{3}{20} = 0.15$, so $-\dfrac{3}{20} = -0.15$.

21. Since –16 is to the right of –24, we have $-16 > -24$.

22. $-\dfrac{3}{22} \approx -0.13636$ and $-\dfrac{2}{15} \approx -0.13333$.

Thus, $-\dfrac{3}{22} < -\dfrac{2}{15}$

23. $9 \le x$ has the same meaning as $x \ge 9$.

24. $|-5.6| = 5.6$

25. 0

Exercise Set 1.5

1. To add –3 + (–6), *add* 3 and 6 and make the answer *negative*.

2. To add –1 + 8, *subtract* 1 from 8 and make the answer *positive*.

3. To add –11 + 5, *subtract* 5 from 11 and make the answer *negative*.

4. The number 0 is called the additive *identity*.

5. The addition –7 + 0 = –7 illustrates the *identity* property of 0.

6. The expressions 5*x* and –9*x* are examples of *like* terms.

7. Choice (f), –3*n*, has the same variable factor as 8*n*.

8. Choice (d), –4*m*, has the same variable factor as 7*m*.

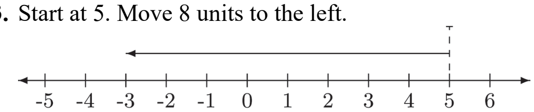
9. Choice (e), 9, is a constant as is 43.

10. Choice (a), –3*z*, has the same variable factor as 28*z*.

11. Choice (b), 5*x*, has the same variable factor as –2*x*.

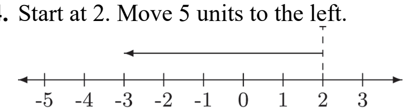
12. Choice (c), 2*t*, has the same variable factor as –9*t*.

13. Start at 5. Move 8 units to the left.



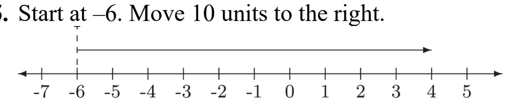
$5 + (-8) = -3$

14. Start at 2. Move 5 units to the left.



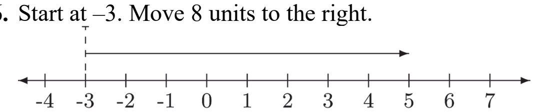
$2 + (-5) = -3$

15. Start at –6. Move 10 units to the right.



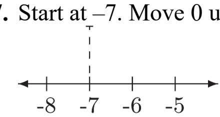
$-6 + 10 = 4$

16. Start at –3. Move 8 units to the right.



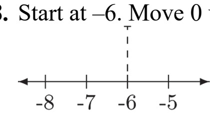
$-3 + 8 = 5$

17. Start at –7. Move 0 units.



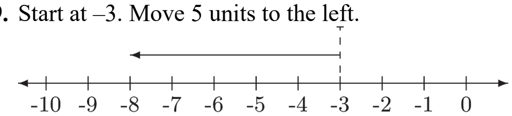
$-7 + 0 = -7$

18. Start at –6. Move 0 units.



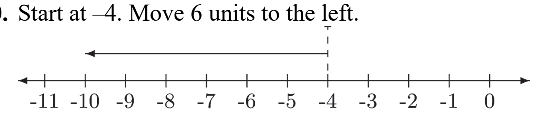
$-6 + 0 = -6$

19. Start at –3. Move 5 units to the left.



$-3 + (-5) = -8$

20. Start at –4. Move 6 units to the left.



$-4 + (-6) = -10$

21. $-6 + (-5)$ Two negatives. Add the absolute values, getting 11. Make the answer negative.
$-6 + (-5) = -11$

22. –20

23. 10 + (–15) The absolute values are 10 and 15. The difference is 15 – 10, or 5. The negative number has the larger absolute value, so the answer is negative.
$10 + (-15) = -5$

24. –10

25. $12+(-12)$ The numbers have the same absolute value. The sum is 0.
$12+(-12)=0$

26. 0

27. $-24+(-17)$ Two negatives. Add the absolute values, getting 41. Make the answer negative.
$-24+(-17)=-41$

28. –42

29. $-13+13$ The numbers have the same absolute value. The sum is 0.
$-13+13=0$

30. 0

31. $20+(-11)$ The absolute values are 20 and 11. The difference is 20 – 11, or 9. The positive number has the larger absolute value, so the answer is positive.
$20+(-11)=9$

32. 3

33. $-36+0$ One number is 0. The answer is the other number.
$-36+0=-36$

34. –74

35. $-3+14$ The absolute values are 3 and 14. The difference is 14 – 3, or 11. The positive number has the larger absolute value, so the answer is positive.
$-3+14=11$

36. 19

37. $-24+(-19)$ Two negatives. Add the absolute values, getting 43. Make the answer negative.
$-24+(-19)=-43$

38. 2

39. $19+(-19)$ The numbers have the same absolute value. The sum is 0.
$19+(-19)=0$

40. -26

41. $23+(-5)$ The absolute values are 23 and 5. The difference is 23 – 5 or 18. The positive number has the larger absolute value, so the answer is positive.
$23+(-5)=18$

42. -22

43. $69+(-85)$ The absolute values are 69 and 85. The difference is $85-69$, or 16. The negative number has the larger absolute value, so the answer is negative.
$69+(-85)=-16$

44. –50

45. $-3.6+2.8$ The absolute values are 3.6 and 2.8. The difference is 3.6 – 2.8, or 0.8. The negative number has the larger absolute value, so the answer is negative.
$-3.6+2.8=-0.8$

46. –1.8

47. $-5.4+(-3.7)$ Two negatives. Add the absolute values, getting 9.1. Make the answer negative.
$-5.4+(-3.7)=-9.1$

48. –13.2

49. $\frac{4}{5}+\left(-\frac{1}{5}\right)$ The absolute values are $\frac{4}{5}$ and $\frac{1}{5}$. The positive number has the larger absolute value, so the answer is positive. $\frac{4}{5}+\left(-\frac{1}{5}\right)=\frac{3}{5}$

50. $\frac{1}{7}$

51. $\frac{-4}{7}+\frac{-2}{7}$ Two negatives. Add the absolute values, getting $\frac{6}{7}$. Make the answer negative.
$\frac{-4}{7}+\frac{-2}{7}=\frac{-6}{7}$

52. $\frac{-7}{9}$

53. $-\frac{2}{5}+\frac{1}{3}$ The absolute values are $\frac{2}{5}$ and $\frac{1}{3}$. The difference is $\frac{6}{15}-\frac{5}{15}$, or $\frac{1}{15}$. The negative number has the larger absolute value, so the answer is negative. $-\frac{2}{5}+\frac{1}{3}=-\frac{1}{15}$

54. $-\frac{4}{13}+\frac{1}{2}=-\frac{8}{26}+\frac{13}{26}=\frac{5}{26}$

55. $\frac{-4}{9}+\frac{2}{3}$ The absolute values are $\frac{4}{9}$ and $\frac{2}{3}$. The difference is $\frac{6}{9}-\frac{4}{9}$, or $\frac{2}{9}$. The positive number has the larger absolute value, so the answer is positive.
$\frac{-4}{9}+\frac{2}{3}=\frac{2}{9}$

56. $\frac{1}{9}+\left(-\frac{1}{3}\right)=\frac{1}{9}+\left(-\frac{3}{9}\right)=-\frac{2}{9}$

57. $35+(-14)+(-19)+(-5)$
$=35+[(-14)+(-19)+(-5)]$ Using the associative law of addition
$=35+(-38)$ Adding the negatives
$=-3$ Adding a positive and a negative

58. $-28+(-44)+17+31+(-94)=48+(-166)$
$=-118$

59. $-4.9+8.5+4.9+(-8.5)$

Note that we have two pairs of numbers with different signs and the same absolute value: –4.9 and 4.9, 8.5, and –8.5. The sum of each pair is 0, so the result is $0+0$, or 0.

60. $24+3.1+(-44)+(-8.2)+63=90.1+(-52.2)$
$=37.9$

61.
$$\begin{array}{lccc}
\text{Rewording:} & \underbrace{\text{First decrease}} & \text{plus} & \underbrace{\text{second decrease}} \\
& \downarrow & \downarrow & \downarrow \\
\text{Translating:} & (-15\text{¢}) & + & (-3\text{¢}) \\
\text{plus} & \underbrace{\text{first increase}} & \text{is} & \underbrace{\text{change in price.}} \\
\downarrow & \downarrow & \downarrow & \downarrow \\
+ & 17\text{¢} & = & \text{change in price}
\end{array}$$

Since $-15+(-3)+17=-18+17$
$=-1,$

the price dropped 1¢ during the given period.

62. Since $(-2\text{¢})+25\text{¢}+(-43\text{¢})=23\text{¢}+(-43\text{¢})$
$=-20\text{¢},$

the price dropped 20¢ during the given period.

63.
$$\begin{array}{lcccc}
\text{Rewording:} & \underbrace{\text{October balance}} & \text{plus} & \underbrace{\text{payment}} & \text{plus} \\
& \downarrow & \downarrow & \downarrow & \downarrow \\
\text{Translating:} & -446 & + & 300 & + \\
& \underbrace{\text{November charges}} & \text{is} & \underbrace{\text{new balance.}} & \\
& \downarrow & \downarrow & \downarrow & \\
& (-127) & = & \text{new balance} &
\end{array}$$

Since $-446+300+(-127)=-146+(-127)$
$=-273,$

Chloe's new balance was \$273.

64. Since $-\$26,500+(-\$10,200)+\$32,400$
$=-\$36,700+\$32,400$
$=-\$4300,$

the loss was \$4300. The profit was \$–4300.

65.
$$\begin{array}{lcccc}
\text{Rewording:} & \underbrace{\text{First try yardage}} & \text{plus} & \underbrace{\text{second try yardage}} \\
& \downarrow & \downarrow & \downarrow \\
\text{Translating:} & (-13) & + & 12 \\
\text{plus} & \underbrace{\text{third try yardage}} & \text{is} & \underbrace{\text{total gain or loss.}} \\
\downarrow & \downarrow & \downarrow & \downarrow \\
+ & 21 & = & \text{total gain or loss}
\end{array}$$

Since $(-13)+12+21$
$=-1+21$
$=20,$

the total gain was 20 yd.

66. Since $450+(-530)+(75)+(-90)$
$=(450+75)+[-530+(-90)]$
$=525+(-620)$
$=-95,$

the balance is –\$95. That is, Omari's account is \$95 overdrawn.

67.
$$\begin{array}{lccccc}
\text{Rewording:} & \underbrace{\text{first drop}} & \text{plus} & \underbrace{\text{rise}} & \text{plus} \\
& \downarrow & \downarrow & \downarrow & \downarrow \\
\text{Translating:} & \left(-\frac{2}{5}\right) & + & \frac{9}{10} & + \\
& \underbrace{\text{drop}} & \text{is} & \underbrace{\text{total level change.}} & \\
& \downarrow & \downarrow & \downarrow & \\
& \left(-\frac{6}{5}\right) & = & \text{total level change} &
\end{array}$$

Since $-\frac{2}{5}+\frac{9}{10}+\left(-\frac{6}{5}\right)=-\frac{4}{10}+\frac{9}{10}+\left(-\frac{12}{10}\right)$
$=\frac{5}{10}-\frac{12}{10},$
$=-\frac{7}{10},$

The lake dropped by $\frac{7}{10}$ ft.

68. Since $-470+45+(-160)+500=-85$, Logan owes \$85 on his credit card.

69.
$$\begin{array}{lccccc}
\text{Rewording:} & \underbrace{\text{Base}} & \text{plus} & \underbrace{\text{height}} & \text{is} & \underbrace{\text{peak}} \\
& \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
\text{Translating:} & -19,864 & + & 33,480 & = & \text{peak}
\end{array}$$

Since $-19,684+33,480=13,796$, the elevation of the peak is 13,796 ft above sea level.

70. Since $-5+8+(-4)=3+(-4)=-1$, the class lost one student.

71. $7a+10a=(7+10)a$ Using the distributive law
$=17a$

72. $11x$

73. $-3x+12x=(-3+12)x$ Using the distributive law
$=9x$

74. $-9m$

75. $7m+(-9m)=[7+(-9)]m=-2m$

76. 0

77. $-8y+(-2y)=[-8+(-2)]y=-10y$

78. $-7n$

79. $-3+8x+4+(-10x)$
$=-3+4+8x+(-10x)$ Using the commutative law of addition
$=(-3+4)+[8+(-10)]x$ Using the distributive law
$=-2x+1$ Adding

80. $8a+5+(-a)+(-3)=8a+(-a)+5+(-3)$
$=7a+2$

81. $6m+9n+(-9n)+(-10m)$
$=9n+(-9n)+6m+(-10m)$ Using the commutative law of addition
$=[9+(-9)]n+[6+(-10)]m$ Using the distributive law
$=-4m$ Adding

82. $-11s+(-8t)+(-3s)+8t=-11s+(-3s)+(-8t)+8t$
$=-14s$

83. $-4x+6.3+(-x)+(-10.2)$
$=-4x+(-x)+6.3+(-10.2)$ Using the commutative law of addition
$=[-4+(-1)]x+6.3+(-10.2)$ Using the distributive law
$=-5x-3.9$ Adding

84. $-7+10.5y+13+(-11.5y)$
$=-7+13+10.5y+(-11.5y)$
$=6-y$

85. Perimeter $=8+5x+9+7x$
$=8+9+5x+7x$
$=(8+9)+(5+7)x$
$=17+12x$

86. $4a+5+6a+8=4a+6a+5+8$
$=10a+13$

87. Perimeter $=3t+3r+7+5t+9+4r$
$=3t+5t+3r+4r+7+9$
$=(3+5)t+(3+4)r+(7+9)$
$=8t+7r+16$

88. $7+5z+3x+8+4z+2x=15+9z+5x$

89. Perimeter $=9+6n+7+8n+4n$
$=9+7+6n+8n+4n$
$=(9+7)+(6+8+4)n$
$=16+18n$

90. $2+6+5n+7n+3+7n=19n+11$

91. *Writing Exercise*. Answers may vary. One possible explanation follows.

Consider performing the addition on a number line. We start to the left of 0 and then move farther left, so the result must be a negative number.

92. *Writing Exercise*. Each nonzero integer from –10 to 10 can be added to its opposite with the sum of each pair being 0. When this is added to the remaining integer, 0, the total is 0.

93. *Writing Exercise*. The sum will be positive when the positive number is greater than the sum of the absolute values of the negative numbers.

94. *Writing Exercise*. No; when we add real numbers with different signs, we must know how to subtract absolute values. That is, we must be able to calculate $a-b$ where a and b are both nonnegative with $a \ge b$.

95. Starting with the final value, we "undo" the deposit and original amount by adding their opposites. The result is the amount of the check.

Rewording: $\underbrace{\text{Final value}}$ plus $\underbrace{\text{opposite of deposit}}$ plus

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

Translating: $-\$42.37 + (-\$152) +$

$\underbrace{\text{opposite of original amount}}$ is $\underbrace{\text{check amount.}}$

$\downarrow$ $\downarrow$ $\downarrow$

$-\$257.33 =$ check amount.

Since $-42.37+(-152)+(-257.33)$
$=(-194.37)+(-257.33)$
$=-451.70,$

So the amount of the check was \$451.70.

96. Since $61+12+(-17.50)=55.50,$ the initial value was \$55.50.

97. $4x+\_\_+(-9x)+(-2y)$
$=4x+(-9x)+\_\_+(-2y)$
$=[4+(-9)]x+\_\_+(-2y)$
$=-5x+\_\_+(-2y)$

This expression is equivalent to $-5x-7y,$ so the missing term is the term which yields $-7y$ when added to $-2y$. Since $-5y+(-2y)=-7y,$ the missing term is $-5y$.

98. $-3a+9b+\_\_+5a=2a+9b+\_\_$

This expression is equivalent to $2a-6b,$ so the missing term is the term which yields $-6b$ when added to $9b$. Since $9b+(-15b)=-6b,$ the missing term is $-15b$.

99. $3m+2n+\_\_+(-2m)$
$=2n+\_\_+(-2m)+3m$
$=2n+\_\_+(-2+3)m$
$=2n+\_\_+m$

This expression is equivalent to $2n+(-6m),$ so the missing term is the term which yields $-6m$ when added to m. Since $-7m+m=-6m,$ the missing term is $-7m$.

100. $\_\_+9x+(-4y)+x=\_\_+10x+(-4y)$

This expression is equivalent to $10x-7y$, so the missing term is the term which yields $-7y$, when added to $-4y$. Since $-3y+(-4y)=-7y,$ the missing term is $-3y$.

101. Note that, in order for the sum to be 0, the two missing terms must be the opposites of the given terms. Thus, the missing terms are $-7t$ and -23.

102. $P = 2l + 2w = 7x + 10$
We know $2l = 2 \cdot 5 = 10$, so $2w$ is $7x$. Then the width is a number which yields $7x$ when added to itself. Since $3.5x + 3.5x = 7x$, the width is $3.5x$, or $\frac{7}{2}x$.

103. $-3 + (-3) + 2 + (-2) + 1 = -5$
Since the total is 5 under par after the five rounds and $-5 = -1 + (-1) + (-1) + (-1) + (-1)$, the golfer was 1 under par on average.

Exercise Set 1.6

1. The numbers 5 and –5 are *opposites* of each other.

2. The number –100 has a negative *sign*.

3. We subtract by adding the *opposite* of the number being subtracted.

4. The word *difference* usually translates to subtraction.

5. $-x$ is read "the opposite of x," so choice (d) is correct.

6. $12 - x$ is read "twelve minus x," so choice (g) is correct.

7. $12 - (-x)$ is read "twelve minus the opposite of x," so choice (f) is correct.

8. $x - 12$ is read "x minus twelve," so choice (h) is correct.

9. $x - (-12)$ is read "x minus negative twelve," so choice (a) is correct.

10. $-x - 12$ is read "the opposite of x minus twelve," so choice (c) is correct.

11. $-x - x$ is read "the opposite of x minus x," so choice (b) is correct.

12. $-x - (-12)$ is read "the opposite of x minus negative 12," so choice (e) is correct.

13. $6 - 10$ is read "six minus ten."

14. $5 - 13$ is read "five minus thirteen."

15. $2 - (-12)$ is read "two minus negative twelve."

16. $4 - (-1)$ is read "four minus negative one."

17. $-x - y$ is read "the opposite of x minus y."

18. $-a - b$ is read "the opposite of a minus b."

19. $-3 - (-n)$ is read "negative three minus the opposite of n."

20. $-7 - (-m)$ is read "negative seven minus the opposite of m."

21. The opposite of 51 is –51 because $51 + (-51) = 0$.

22. 17

23. The opposite of $-\frac{11}{3}$ is $\frac{11}{3}$ because $-\frac{11}{3} + \frac{11}{3} = 0$.

24. $-\frac{7}{2}$

25. The opposite of –3.14 is 3.14 because $-3.14 + 3.14 = 0$.

26. –48.2

27. If $x = -45$, then $-x = -(-45) = 45$. (The opposite of –45 is 45.)

28. –26

29. If $x = -\frac{14}{3}$, then $-x = -\left(-\frac{14}{3}\right) = \frac{14}{3}$.
(The opposite of $-\frac{14}{3}$ is $\frac{14}{3}$.)

30. $-\frac{1}{328}$

31. If $x = 0.101$, then $-x = -(0.101) = -0.101$.
(The opposite of 0.101 is –0.101.)

32. 0

33. If $x = 37$, then $-(-x) = -(-37) = 37$
(The opposite of the opposite of 37 is 37.)

34. 29

35. If $x = -\frac{2}{5}$, then $-(-x) = -\left[-\left(-\frac{2}{5}\right)\right] = -\frac{2}{5}$.
(The opposite of the opposite of $-\frac{2}{5}$ is $-\frac{2}{5}$.)

36. –9.1

37. When we change the sign of –1 we obtain 1.

38. 7

39. When we change the sign we obtain –15.

40. –10

41. $7 - 10 = 7 + (-10) = -3$

42. $4 - 13 = 4 + (-13) = -9$

43. $0 - 6 = 0 + (-6) = -6$

44. $0 - 8 = 0 + (-8) = -8$

45. $-4 - 3 = -4 + (-3) = -7$

46. $-5 - 6 = -5 + (-6) = -11$

47. $-9 - (-3) = -9 + 3 = -6$

48. $-9 - (-5) = -9 + 5 = -4$

49. Note that we are subtracting a number from itself. The result is 0. We could also do this exercise as follows:
$-8 - (-8) = -8 + 8 = 0$

50. $-10-(-10)=-10+10=0$
(See Exercise 49.)

51. $14-19=14+(-19)=-5$

52. $12-16=12+(-16)=-4$

53. $30-40=30+(-40)=-10$

54. $20-27=20+(-27)=-7$

55. $-9-(-9)=-9+9=0$
(See Exercise 49.)

56. $-40-(-40)=-40+40=0$
(See Exercise 49.)

57. $5-5=5+(-5)=0$
(See Exercise 49.)

58. $7-7=7+(-7)=0$
(See Exercise 49.)

59. $4-(-4)=4+4=8$

60. $6-(-6)=6+6=12$

61. $-7-4=-7+(-4)=-11$

62. $-6-8=-6+(-8)=-14$

63. $6-(-10)=6+10=16$

64. $3-(-12)=3+12=15$

65. $-4-15=-4+(-15)=-19$

66. $-14-2=-14+(-2)=-16$

67. $-6-(-5)=-6+5=-1$

68. $-4-(-1)=-4+1=-3$

69. $5-(-12)=5+12=17$

70. $5-(-6)=5+6=11$

71. $0-(-3)=0+3=3$

72. $0-(-5)=0+5=5$

73. $-5-(-2)=-5+2=-3$

74. $-3-(-1)=-3+1=-2$

75. $-7-14=-7+(-14)=-21$

76. $-9-16=-9+(-16)=-25$

77. $0-11=0+(-11)=-11$

78. $0-31=0+(-31)=-31$

79. $-8-0=-8+0=-8$

80. $-9-0=-9+0=-9$

81. $-52-8=-52+(-8)=-60$

82. $-63-11=-63+(-11)=-74$

83. $2-25=2+(-25)=-23$

84. $18-63=18+(-63)=-45$

85. $-4.2-3.1=-4.2+(-3.1)=-7.3$

86. $-10.1-2.6=-10.1+(-2.6)=-12.7$

87. $-1.3-(-2.4)=-1.3+2.4=1.1$

88. $-5.8-(-7.3)=-5.8+7.3=1.5$

89. $3.2-8.7=3.2+(-8.7)=-5.5$

90. $1.5-9.4=1.5+(-9.4)=-7.9$

91. $0.072-1=0.072+(-1)=-0.928$

92. $0.825-1=0.825+(-1)=-0.175$

93. $\frac{2}{11}-\frac{9}{11}=\frac{2}{11}+\left(-\frac{9}{11}\right)=-\frac{7}{11}$

94. $\frac{3}{7}-\frac{5}{7}=\frac{3}{7}+\left(-\frac{5}{7}\right)=-\frac{2}{7}$

95. $\frac{-1}{5}-\frac{3}{5}=\frac{-1}{5}+\left(\frac{-3}{5}\right)=\frac{-4}{5}$, or $-\frac{4}{5}$

96. $\frac{-2}{9}-\frac{5}{9}=\frac{-2}{9}+\left(\frac{-5}{9}\right)=\frac{-7}{9}$, or $-\frac{7}{9}$

97. $-\frac{4}{17}-\left(-\frac{9}{17}\right)=-\frac{4}{17}+\frac{9}{17}=\frac{5}{17}$

98. $-\frac{2}{13}-\left(-\frac{5}{13}\right)=-\frac{2}{13}+\frac{5}{13}=\frac{3}{13}$

99. $16-(-12)-1-(-2)+3=16+12+(-1)+2+3$
$=32$

100. $22-(-18)+7+(-42)-27$
$=22+18+7+(-42)+(-27)=-22$

101. $-31+(-28)-(-14)-17$
$=(-31)+(-28)+14+(-17)$
$=-62$

102. $-43-(-19)-(-21)+25=-43+19+21+25$
$=22$

103. $-34-28+(-33)-44$
$=(-34)+(-28)+(-33)+(-44)$
$=-139$

104. $39+(-88)-29-(-83)$
$=39+(-88)+(-29)+83=5$

105. $-93+(-84)-(-93)-(-84)$
Note that we are subtracting –93 from –93 and –84 from –84. Thus, the result will be 0. We could also do

this exercise as follows:
$-93+(-84)-(-93)-(-84)$
$=-93+(-84)+93+84$
$=0$

106. $84+(-99)+44-(-18)-43$
$=84+(-99)+44+18+(-43)=4$

107. $-3y-8x=-3y+(-8x)$, so the terms are $-3y$ and $-8x$.

108. $7a$, $-9b$

109. $9-5t-3st=9+(-5t)+(-3st)$, so the terms are 9, $-5t$, and $-3st$.

110. -4, $-3x$, $2xy$

111. $10x-13x$
$=10x+(-13x)$ Adding the opposite
$=(10+(-13))x$ Using the distributive law
$=-3x$

112. $-11a$

113. $7a-12a+4$
$=7a+(-12a)+4$ Adding the opposite
$=(7+(-12))a+4$ Using the distributive law
$=-5a+4$

114. $-22x+7$

115. $-8n-9+7n$
$=-8n+(-9)+7n$ Adding the opposite
$=-8n+7n+(-9)$ Using the commutative law of addition
$=-n-9$ Adding like terms

116. $n-7$

117. $5-3x-11=5+(-3x)+(-11)$
$=-3x+5+(-11)$
$=-3x-6$

118. $3a-5$

119. $2-6t-9-2t=2+(-6t)+(-9)+(-2t)$
$=2+(-9)+(-6t)+(-2t)$
$=-7-8t$

120. $-b-12$

121. $5y+(-3x)-9x+1-2y+8$
$=5y+(-3x)+(-9x)+1+(-2y)+8$
$=5y+(-2y)+(-3x)+(-9x)+1+8$
$=3y-12x+9$

122. $3x+6z+46$

123. $13x-(-2x)+45-(-21)-7x$
$=13x+2x+45+21+(-7x)$
$=13x+2x+(-7x)+45+21$
$=8x+66$

124. $6t+39$

125. $-8-32=-8+(-32)=-40$

126. $-7-19=-26$

127. $18-(-25)=18+25=43$

128. $-5-(-31)=26$

129. We subtract –5.2 from 3.8.
Translate: 3.8 – (–5.2)
Simplify: 3.8 – (–5.2) = 3.8 + 5.2 = 9

130. –2.1 – (–5.9); 3.8

131. We subtract –79 from 114.
Translate: 114 – (–79)
Simplify: 114 – (–79) = 114 + 79 = 193

132. 23 – (–17); 40

133. We subtract the lower elevation from the higher elevation:
$550-(-400)=550+400=950$
It falls 950 m.

134. 14,505 + (–282) = 14,787 ft

135. We subtract the lower temperature from the higher temperature:
$134-(-79.8)=134+79.8=213.8$
The temperature range is 213.8°F.

136. $49-(-54)=49+54=103$°F

137. We subtract the lower differential from the higher differential:
$7.3-(-1.6)=7.3+1.6=8.9$
The differential improved by 8.9 points.

138. –8380 – (–10,994) = 2614 m

139. *Writing Exercise*. Yes; rewrite subtraction as addition of the opposite.

140. *Writing Exercise*. $-a+b$ is the opposite of $a+(-b)$ since $a+(-b)+(-a+b)=a+(-a)+ \ (-b)+b=0$.

141. Answers will vary. The symbol "–" can represent the opposite, or a negative, or subtraction.

142. *Writing Exercise*. For two negative numbers a and b, $a-b$ is negative when $|a|>|b|$.

143. If the clock reads 8:00 A.M. on the day following the blackout when the actual time is 3:00 P.M., then the clock is 7 hr behind the actual time. This indicates that the power outage lasted 7 hr, so power was restored 7 hr after 4:00 P.M., or at 11:00 P.M. on August 14.

144. True. For example, for $m=5$ and $n=3, 5>3$ and $5-3>0$, or $2>0$. For $m=-4$ and $n=-9$, $-4>-9$ and $-4-(-9)>0$, or $5>0$.

145. False. For example, let $m=-3$ and $n=-5$. Then $-3>-5$, but $-3+(-5)=-8 \not> 0$.

146. False. For example, let $m = 2$ and $n = -2$. Then 2 and -2 are opposites, but $2 - (-2) = 4 \neq 0$.

147. True. For example, for $m = 4$ and $n = -4$, $4 = -(-4)$ and $4 + (-4) = 0$; for $m = -3$ and $n = 3$, $-3 = -3$ and $-3 + 3 = 0$.

148. *Writing Exercise*. After the second "double or nothing" wager, the debt was \$20, so the debt before that wager (that is, after the first "double or nothing" wager) was \$10. Then the debt before the first "double or nothing" wager (that is, after the original wager) was \$5. Thus, the gambler originally bet \$5, this debt was doubled to \$10, and that debt was doubled to \$20.

149. $\boxed{(-)}\ \boxed{9}\ \boxed{-}\ \boxed{(-)}\ \boxed{7}\ \boxed{\text{ENTER}}$

150. *Writing Exercise*. If *n* is positive and *m* is negative, then $-m$ is positive and $n + (-m)$ is positive.

Connecting the Concepts

1. $-8 + (-2) = -10$
2. $-8 \cdot (-2) = 16$
3. $-8 \div (-2) = \frac{-8}{-2} = 4$
4. $-8 - (-2) = -8 + 2 = -6$
5. $\frac{3}{5} - \frac{8}{5} = -\frac{5}{5} = -1$
6. $\frac{12}{5} + \left(\frac{-7}{5}\right) = \frac{5}{5} = 1$
7. $(1.3)(-2.9) = -3.77$
8. $-44.1 \div 6.3 = -7$
9. $-38 - (-38) = 0$
10. $-46 - 46 = -92$

Exercise Set 1.7

1. The product of two negative numbers is *positive*.
2. The product of an *odd* number of negative numbes is negative.
3. Division by zero is *undefined*.
4. To divide by a fraction, multiply by its *reciprocal*.
5. The *opposite* of a negative number is positive.
6. The *reciprocal* of a negative number is negative.
7. The product of two reciprocals is 1.
8. The sum of a pair of opposites is 0.
9. The sum of a pair of additive inverses is 0.
10. The product of two multiplicative inverses is 1.
11. The number 0 has no reciprocal.
12. The number 1 is its own reciprocal.
13. The number 1 is the multiplicative identity.
14. The number 0 is the additive identity.
15. A nonzero number divided by itself is 1.
16. Division by 0 is undefined.
17. $-4 \cdot 10 = -40$ Think: $4 \cdot 10 = 40$, make the answer negative.
18. -30
19. $-8 \cdot 7 = -56$ Think: $8 \cdot 7 = 56$, make the answer negative.
20. -18
21. $4 \cdot (-10) = -40$
22. -45
23. $-9 \cdot (-8) = 72$ Multiplying absolute values; the answer is positive.
24. 110
25. $-19 \cdot (-10) = 190$
26. 120
27. $11 \cdot (-12) = -132$
28. -645
29. $4.5 \cdot (-28) = -126$
30. 102.9
31. $-5 \cdot (-2.3) = 11.5$
32. $-6 \cdot 4.8 = -28.8$
33. $-(25) \cdot 0 = 0$ The product of 0 and any real number is 0.
34. 0
35. $\frac{2}{5} \cdot \left(-\frac{5}{7}\right) = -\left(\frac{2 \cdot 5}{5 \cdot 7}\right) = -\left(\frac{2}{7} \cdot \frac{5}{5}\right) = \frac{-2}{7}$
36. $-\frac{10}{21}$
37. $-\frac{3}{8} \cdot \left(-\frac{2}{9}\right) = \frac{\cancel{3} \cdot \cancel{2} \cdot 1}{4 \cdot \cancel{2} \cdot \cancel{3} \cdot 3} = \frac{1}{12}$
38. $\frac{1}{4}$
39. $(-5.3)(2.1) = -11.13$
40. -35.15

41. $-\frac{5}{9}\cdot\frac{3}{4}=-\frac{5\cdot\cancel{3}}{\cancel{3}\cdot 3\cdot 4}=-\frac{5}{12}$

42. -6

43. $3\cdot(-7)\cdot(-2)\cdot 6$
$=-21\cdot(-12)$ Multiplying the first two numbers and the last two numbers
$=252$

44. $9\cdot(-2)\cdot(-6)\cdot 7=-18\cdot(-42)=756$

45. 0, The product of 0 and any real number is 0.

46. 0 (See Exercise 45.)

47. $-\frac{1}{3}\cdot\frac{1}{4}\cdot\left(-\frac{3}{7}\right)=-\frac{1}{12}\cdot\left(-\frac{3}{7}\right)=\frac{3}{12\cdot 7}$
$=\frac{\cancel{3}\cdot 1}{\cancel{3}\cdot 4\cdot 7}=\frac{1}{28}$

48. $-\frac{1}{2}\cdot\frac{3}{5}\cdot\left(-\frac{2}{7}\right)=-\frac{3}{10}\cdot\left(-\frac{2}{7}\right)=\frac{3\cdot\cancel{2}}{\cancel{2}\cdot 5\cdot 7}=\frac{3}{35}$

49. $-2\cdot(-5)\cdot(-3)\cdot(-5)=10\cdot 15=150$

50. $-3\cdot(-5)\cdot(-2)\cdot(-1)=15\cdot 2=30$

51. 0, The product of 0 and any real number is 0.

52. 0 (See Exercise 51.)

53. $(-8)(-9)(-10)=72(-10)=-720$

54. $(-7)(-8)(-9)(-10)=56\cdot 90=5040$

55. $(-6)(-7)(-8)(-9)(-10)=42\cdot 72\cdot(-10)$
$=3024\cdot(-10)=-30{,}240$

56. $(-5)(-6)(-7)(-8)(-9)(-10)=30\cdot 56\cdot 90$
$=1680\cdot 90=151{,}200$

57. $18\div(-2)=-9$ Check: $-9\cdot(-2)=18$

58. -8

59. $\frac{36}{-9}=-4$ Check: $-4\cdot(-9)=36$

60. -2

61. $\frac{-56}{8}=-7$ Check: $-7\cdot 8=-56$

62. 5

63. $\frac{-48}{-12}=4$ Check: $4(-12)=-48$

64. 7

65. $-72\div 8=-9$ Check: $-9\cdot 8=-72$

66. -2

67. $-10.2\div(-2)=5.1$ Check: $5.1(-2)=-10.2$

68. -2.5

69. $-100\div(-11)=\frac{100}{11}$

70. $\frac{64}{7}$

71. $\frac{400}{-50}=-8$ Check: $-8\cdot(-50)=400$

72. $\frac{300}{13}$

73. Undefined

74. 0

75. $-4.8\div 1.2=-4$ Check: $-4(1.2)=-4.8$

76. -3

77. $\frac{0}{-9}=0$

78. 0

79. $\frac{9.7(-2.8)0}{4.3}$
Since the numerator has a factor of 0, the product in the numerator is 0. The denominator is nonzero, so the quotient is 0.

80. Undefined

81. $\frac{-8}{3}=\frac{8}{-3}$ and $\frac{-8}{3}=-\frac{8}{3}$

82. $-\frac{10}{3};\frac{10}{-3}$

83. $\frac{29}{-35}=\frac{-29}{35}$ and $\frac{29}{-35}=-\frac{29}{35}$

84. $-\frac{18}{7};\frac{-18}{7}$

85. $-\frac{7}{3}=\frac{-7}{3}$ and $-\frac{7}{3}=\frac{7}{-3}$

86. $\frac{-4}{15};\frac{4}{-15}$

87. $\frac{-x}{2}=\frac{x}{-2}$ and $\frac{-x}{2}=-\frac{x}{2}$

88. $\frac{-9}{a};-\frac{9}{a}$

89. The reciprocal of $\frac{4}{-5}$ is $\frac{-5}{4}\left(\text{or equivalently, }-\frac{5}{4}\right)$ because $\frac{4}{-5}\cdot\frac{-5}{4}=1$.

90. $\frac{-11}{13}$

91. The reciprocal of $\frac{51}{-10}$ is $-\frac{10}{51}$ because $\frac{51}{-10}\cdot\left(-\frac{10}{51}\right)=1.$

92. $-\frac{24}{43}$

93. The reciprocal of -10 is $\frac{1}{-10}\left(\text{or equivalently, } -\frac{1}{10}\right)$ because $-10\left(\frac{1}{-10}\right)=1.$

94. $\frac{1}{34}$

95. The reciprocal of 4.3 is $\frac{1}{4.3}$ because $4.3\left(\frac{1}{4.3}\right)=1.$ Since $\frac{1}{4.3}=\frac{1}{4.3}\cdot\frac{10}{10}=\frac{10}{43},$ the reciprocal can also be expressed as $\frac{10}{43}$.

96. $\frac{1}{-1.7}$, or $-\frac{1}{1.7}$; since $-\frac{1}{1.7}=-\frac{10}{17}$, the reciprocal can also be expressed as $-\frac{10}{17}$.

97. The reciprocal of $\frac{-1}{4}$ is $\frac{4}{-1}(\text{or equivalently}, -4)$ because $\frac{-1}{4}\cdot\frac{4}{-1}=1.$

98. $\frac{11}{-1}$, or -11

99. The reciprocal of 0 does not exist. (There is no number n for which $0\cdot n=1$.)

100. –1

101. $\left(\frac{-7}{4}\right)\left(-\frac{3}{5}\right)=\left(-\frac{7}{4}\right)\left(-\frac{3}{5}\right)$ Rewriting $\frac{-7}{4}$ as $-\frac{7}{4}$
$=\frac{21}{20}$

102. $\frac{5}{18}$

103. $\frac{-3}{8}+\frac{-5}{8}=\frac{-8}{8}=-1$

104. $\frac{3}{5}$

105. $\left(\frac{-9}{5}\right)\left(\frac{5}{-9}\right)$
Note that this is the product of reciprocals. Thus, the result is 1.

106. $\left(-\frac{2}{7}\right)\left(\frac{5}{-8}\right)=\left(-\frac{2}{7}\right)\left(-\frac{5}{8}\right)=\frac{\cancel{2}\cdot5}{7\cdot\cancel{2}\cdot4}=\frac{5}{28}$

107. $\left(-\frac{3}{11}\right)-\left(-\frac{6}{11}\right)=\frac{-3}{11}+\frac{6}{11}=\frac{3}{11}$

108. $\left(\frac{-4}{7}\right)-\left(-\frac{2}{7}\right)=\frac{-4}{7}+\frac{2}{7}=\frac{-2}{7}$

109. $\frac{7}{8}\div\left(-\frac{1}{2}\right)=\frac{7}{8}\cdot\left(-\frac{2}{1}\right)=-\frac{14}{8}=-\frac{7\cdot\cancel{2}}{\cancel{2}\cdot4\cdot1}=-\frac{7}{4}$

110. $\frac{3}{4}\div\left(-\frac{2}{3}\right)=\frac{3}{4}\cdot\left(-\frac{3}{2}\right)=-\frac{9}{8}$

111. $\frac{9}{5}\cdot\frac{-20}{3}=-\frac{9}{5}\cdot\frac{20}{3}=-\frac{\cancel{3}\cdot3\cdot4\cdot\cancel{5}}{\cancel{5}\cdot\cancel{3}}=-12$

112. $\frac{-5}{12}\cdot\frac{7}{15}=-\frac{5}{12}\cdot\frac{7}{15}=-\frac{\cancel{5}\cdot7}{3\cdot4\cdot3\cdot\cancel{5}}=-\frac{7}{36}$

113. $\left(-\frac{18}{7}\right)+\left(\frac{3}{-7}\right)=\frac{-18}{7}+\frac{-3}{7}=\frac{-21}{7}=-3$

114. $\left(\frac{12}{-5}\right)+\left(-\frac{3}{5}\right)=\frac{-12}{5}+\frac{-3}{5}=\frac{-15}{5}=-3$

115. $-\frac{5}{9}\div\left(-\frac{5}{9}\right)$
Note that we have a number divided by itself. Thus, the result is 1.

116. $\frac{-5}{12}\div\frac{15}{7}=-\frac{5}{12}\cdot\frac{7}{15}=-\frac{7}{36}$

117. $\frac{-3}{10}+\frac{2}{5}=\frac{-3}{10}+\frac{2}{5}\cdot\frac{2}{2}=\frac{-3}{10}+\frac{4}{10}=\frac{1}{10}$

118. $\frac{-5}{9}+\frac{2}{3}=\frac{-5}{9}+\frac{6}{9}=\frac{1}{9}$

119. $\frac{7}{10}\div\left(\frac{-3}{5}\right)=\frac{7}{10}\div\left(-\frac{3}{5}\right)=\frac{7}{10}\cdot\left(-\frac{5}{3}\right)=-\frac{35}{30}$
$=-\frac{7\cdot\cancel{5}}{2\cdot\cancel{5}\cdot3}=-\frac{7}{6}$

120. $\left(\frac{-3}{5}\right)\div\frac{6}{15}=-\frac{3}{5}\cdot\frac{15}{6}=-\frac{\cancel{3}\cdot3\cdot\cancel{5}}{\cancel{5}\cdot2\cdot\cancel{3}}=-\frac{3}{2}$

121. $\frac{14}{-9}\div\frac{0}{3}=\frac{14}{-9}\cdot\frac{3}{0}$ Undefined

122. 0

123. $\frac{-4}{15}+\frac{2}{-3}=\frac{-4}{15}+\frac{-2}{3}=\frac{-4}{15}+\frac{-2}{3}\cdot\frac{5}{5}=\frac{-4}{15}+\frac{-10}{15}$
$=\frac{-14}{15}$, or $-\frac{14}{15}$

124. $\frac{3}{-10}+\frac{-1}{5}=\frac{-3}{10}+\frac{-1}{5}=\frac{-3}{10}+\frac{-2}{10}$
$=\frac{-5}{10}=\frac{-1}{2}$, or $-\frac{1}{2}$

125. *Writing Exercise*. You get the original number. The reciprocal of the reciprocal of a number is the original number.

126. *Writing Exercise*. Think of $3\cdot(-5)$ as $-5+(-5)+(-5)$. Start at –5 and move to the left 5 units to -10; then move to the left another 5 units to –15. Thus, $3\cdot(-5)=-15$.

127. *Writing Exercise*. Yes; consider $n(n\neq 0)$ and its opposite –*n*. The reciprocals of these numbers are $\frac{1}{n}$ and $\frac{1}{-n}$. Now $\frac{1}{n}+\frac{1}{-n}=\frac{1}{n}+\frac{-1}{n}=0$, so the reciprocals are also opposites.

128. *Writing Exercise*. Yes; consider *n* and its reciprocal $\frac{1}{n}$. The opposites of these numbers are –*n* and $-\frac{1}{n}$. Now $(-n)\left(-\frac{1}{n}\right)=1$, so the opposite are also reciprocals.

129. Let *a* and *b* represent the numbers. The $a+b$ is the sum and $\frac{1}{a+b}$ is the reciprocal of the sum.

130. Let *a* and *b* represent the numbers. The the reciprocals of those numbers are $\frac{1}{a}$ and $\frac{1}{b}$ and the sum of the reciprocals is $\frac{1}{a}+\frac{1}{b}$.

131. Let *a* and *b* represent the numbers. Then $a+b$ is the sum and $-(a+b)$ is the opposite of the sum.

132. Let *a* and *b* represent the numbers. Then –*a* and –*b* are the opposites and $-a+(-b)$ is the sum of the opposites.

133. Let *x* represent a real number. $x=-x$

134. Let *x* represent a real number. $x=\frac{1}{x}$

135. For 2 and 3, the reciprocal of the sum is $\frac{1}{(2+3)}$ or $\frac{1}{5}$. But $\frac{1}{5}\neq\frac{1}{2}+\frac{1}{3}$.

136. –1, 1

137. The starting temperature is $-3°$F.

$$\text{Rewording: } \underbrace{\text{starting temp.}}\ \text{rise } 2°\ \text{for } \underbrace{3\text{hr}}\ \text{rise } 3°\ \text{for } \underbrace{6\text{hr}}$$

$$\text{Translating: } -3 \quad + \quad 2 \quad \cdot \quad 3 \quad + \quad 3 \quad \cdot \quad 6$$

$$\underbrace{\text{fall}}\ 2°\ \text{for } 3\text{hr fall } 5°\ \text{for } 2\text{hr}$$

$$- \quad 2 \quad \cdot \quad 3 \quad - \quad 5 \quad \cdot \quad 2$$

Since $-3+2\cdot3+3\cdot6-2\cdot3-5\cdot2$
$=-3+6+18-6-10=5.$

The temperature forecast at 8PM is 5°F.

138. When *n* is negative, –*n* is positive, so $\frac{m}{-n}$ is the quotient of a negative and a positive number and, thus, is negative.

139. –*n* and –*m* are both positive, so $\frac{-n}{-m}$ is positive.

140. When *n* is negative, $-n$ is positive, so $\frac{-n}{m}$ is the quotient of a positive and a negative number and, thus, is negative. When *m* is negative, –*m* is positive, so $-m\cdot\left(\frac{-n}{m}\right)$ is the product of a positive and a negative number and, thus, is negative.

141. –*m* is positive, so $\frac{n}{-m}$ is negative and $-\left(\frac{n}{-m}\right)$ is positive.

142. $m+n$ is the sum of two negative numbers, so it is negative; $\frac{m}{n}$ is the quotient of two negative numbers, so it is positive. Then $(m+n)\cdot\frac{m}{n}$ is the product of a negative and a positive number and, thus, is negative.

143. –*n* and –*m* are both positive, so $-n-m$, or $-n+(-m)$ is positive; $\frac{n}{m}$ is also positive, so $(-n-m)\frac{n}{m}$ is positive.

144. a) *m* and *n* have different signs;
b) either *m* or *n* is zero;
c) *m* and *n* have the same sign

145.

$a(-b)+ab=a[-b+b]$	Distributive law
$=a(0)$	Law of opposites
$=0$	Multiplicative property of 0

Therefore, $a(-b)$ is the opposite of *ab* by the law of opposites.

146. *Writing Exercise*. No; if $a>0$ and $b<0$, then $a>b$ but since $\frac{1}{a}>0$ and $\frac{1}{b}<0$, $\frac{1}{a}>\frac{1}{b}$.

Exercise Set 1.8

1. Since 6 is the exponent in x^6, the correct choice is (c).

2. Since 4 is the base in 4^3, the correct choice is (b).

3. Since 2 is the exponent that indicates a square, as in 10^2, the correct choice is (a).

4. Since 3 is the exponent that indicates a cube, as in 8^3, the correct choice is (f).

5. Since the terms $3y^4$ and $-y^4$ have the same base and exponent, they are like terms. The correct choice is (e).

6. The opposite of $x+3$ is $-(x+3)$, which simplifies to $-x-3$. The correct choice is (d).

7. $4+8\div 2\cdot 2$
There are no grouping symbols or exponential expressions, so we multiply and divide from left to right. This means that we divide first.

8. Subtraction

9. $5-2(3+4)$
We perform the operation in the parentheses first. This means that we add first.

10. Multiplication

11. $18-2[4+(3-2)]$
We perform the operation in the innermost grouping symbols first. This means that we perform the subtraction in the parentheses first.

12. Multiplication

13. $\underbrace{x\cdot x\cdot x\cdot x\cdot x\cdot x}_{\text{6 factors}}=x^6$

14. y^6

15. $\underbrace{(-5)(-5)(-5)}_{\text{3 factors}}=(-5)^3$

16. $(-7)^4$

17. $3t\cdot 3t\cdot 3t\cdot 3t\cdot 3t=(3t)^5$

18. $(5m)^5$

19. $2\cdot\underbrace{n\cdot n\cdot n\cdot n}_{\text{4 factors}}=2n^4$

20. $8a^3$

21. $4^2=4\cdot 4=16$

22. 125

23. $(-3)^2=(-3)(-3)=9$

24. 49

25. $-3^2=-(3\cdot 3)=-9$

26. –49

27. $4^3=4\cdot 4\cdot 4=16\cdot 4=64$

28. 9

29. $(-5)^4=(-5)(-5)(-5)(-5)=25\cdot 25=625$

30. 625

31. $7^1=7$ (1 factor)

32. –1

33. $(-2)^5=(-2)(-2)(-2)(-2)(-2)=-32$

34. –32

35. $(3t)^4=(3t)(3t)(3t)(3t)$
$=3\cdot 3\cdot 3\cdot 3\cdot t\cdot t\cdot t\cdot t=81t^4$

36. $25t^2$

37. $(-7x)^3=(-7x)(-7x)(-7x)$
$=(-7)(-7)(-7)(x)(x)(x)=-343x^3$

38. $625x^4$

39. $5+3\cdot 7=5+21$ Multiplying
$=26$ Adding

40. $3-4\cdot 2=3-8=-5$

41. $10\cdot 5+1\cdot 1=50+1$ Multiplying
$=51$ Adding

42. $19-5\cdot 4+3=19-20+3=-1+3=2$

43. $5-50\div 5\cdot 2=5-10\cdot 2$ Dividing
$=5-20$ Multiplying
$=-15$ Subtracting

44. $12\div 3+18\div 2=4+9=13$

45. $14\cdot 19\div(19\cdot 14)$
Since $14\cdot 19$ and $19\cdot 14$ are equivalent, we are dividing the product $14\cdot 19$ by itself. Thus the result is 1.

46. $18-6\div 3\cdot 2+7=18-2\cdot 2+7$
$=18-4+7$
$=14+7=21$

47. $3(-10)^2-8\div 2^2$
$=3(100)-8\div 4$ Simplifying the exponential expressions
$=300-8\div 4$ Multiplying and dividing
$=300-2$ from left to right
$=298$ Subtracting

48. $9-3^2\div 9(-1)=9-9\div 9(-1)=9-1(-1)$
$=9+1=10$

49. $8-(2\cdot 3-9)$
$=8-(6-9)$ Multiplying inside the parentheses
$=8-(-3)$ Subtracting inside the parentheses
$=8+3$ Removing parentheses
$=11$ Adding

50. $(8-2\cdot 3)-9=(8-6)-9=2-9=-7$

51. $(8-2)(3-9)$

$= 6(-6)$ Subtracting inside the parentheses

$= -36$ Multiplying

52. $32 \div (-2)^2 \cdot 4 = 32 \div 4 \cdot 4 = 8 \cdot 4 = 32$

53. $13(-10)^2 + 45 \div (-5)$

$= 13(100) + 45 \div (-5)$ Simplifying the exponential expression

$= 1300 + 45 \div (-5)$ Multiplying and dividing

$= 1300 - 9$ from left to right

$= 1291$ Subtracting

54. $2^4 + 2^3 - 10 \div (-1)^4 = 16 + 8 - 10 = 14$

55. $5 + 3(2-9)^2 = 5 + 3(-7)^2$

$= 5 + 3 \cdot 49 = 5 + 147 = 152$

56. $9 - (3-5)^3 - 4 = 9 - (-2)^3 - 4$

$= 9 - (-8) - 4 = 17 - 4 = 13$

57. $[2 \cdot (5-8)]^2 - 12 = [2 \cdot (-3)]^2 - 12$

$= (-6)^2 - 12$

$= 36 - 12$

$= 24$

58. $2^3 + 2^4 - 5[8 - 4(9-10)^2]$

$= 8 + 16 - 5[8 - 4(-1)^2]$

$= 24 - 5[8 - 4(1)]$

$= 24 - 5[8 - 4]$

$= 24 - 5[4]$

$= 24 - 20$

$= 4$

59. $\dfrac{7+2}{5^2 - 4^2} = \dfrac{9}{25-16} = \dfrac{9}{9} = 1$

60. $\dfrac{(5^2 - 3^2)^2}{2 \cdot 6 - 4} = \dfrac{(25-9)^2}{12-4} = \dfrac{16^2}{8} = \dfrac{256}{8} = 32$

61. $8(-7) + |3(-4)| = -56 + |-12| = -56 + 12 = -44$

62. $|10(-5)| + 1(-1) = |-50| - 1 = 50 - 1 = 49$

63. $36 \div (-2)^2 + 4[5 - 3(8-9)^5]$

$= 36 \div 4 + 4[5 - 3(-1)^5]$

$= 9 + 4[5 + 3]$

$= 9 + 32$

$= 41$

64. $-48 \div (7-9)^3 - 2[1 - 5(2-6) + 3^2]$

$= -48 \div (-8) - 2[1 + 20 + 9]$

$= 6 - 60$

$= -54$

65. $\dfrac{(7)^2 - (-1)^7}{5 \cdot 7 - 4 \cdot 3^2 - 2^2} = \dfrac{49 - (-1)}{35 - 4 \cdot 9 - 4}$

$= \dfrac{49+1}{35 - 36 - 4} = \dfrac{50}{-5} = -10$

66. $\dfrac{(-2)^3 + 4^2}{2 \cdot 3 - 5^2 + 3 \cdot 7} = \dfrac{-8+16}{6 - 25 + 21} = \dfrac{8}{2} = 4$

67. $\dfrac{-3^3 - 2 \cdot 3^2}{8 \div 2^2 - (6 - |2-15|)} = \dfrac{-27 - 2 \cdot 9}{8 \div 4 - (6 - |-13|)}$

$= \dfrac{-27-18}{2-(6-13)} = \dfrac{-45}{2-(-7)}$

$= \dfrac{-45}{9} = -5$

68. $\dfrac{(-5)^2 - 3 \cdot 5}{3^2 + 4 \cdot |6-7| \cdot (-1)^5} = \dfrac{25-15}{9 + 4 \cdot 1 \cdot (-1)} = \dfrac{10}{9-4} = 2$

69. $9 - 4x = 9 - 4 \cdot 7$ Substituting 7 for x

$= 9 - 28$ Multiplying

$= -19$

70. $1 + (-2)^3 = 1 - 8 = -7$

71. $24 \div t^3 = 24 \div (-2)^3$ Substituting -2 for t

$= 24 \div (-8)$ Simplifying the exponential expression

$= -3$ Dividing

72. $-100 \div (-5)^2 = -100 \div 25 = -4$

73. $45 \div a \cdot 5 = 45 \div (-3) \cdot 5$ Substituting -3 for a

$= -15 \cdot 5$ Multiplying and dividing in order from left to right

$= -75$

74. $50 \div 2 \cdot 5 = 25 \cdot 5 = 125$

75. $5x \div 15x^2$

$= 5 \cdot 3 \div 15(3)^2$ Substituting 3 for x

$= 5 \cdot 3 \div 15 \cdot 9$ Simplifying the exponential expression

$= 15 \div 15 \cdot 9$ Multiplying and dividing

$= 1 \cdot 9$ in order from

$= 9$ left to right

76. $6 \cdot 2 \div 12(2)^3 = 6 \cdot 2 \div 12 \cdot 8 = 12 \div 12 \cdot 8 = 1 \cdot 8 = 8$

77. $45 \div 3^2 x(x-1)$

$= 45 \div 3^2 \cdot 3(3-1)$ Substituting 3 for x

$= 45 \div 3^2 \cdot 3(2)$ Subtracting inside the parentheses

$= 45 \div 9 \cdot 3(2)$ Evaluating the exponential expression

$= 5 \cdot 3(2)$ Dividing and

$= 15(2)$ multiplying from

$= 30$ left to right

78. $-30 \div (-6)((-6)+4)^2 = -30 \div (-6)(-2)^2$

$= -30 \div (-6) \cdot 4 = 5 \cdot 4 = 20$

79. $-x^2 - 5x = -(-3)^2 - 5(-3) = -9 - 5(-3)$

$= -9 + 15 = 6$

80. $(-(-3))^2 - 5(-3) = (3)^2 - 5(-3)$
$= 9 - 5(-3) = 9 + 15 = 24$

81. $\dfrac{3a - 4a^2}{a^2 - 20} = \dfrac{3 \cdot 5 - 4(5)^2}{(5)^2 - 20} = \dfrac{3 \cdot 5 - 4 \cdot 25}{25 - 20}$
$= \dfrac{15 - 100}{5} = \dfrac{-85}{5} = -17$

82. $\dfrac{(-2)^3 - 4(-2)}{-2(-2-3)} = \dfrac{-8 - 4(-2)}{-2(-5)}$
$= \dfrac{-8+8}{10} = \dfrac{0}{10} = 0$

83. $3x - 2 + 5(2x + 7)$
$= 3x - 2 + 10x + 35$ Distributing the 5
$= 13x + 33$ Adding like terms

84. $8n + 3(5n + 2) + 1$
$= 8n + 15n + 6 + 1$
$= 23n + 7$

85. $2x^2 + 5(3x^2 - x) - 12x$
$= 2x^2 + 15x^2 - 5x - 12x$ Distributing the 5
$= 17x^2 - 17x$ Adding like terms

86. $6x^2 - x + 4(7x^2 + 3x)$
$= 6x^2 - x + 28x^2 + 12x$
$= 34x^2 + 11x$

87. $9t - 7r + 2(3r + 6t)$
$= 9t - 7r + 6r + 12t$ Distributing the 2
$= 21t - r$ Adding like terms

88. $4m - 9n + 3(2m - n)$
$= 4m - 9n + 6m - 3n$
$= 10m - 12n$

89. $5t^3 + t + 3(t - 2t^3)$
$= 5t^3 + t + 3t - 6t^3$ Distributing the 3
$= -t^3 + 4t$ Adding like terms

90. $8n^2 - 3n + 2(n - 4n^2)$
$= 8n^2 - 3n + 2n - 8n^2$
$= -n$

91. $-(9x + 1) = -9x - 1$ Removing parentheses and changing the sign of each term

92. $-3x - 5$

93. $-[7n + 8] = 7n - 8$ Removing grouping symbols and changing the sign of each term

94. $-6x + 7$

95. $-(4a - 3b + 7c) = -4a + 3b - 7c$

96. $-[5n - m - 2p] = -5n + m + 2p$

97. $-(3x^2 + 5x - 1) = -3x^2 - 5x + 1$

98. $-(-9x^3 + 8x + 10) = 9x^3 - 8x - 10$

99. $8x - (6x + 7)$
$= 8x - 6x - 7$ Removing parentheses and changing the sign of each term
$= 2x - 7$ Collecting like terms

100. $2a - (5a - 9) = 2a - 5a + 9 = -3a + 9$

101. $2x - 7x - (4x - 6) = 2x - 7x - 4x + 6 = -9x + 6$

102. $2a + 5a - (6a + 8) = 2a + 5a - 6a - 8 = a - 8$

103. $15x - y - 5(3x - 2y + 5z)$
$= 15x - y - 15x + 10y - 25z$ Multiplying each term in parentheses by -5
$= 9y - 25z$

104. $4a - b - 4(5a - 7b + 8c)$
$= 4a - b - 20a + 28b - 32c$
$= -16a + 27b - 32c$

105. $3x^2 + 11 - (2x^2 + 5) = 3x^2 + 11 - 2x^2 - 5$
$= x^2 + 6$

106. Note that we have the expression $5x^4 + 3x$ subtracted from itself. Thus, the result is 0.

107. $12a^2 - 3ab + 5b^2 - 5(-5a^2 + 4ab - 6b^2)$
$= 12a^2 - 3ab + 5b^2 + 25a^2 - 20ab + 30b^2$
$= 37a^2 - 23ab + 35b^2$

108. $-8a^2 + 5ab - 12b^2 - 6(2a^2 - 4ab - 10b^2)$
$= -8a^2 + 5ab - 12b^2 - 12a^2 + 24ab + 60b^2$
$= -20a^2 + 29ab + 48b^2$

109. $-7t^3 - t^2 - 3(5t^3 - 3t)$
$= -7t^3 - t^2 - 15t^3 + 9t$
$= -22t^3 - t^2 + 9t$

110. $9t^4 + 7t - 5(9t^3 - 2t) = 9t^4 + 7t - 45t^3 + 10t$
$= 9t^4 - 45t^3 + 17t$

111. $5(2x - 7) - [4(2x - 3) + 2]$
$= 5(2x - 7) - [8x - 12 + 2]$
$= 5(2x - 7) - [8x - 10]$
$= 10x - 35 - 8x + 10$
$= 2x - 25$

112. $3(6x - 5) - [3(1 - 8x) + 5]$
$= 3(6x - 5) - [3 - 24x + 5]$
$= 3(6x - 5) - [8 - 24x]$
$= 18x - 15 - 8 + 24x$
$= 42x - 23$

113. *Writing Exercise*. Operations should be performed in the following order.
1. Parentheses.
2. Exponents.
3. Multiply.
4. Divide.
5. Add.
6. Subtract.

Since Multiply occurs before Divide, and Add occurs before Subtract in the mnemonic, a student could conclude that must be the only order of operations.

114. *Writing Exercise*. Jake probably intends to perform the calculation $18 \div (2 \cdot 3) = 18 \div 6 = 3$. Instead, the calculator will use the rules for order of operations, dividing and multiplying from left to right: $18/2 \cdot 3 = 9 \cdot 3 = 27$.

115. *Writing Exercise*. Finding the opposite of a number and then squaring it is not equivalent to squaring the number and then finding the opposite of the result.

$$\begin{aligned}(-x)^2 &= (-1 \cdot x)^2 \\ &= (-1 \cdot x)(-1 \cdot x) \\ &= (-1)(-1)(x)(x) \\ &= x^2 \neq -x^2 \text{ for } x \neq 0.\end{aligned}$$

116. *Writing Exercise*. The opposite of the absolute value of a number is not equivalent to the opposite of the number. If $x < 0$ then $-|x| = -(-x) = x \neq -x$.

117.
$$\begin{aligned}&5t - \{7t - [4r - 3(t-7)] + 6r\} - 4r \\ &= 5t - \{7t - [4r - 3t + 21] + 6r\} - 4r \\ &= 5t - \{7t - 4r + 3t - 21 + 6r\} - 4r \\ &= 5t - \{10t + 2r - 21\} - 4r \\ &= 5t - 10t - 2r + 21 - 4r \\ &= -5t - 6r + 21\end{aligned}$$

118.
$$\begin{aligned}&z - \{2z - [3z - (4z - 5z) - 6z] - 7z\} - 8z \\ &= z - \{2z - [3z - (-z) - 6z] - 7z\} - 8z \\ &= z - \{2z - [3z + z - 6z] - 7z\} - 8z \\ &= z - \{2z - [-2z] - 7z\} - 8z \\ &= z - \{2z + 2z - 7z\} - 8z \\ &= z - \{-3z\} - 8z \\ &= z + 3z - 8z \\ &= -4z\end{aligned}$$

119.
$$\begin{aligned}&\{x - [f - (f - x)] + [x - f]\} - 3x \\ &= \{x - [f - f + x] + [x - f]\} - 3x \\ &= \{x - [x] + [x - f]\} - 3x \\ &= \{x - x + x - f\} - 3x \\ &= x - f - 3x \\ &= -2x - f\end{aligned}$$

120. *Writing Exercise*. Yes;
$$\begin{aligned}ab = 1 \cdot ab &= (-1)(-1)(ab) \\ &= (-1) \cdot a \cdot (-1) \cdot b \\ &= (-a)(-b).\end{aligned}$$

121. *Writing Exercise*. No; let $a = 2, b = -1,$ and $c = 3$. Then $a|b - c| = 2|-1-3| = 2|-4| = 2 \cdot 4 = 8$, but $ab - ac = 2(-1) - 2 \cdot 3 = -2 - 6 = -8$.

122. False; let $m = 1$ and $n = 2$. Then $-2 + 1 = -(2-1)$ $= -1$, but $-(2+1) = -3$.

123. True; $m - n = -n + m = -(n - m)$

124. False; let $m = 2$ and $n = 3$. Then $3(-3-2)$ $= 3(-5) = -15$, but $-3^2 + 3 \cdot 2 = -9 + 6 = -3$.

125. False; let $m = 2$ and $n = 1$. Then $-2(1-2)$ $= -2(-1) = 2$, but $-(2 \cdot 1 + 2^2) = -(2+4) = -6$.

126. True; $-n(-n - m) = n^2 + nm = n(n + m)$

127. $[x + 3(2 - 5x) \div 7 + x](x - 3)$
When $x = 3$, the factor $x - 3$ is 0, so the product is 0.

128. $[x + 2 \div 3x] \div [x + 2 \div 3x]$
Note that we have the expression $x + 2 \div 3x$ divided by itself, so the result is 1.

129. $\dfrac{x^2 + 2^x}{x^2 - 2^x} = \dfrac{3^2 + 2^3}{3^2 - 2^3} = \dfrac{9+8}{9-8} = \dfrac{17}{1} = 17$

130. $\dfrac{x^2 + 2^x}{x^2 - 2^x} = \dfrac{2^2 + 2^2}{2^2 - 2^2} = \dfrac{4+4}{4-4} = \dfrac{8}{0}$ Undefined

131.
$$\begin{aligned}&4 \cdot 20^3 + 17 \cdot 20^2 + 10 \cdot 20 + 0 \cdot 1 \\ &= 4 \cdot 8000 + 17 \cdot 400 + 10 \cdot 20 + 0 \cdot 1 \\ &= 32{,}000 + 6800 + 200 + 0 \\ &= 39{,}000\end{aligned}$$

132. These symbols represent 1, 5, and 0, respectively.

133. The tower is composed of cubes with sides of length x. The volume of each cube is $x \cdot x \cdot x$, or x^3. Now we count the number of cubes in the tower. The two lowest levels each contain 3×3, or 9 cubes. The next level contains one cube less than the two lowest levels, so it has $9 - 1$, or 8 cubes. The fourth level from the bottom contains one cube less than the level below it, so it has $8 - 1$, or 7 cubes. The fifth level from the bottom contains one cube less than the level below it, so it has $7 - 1$, or 6 cubes. Finally, the top level contains one cube less than the level below it, so it has $6 - 1$, or 5 cubes. All together there are $9 + 9 + 8 + 7 + 6 + 5$, or 44 cubes, each with volume x^3, so the volume of the tower is $44x^3$.

Chapter 1 Review

1. True

2. True

3. False

4. True

5. False

6. False

7. True

8. False

9. False

10. True

11. $8t = 8 \cdot 3 = 24$ Substitute 3 for t

12. $$\begin{aligned} 9 - y^2 &= 9 - (-5)^2 \quad \text{Substitute} -5 \text{ for } y \\ &= 9 - 25 \\ &= -16 \end{aligned}$$

13. $$\begin{aligned} &-10 + a^2 \div (b+1) \quad \text{Substitute 5 for } a \text{ and } -6 \text{ for } b \\ &= -10 + 5^2 \div (-6+1) \\ &= -10 + 25 \div (-5) \\ &= -10 + (-5) \\ &= -15 \end{aligned}$$

14. $y - 7$

15. $xz + 10$, or $10 + xz$

16. Let b represent Brandt's speed and w represent the wind speed; $15(b - w)$

17. Let m represent the number of miles that Ria's truck travels on one gallon of gasoline.

$$\begin{array}{lccccc} & m & \text{is} & \text{one-half} & \text{times} & 52. \\ \text{Translating:} & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ & m & = & \frac{1}{2} & \cdot & 52 \end{array}$$

or $m = \frac{1}{2} \cdot 52$

18. Let p represent the number of points earned and k represent the number kilometers driven.
$p = 3.2k$

19. $3t + 5 = t \cdot 3 + 5$

20. $(2x + y) + z = 2x + (y + z)$

21. Answers may vary.
$4(xy) = (4x)y$
$4(xy) = 4(yx)$
$4(xy) = 4(yx) = (4y)x$

22. $$\begin{aligned} 6(3x + 5y) &= 6 \cdot 3x + 6 \cdot 5y \\ &= 18x + 30y \end{aligned}$$

23. $$\begin{aligned} 8(5x + 3y + 2) &= 8 \cdot 5x + 8 \cdot 3y + 8 \cdot 2 \\ &= 40x + 24y + 16 \end{aligned}$$

24. $$\begin{aligned} 21x + 15y &= 3 \cdot 7x + 3 \cdot 5y \\ &= 3(7x + 5y) \end{aligned}$$

25. $$\begin{aligned} 22a + 99b + 11 &= 11 \cdot 2a + 11 \cdot 9b + 11 \cdot 1 \\ &= 11(2a + 9b + 1) \end{aligned}$$

26. $56 = 7 \cdot 8 = 7 \cdot 2 \cdot 2 \cdot 2$ or $2 \cdot 2 \cdot 2 \cdot 7$

27. $\frac{20}{48} = \frac{\cancel{2} \cdot \cancel{2} \cdot 5}{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 3} = \frac{5}{12}$

28. $\frac{18}{8} = \frac{\cancel{2} \cdot 3 \cdot 3}{\cancel{2} \cdot 2 \cdot 2} = \frac{9}{4}$

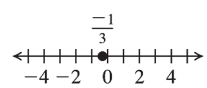
29. $\frac{5}{12} + \frac{3}{8} = \frac{10}{24} + \frac{9}{24} = \frac{10+9}{24} = \frac{19}{24}$

30. $$\begin{aligned} \frac{9}{16} \div 3 &= \frac{9}{16} \cdot \frac{1}{3} \quad \text{Multiply by the reciprocal of the divisor.} \\ &= \frac{\cancel{3} \cdot 3 \cdot 1}{16 \cdot \cancel{3}} \\ &= \frac{3}{16} \end{aligned}$$

31. $$\begin{aligned} \frac{2}{3} - \frac{1}{15} &= \frac{5}{5} \cdot \frac{2}{3} - \frac{1}{15} \quad \text{Use 15 as the common denominator.} \\ &= \frac{5 \cdot 2}{15} - \frac{1}{15} \\ &= \frac{10 - 1}{15} = \frac{9}{15} \\ &= \frac{\cancel{3} \cdot 3}{\cancel{3} \cdot 5} = \frac{3}{5} \end{aligned}$$

32. $\frac{9}{10} \cdot \frac{6}{5} = \frac{9 \cdot 6}{10 \cdot 5} = \frac{9 \cdot \cancel{2} \cdot 3}{\cancel{2} \cdot 5 \cdot 5} = \frac{27}{25}$

33. The real number –9800 corresponds to borrowing \$9800. The real number 1350 corresponds to the savings account value of \$1350.

34.


35. $-3 < x$ has the same meaning as $x > -3$.

36. $-10 < 0$

37. $-\frac{4}{9} = -\left(\frac{4}{9}\right) = -(4 \div 9)$, so we divide.

$$\begin{array}{r} 0.444 \\ 9\overline{)\,4000} \\ \underline{36} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

$\frac{4}{9} = 0.\overline{4}$, so $-\frac{4}{9} = -0.\overline{4}$

38. $|-1| = 1$, since –1 is 1 unit from 0.

39. $$\begin{aligned} -(-x) &= -(-(-12)) \quad \text{Substitute } -12 \text{ for } x \\ &= -(12) = -12 \end{aligned}$$

40. $-3 + (-7) = -10$

41. $-\frac{2}{3}+\frac{1}{12}=\frac{-8}{12}+\frac{1}{12}$ The absolute values are $\frac{8}{12}$ and $\frac{1}{12}$. The difference is $\frac{8}{12}-\frac{1}{12}$, or $\frac{8-1}{12}=\frac{7}{12}$. The negative number has the greater absolute value, so the answer is negative. $-\frac{2}{3}+\frac{1}{12}=-\frac{7}{12}$

42. $-3.8+5.1+(-12)+(-4.3)+10$
$=(5.1+10)+[(-3.8)+(-12)+(-4.3)]$
$=15.1+[-20.1]$ Adding positive numbers and adding negative numbers
$=-5$ Adding a positive and a negative number.

43. $-2-(-10)=-2+10=8$

44. $\frac{-9}{10}-\frac{1}{2}=\frac{-9}{10}+\left(\frac{-1}{2}\right)$
$=\frac{-9}{10}+\left(\frac{-5}{10}\right)=\frac{-14}{10}=\frac{-7}{5}$

45. $-2.7(3.4)=-(2.7\cdot 3.4)=-9.18$

46. $\frac{2}{3}\cdot\left(-\frac{3}{7}\right)=-\left(\frac{2}{3}\cdot\frac{3}{7}\right)=-\left(\frac{2\cdot 3}{3\cdot 7}\right)=-\frac{2}{7}$

47. $2\cdot(-7)\cdot(-2)\cdot(-5)=-14\cdot 10=-140$

48. $35\div(-5)=-7$

49. $-5.1\div 1.7=-3$

50. $-\frac{3}{5}\div\left(-\frac{4}{15}\right)=\frac{3}{5}\cdot\frac{15}{4}=\frac{3\cdot 3\cdot \cancel{5}}{\cancel{5}\cdot 4}=\frac{9}{4}$

51. $120-6^2\div 4\cdot 8=120-36\div 4\cdot 8$
$=120-9\cdot 8$
$=120-72$
$=48$

52. $(120-6^2)\div 4\cdot 8=(120-36)\div 4\cdot 8$
$=84\div 4\cdot 8$
$=21\cdot 8$
$=168$

53. $(120-6^2)\div(4\cdot 8)=(120-36)\div(4\cdot 8)$
$=84\div 32$
$=\frac{84}{32}$
$=\frac{\cancel{4}\cdot 21}{\cancel{4}\cdot 8}$
$=\frac{21}{8}$

54. $16\div(-2)^3-5[3-1+2(4-7)]$
$=16\div(-8)-5[3-1+2(4-7)]$
$=-2-5[3-1+2(-3)]$
$=-2-5[-4]$
$=-2+20$
$=18$

55. $|-3\cdot 5-4\cdot 8|-3(-2)=|-15-32|+6$
$=|-47|+6=53$

56. $\frac{4(18-8)+7\cdot 9}{9^2-8^2}=\frac{4(18-8)+7\cdot 9}{81-64}$
$=\frac{4(10)+7\cdot 9}{81-64}$
$=\frac{40+63}{81-64}$
$=\frac{103}{17}$

57. $11a+2b+(-4a)+(-3b)$
$=11a+(-4a)+2b+(-3b)$
$=[11+(-4)]a=[2+(-3)]b$
$=7a-b$

58. $7x-3y-11x+8y$
$=7x+(-11x)+(-3y)+8y$
$=[7+(-11)]x+(-3+8)y$
$=-4x+5y$

59. The opposite of -7 is 7, since $7+(-7)=0$.

60. The reciprocal of -7 is $-\frac{1}{7}$, since $-\frac{1}{7}\cdot -7=1$.

61. $\underbrace{2x\cdot 2x\cdot 2x\cdot 2x}_{\text{4 Factors}}=(2x)^4$

62. $(-5x)^3=-5x\cdot(-5x)\cdot(-5x)$
$=-5\cdot(-5)\cdot(-5)\cdot x\cdot x\cdot x$
$=-125x^3$

63. $2a-(5a-9)=2a+(-5a)+9$
$=[2+(-5)]a+9=-3a+9$

64. $11x^4+2x+8(x-x^4)=11x^4+2x+8\cdot x-8x^4$
$=11x^4-8x^4+2x+8x$
$=3x^4+10x$

65. $2n^2-5(-3n^2+m^2-4mn)+6m^2$
$=2n^2+15n^2-5m^2+20mn+6m^2$
$=(2+15)n^2+(-5+6)m^2+20mn$
$=17n^2+m^2+20mn$

66. $8(x+4)-6-[3(x-2)+4]$
$=8x+32-6-[3x-6+4]$
$=8x+32-6-[3x-2]$
$=8x+26-3x+2$
$=(8-3)x+26+2$
$=5x+28$

67. *Writing Exercise*. The value of a constant never varies. A variable can represent a variety of numbers.

68. *Writing Exercise*. A term is one of the parts of an expression that is separated from the other parts by plus signs. A factor is part of a product.

69. *Writing Exercise*. The distributive law is used in factoring algebraic expressions, multiplying algebraic expressions, combining like terms, finding the opposite of a sum, and subtracting algebraic expressions.

70. *Writing Exercise*. A negative number raised to an even exponent is positive; a negative number raised to an odd exponent is negative.

71. Substitute 1 for *a*, 2 for *b*, and evaluate:

$$\begin{aligned} a^{50}-20a^{25}b^4+100b^8 &= 1^{50}-20\cdot 1^{25}2^4+100\cdot 2^8 \\ &= 1-20\cdot 1\cdot 16+100\cdot 256 \\ &= 1-320+25{,}600 \\ &= 25{,}281 \end{aligned}$$

72. a. Since $0.090909\ldots+0.181818\ldots=0.272727\ldots$

we have $\frac{1}{11}+\frac{2}{11}=\frac{3}{11}$; $0.272727\ldots=\frac{3}{11}$

b. Since $10\cdot 0.090909\ldots=0.909090\ldots$,

we have $10\cdot\frac{1}{11}=\frac{10}{11}$; $0.909090\ldots=\frac{10}{11}$

73. $-\left|\frac{7}{8}-\left(-\frac{1}{2}\right)-\frac{3}{4}\right|$ Use 8: the common denominator

$$\begin{aligned} &= -\left|\frac{7}{8}-\left(-\frac{4}{8}\right)-\frac{6}{8}\right| \\ &= -\left|\frac{7}{8}+\frac{4}{8}+\frac{-6}{8}\right| \\ &= -\left|\frac{5}{8}\right| = -\frac{5}{8} \end{aligned}$$

74.

$$\begin{aligned} &(|2.7-3|+3^2-|-3|)\div(-3) \\ &= (|2.7-3|+9-|-3|)\div(-3) \\ &= (|-0.3|+9-|-3|)\div(-3) \\ &= (0.3+9-3)\div(-3) \\ &= 6.3\div(-3) \\ &= -2.1 \end{aligned}$$

75. i

76. j

77. a

78. h

79. k

80. b

81. c

82. e

83. d

84. f

85. g

Chapter 1 Test

1. Substitute 10 for *x* and 5 for *y*.

$$\frac{2x}{y}=\frac{2\cdot 10}{5}=\frac{2\cdot 2\cdot \cancel{5}}{\cancel{5}}=4$$

2. Let *x* and *y* represent the numbers; $xy-9$

3.

$$\begin{aligned} A &= \frac{1}{2}\cdot b\cdot h \\ &= \frac{1}{2}\cdot(16\text{ ft})\cdot(30\text{ ft}) \\ &= \frac{1}{2}\cdot 16\cdot 30\cdot \text{ft}\cdot\text{ft} \\ &= 8\cdot 30\text{ ft}\cdot\text{ft} \\ &= 240\text{ ft}^2,\text{ or 240 square feet} \end{aligned}$$

4. $3p+q=q+3p$

5. $x\cdot(4\cdot y)=(x\cdot 4)\cdot y$

6. Let *t* represent the number of golden lion tamarins living in zoos; $2500=5t$

7. $7(5+x)=7\cdot 5+7\cdot x=35+7x$

8. $-5(y-2)=-5\cdot y-5(-2)=-5y+10$

9. $11+44x=11\cdot 1+11\cdot 4x=11(1+4x)$

10.

$$\begin{aligned} 7x+7+49y &= 7\cdot x+7\cdot 1+7\cdot 2y \\ &= 7(x+1+7y) \end{aligned}$$

11.

$$\begin{aligned} 300 &= 2\cdot 150=2\cdot 2\cdot 75 \\ &= 2\cdot 2\cdot 3\cdot 25=2\cdot 2\cdot 3\cdot 5\cdot 5 \end{aligned}$$

12. $\frac{24}{56}=\frac{3\cdot\cancel{8}}{7\cdot\cancel{8}}=\frac{3}{7}$

13. $-4<0$, since -4 is left of 0.

14. $-3>-8$, since -3 is right of -8.

15. $\left|\frac{9}{4}\right|=\frac{9}{4}$, since $\frac{9}{4}$ is $\frac{9}{4}$ units from 0.

16. $|-3.8|=3.8$, since -3.8 is 3.8 units from 0.

17. $\frac{2}{3}$, since $\frac{2}{3}+\frac{-2}{3}=0$

18. $\frac{-7}{4}$, since $\frac{-7}{4}\cdot\frac{-4}{7}=1$

19. Let $x=-10$, then $-x=-(-10)=10$.

20. $-5\ge x$ has the same meaning as $x\le -5$.

21. $3.1-(-4.7)=3.1+4.7=7.8$

22.

$$\begin{aligned} -8+4+(-7)+3 &= [-8+(-7)]+[4+3] \\ &= -15+7=-8 \end{aligned}$$

23. $-\frac{1}{8}-\frac{3}{4}=-\frac{1}{8}-\frac{2}{2}\cdot\frac{3}{4}$
$=-\frac{1}{8}-\frac{6}{8}=\frac{-1-6}{8}=-\frac{7}{8}$

24. $4\cdot(-12)=-(4\cdot 12)=-48$

25. $-\frac{1}{2}\cdot\left(-\frac{4}{9}\right)=\frac{1}{2}\cdot\frac{4}{9}=\frac{1\cdot\cancel{2}\cdot 2}{\cancel{2}\cdot 9}=\frac{2}{9}$

26. $-\frac{3}{5}\div\left(-\frac{4}{5}\right)=\frac{3}{5}\cdot\frac{5}{4}=\frac{3\cdot\cancel{5}}{\cancel{5}\cdot 4}=\frac{3}{4}$

27. $4.864\div(-0.5)=-9.728$

28. $10-2(-16)\div 4^2+|2-10|$
$=10-2(-16)\div 16+|2-10|$
$=10+32\div 16+|-8|$
$=10+2+8$
$=20$

29. $9+7-4-(-3)=9+7+(-4)+3$
$=9+7+3+(-4)$
$=19+(-4)=15$

30. $256\div(-16)\cdot 4=-16\cdot 4=-64$

31. $2^3-10[4-3(-2+18)]=8-10[4-3(-2+18)]$
$=8-10[4-16\cdot 3]$
$=8-10[4-48]$
$=8-10[4+(-48)]$
$=8-10[-44]$
$=8-(-440)$
$=8+440=448$

32. $18y+30a-9a+4y=30a+(-9a)+18y+4y$
$=[30+(-9)]a+(18+4)y$
$=21a+22y$

33. $(-2x)^4=-2x\cdot(-2x)\cdot(-2x)\cdot(-2x)$
$=-2\cdot(-2)\cdot(-2)\cdot(-2)\cdot x\cdot x\cdot x\cdot x=16x^4$

34. $4x-(3x-7)=4x-3x+7$
$=(4-3)x+7=x+7$

35. $4(2a-3b)+a-7=8a-12b+a-7$
$=(8a+a)-12b-7$
$=(8+1)a-12b-7$
$=9a-12b-7$

36. $3[5(y-3)+9]-2(8y-1)$
$=3[5y-15+9]-16y+2$
$=3[5y-6]-16y+2$
$=15y-18-16y+2$
$=-y-16$

37. y is 4 less than half of x

Rewording: $\underbrace{\text{half of } x}\quad\underbrace{\text{less } 4}\quad \text{is}\quad y$

$\downarrow\quad\downarrow\quad\downarrow\quad\downarrow$

Translating: $\frac{1}{2}x\quad -4\quad =\quad y$

Since $x=20$, $\frac{1}{2}(20)-4=y$
$10-4=y$
$y=6$

Substitute 20 for x and 6 for y.
$\frac{5y-x}{2}=\frac{5\cdot 6-20}{2}$
$=\frac{30-20}{2}=\frac{10}{2}=5$

38. $9-(3-4)+5=15$

39. $n\ge 0$

40. $a-\{3a-[4a-(2a-4a)]\}$
$=a-\{3a-[4a-(-2a)]\}$
$=a-\{3a-[4a+2a]\}$
$=a-\{3a-6a\}$
$=a-\{-3a\}$
$=a+3a=4a$

41. Let $a=2, b=3, c=4$.
$a|b-c|=|ab|-|ac|$
$2|3-4|\stackrel{?}{=}|2\cdot 3|-|2\cdot 4|$
$2|-1|\stackrel{?}{=}|6|-|8|$
$2\cdot 1\stackrel{?}{=}6-8$
$2\ne -2$
False.

Chapter 2

Equations, Inequalities, and Problem Solving

Exercise Set 2.1

1. The equations $x+3=7$ and $6x=24$ are *equivalent equations*. Choice (c) is correct.

2. The expressions $3(x-2)$ and $3x-6$ are *equivalent expressions*. Choice (b) is correct.

3. A *solution* is a replacement that makes an equation true. Choice (f) is correct.

4. The 9 in $9ab$ is a *coefficient*. Choice (a) is correct.

5. *The multiplication principle* is used to solve $\frac{2}{3}\cdot x=-4.$ Choice (d) is correct.

6. *The addition principle* is used to solve $\frac{2}{3}+x=-4.$

   Choice (e) is correct.

7. Substitute 4 for x.

$$\begin{array}{l|l} \multicolumn{2}{c}{6-x=-2} \\ \hline 6-4 & -2 \end{array}$$
$$2\stackrel{?}{=}-2 \quad \text{FALSE}$$

Since the left-hand and right-hand sides differ, 4 is not a solution.

8.
$$\begin{array}{l|l} \multicolumn{2}{c}{6-x=-2} \\ \hline 6-8 & -2 \end{array}$$
$$-2\stackrel{?}{=}-2 \quad \text{TRUE}$$

9. Substitute 18 for t.

$$\begin{array}{l|l} \multicolumn{2}{c}{\frac{2}{3}t=12} \\ \hline \frac{2}{3}(18) & 12 \end{array}$$
$$12\stackrel{?}{=}12 \quad \text{TRUE}$$

Since the left-hand and right-hand sides are the same, 18 is a solution.

10.
$$\begin{array}{l|l} \multicolumn{2}{c}{\frac{2}{3}t=12} \\ \hline \frac{2}{3}(8) & 12 \end{array}$$
$$\frac{16}{3}\stackrel{?}{=}12 \quad \text{FALSE}$$

11. Substitute –2 for x.

$$\begin{array}{l|l} \multicolumn{2}{c}{x+7=3-x} \\ \hline -2+7 & 3-(-2) \\ & 3+2 \end{array}$$
$$5\stackrel{?}{=}5 \quad \text{TRUE}$$

Since the left-hand and right-hand sides are the same, –2 is a solution.

12.
$$\begin{array}{l|l} \multicolumn{2}{c}{-4+x=5x} \\ \hline -4+(-1) & 5(-1) \end{array}$$
$$-5\stackrel{?}{=}-5 \quad \text{TRUE}$$

13. Substitute –20 for n.

$$\begin{array}{l|l} \multicolumn{2}{c}{4-\frac{1}{5}n=8} \\ \hline 4-\frac{1}{5}(-20) & 8 \\ 4+4 & \end{array}$$
$$8\stackrel{?}{=}8 \quad \text{TRUE}$$

Since the left-hand and right-hand sides are the same, –20 is a solution.

14.
$$\begin{array}{l|l} \multicolumn{2}{c}{-3=5-\frac{n}{2}} \\ \hline -3 & 5-\frac{4}{2} \\ & 5-2 \end{array}$$
$$-3\stackrel{?}{=}3 \quad \text{FALSE}$$

15.
$$\begin{aligned} x+10&=21 \\ x+10-10&=21-10 \\ x&=11 \end{aligned}$$

Check:
$$\begin{array}{l|l} \multicolumn{2}{c}{x+10=21} \\ \hline 11+10 & 21 \end{array}$$
$$21\stackrel{?}{=}21 \qquad \text{TRUE}$$

The solution is 11.

16. 38

17.
$$\begin{aligned} y+7&=-18 \\ y+7-7&=-18-7 \\ y&=-25 \end{aligned}$$

Check:
$$\begin{array}{l|l} \multicolumn{2}{c}{y+7=-18} \\ \hline -25+7 & -18 \end{array}$$
$$-18\stackrel{?}{=}-18 \quad \text{TRUE}$$

The solution is –25.

18. –19

19.
$$\begin{aligned} -6&=y+25 \\ -6-25&=y+25-25 \\ -31&=y \end{aligned}$$

Check:
$$\begin{array}{l|l} \multicolumn{2}{c}{-6=y+25} \\ \hline -6 & -31+25 \end{array}$$
$$-6\stackrel{?}{=}-6 \quad \text{TRUE}$$

The solution is –31.

20. –13

21.
$$\begin{aligned} x-18&=23\\ x-18+18&=23+18\\ x&=41 \end{aligned}$$
Check: $$\begin{array}{c|c} \multicolumn{2}{c}{x-18=23}\\ \hline 41-18 & 23 \end{array}$$
$$23 \stackrel{?}{=} 23 \quad \text{TRUE}$$
The solution is 41.

22. 35

23.
$$\begin{aligned} 12&=-7+y\\ 7+12&=7+(-7)+y\\ 19&=y \end{aligned}$$
Check: $$\begin{array}{c|c} \multicolumn{2}{c}{12=-7+y}\\ \hline 12 & -7+19 \end{array}$$
$$12 \stackrel{?}{=} 12 \quad \text{TRUE}$$
The solution is 19.

24. 23

25.
$$\begin{aligned} -5+t&=-11\\ 5+(-5)+t&=5+(-11)\\ t&=-6 \end{aligned}$$
Check: $$\begin{array}{c|c} \multicolumn{2}{c}{-5+t=-11}\\ \hline -5+(-6) & -11 \end{array}$$
$$-11 \stackrel{?}{=} -11 \quad \text{TRUE}$$
The solution is –6.

26. –15

27.
$$\begin{aligned} r+\frac{1}{3}&=\frac{8}{3}\\ r+\frac{1}{3}-\frac{1}{3}&=\frac{8}{3}-\frac{1}{3}\\ r&=\frac{7}{3} \end{aligned}$$
Check: $$\begin{array}{c|c} \multicolumn{2}{c}{r+\frac{1}{3}=\frac{8}{3}}\\ \hline \frac{7}{3}+\frac{1}{3} & \frac{8}{3} \end{array}$$
$$\frac{8}{3} \stackrel{?}{=} \frac{8}{3} \quad \text{TRUE}$$
The solution is $\frac{7}{3}$.

28. $\frac{1}{4}$

29.
$$\begin{aligned} x-\frac{3}{5}&=-\frac{7}{10}\\ x-\frac{3}{5}+\frac{3}{5}&=-\frac{7}{10}+\frac{3}{5}\\ x&=-\frac{7}{10}+\frac{3}{5}\cdot\frac{2}{2}\\ x&=-\frac{7}{10}+\frac{6}{10}\\ x&=-\frac{1}{10} \end{aligned}$$
Check: $$\begin{array}{c|c} \multicolumn{2}{c}{x-\frac{3}{5}=-\frac{7}{10}}\\ \hline -\frac{1}{10}-\frac{3}{5} & -\frac{7}{10}\\ -\frac{1}{10}-\frac{6}{10} & \end{array}$$
$$-\frac{7}{10} \stackrel{?}{=} -\frac{7}{10} \quad \text{TRUE}$$
The solution is $-\frac{1}{10}$.

30.
$$\begin{aligned} x-\frac{2}{3}&=-\frac{5}{6}\\ x&=-\frac{5}{6}+\frac{4}{6}\\ x&=-\frac{1}{6} \end{aligned}$$

31.
$$\begin{aligned} x-\frac{5}{6}&=\frac{7}{8}\\ x-\frac{5}{6}+\frac{5}{6}&=\frac{7}{8}+\frac{5}{6}\\ x&=\frac{7}{8}\cdot\frac{3}{3}+\frac{5}{6}\cdot\frac{4}{4}\\ x&=\frac{21}{24}+\frac{20}{24}\\ x&=\frac{41}{24} \end{aligned}$$
Check: $$\begin{array}{c|c} \multicolumn{2}{c}{x-\frac{5}{6}=\frac{7}{8}}\\ \hline \frac{41}{24}-\frac{5}{6} & \frac{7}{8}\\ \frac{41}{24}-\frac{20}{24} & \frac{21}{24} \end{array}$$
$$\frac{21}{24} \stackrel{?}{=} \frac{21}{24} \quad \text{TRUE}$$
The solution is $\frac{41}{24}$.

32.
$$\begin{aligned} y-\frac{3}{4}&=\frac{5}{6}\\ y&=\frac{10}{12}+\frac{9}{12}\\ y&=\frac{19}{12} \end{aligned}$$

33.
$$\begin{aligned} -\frac{1}{5}+z&=-\frac{1}{4}\\ \frac{1}{5}-\frac{1}{5}+z&=\frac{1}{5}-\frac{1}{4}\\ z&=\frac{1}{5}\cdot\frac{4}{4}-\frac{1}{4}\cdot\frac{5}{5}\\ z&=\frac{4}{20}-\frac{5}{20}\\ z&=-\frac{1}{20} \end{aligned}$$

Check:
$$\begin{array}{c|c}
\multicolumn{2}{c}{-\frac{1}{5}+z=-\frac{1}{4}} \\ \hline
-\frac{1}{5}+\left(-\frac{1}{20}\right) & -\frac{1}{4} \\
-\frac{4}{20}+\left(-\frac{1}{20}\right) & -\frac{5}{20}
\end{array}$$
$$-\frac{5}{20}\stackrel{?}{=}-\frac{5}{20} \quad \text{TRUE}$$

The solution is $-\frac{1}{20}$.

34. $-\frac{2}{3}+y=-\frac{3}{4}$
$$y=-\frac{9}{12}+\frac{8}{12}$$
$$y=-\frac{1}{12}$$

35.
$$m-2.8=6.3$$
$$m-2.8+2.8=6.3+2.8$$
$$m=9.1$$

Check:
$$\begin{array}{c|c}
\multicolumn{2}{c}{m-2.8=6.3} \\ \hline
9.1-2.8 & 6.3
\end{array}$$
$$6.3\stackrel{?}{=}6.3 \quad \text{TRUE}$$

The solution is 9.1.

36. 14

37.
$$-9.7=-4.7+y$$
$$4.7+(-9.7)=4.7+(-4.7)+y$$
$$-5=y$$

Check:
$$\begin{array}{c|c}
\multicolumn{2}{c}{-9.7=-4.7+y} \\ \hline
-9.7 & -4.7+(-5)
\end{array}$$
$$-9.7\stackrel{?}{=}-9.7 \quad \text{TRUE}$$

The solution is –5.

38. –10.6

39. $8a=56$

$\frac{8a}{8}=\frac{56}{8}$ Dividing both sides by 8

$1\cdot a=7$ Simplifying

$a=7$ Identity property of 1

Check:
$$\begin{array}{c|c}
\multicolumn{2}{c}{8a=56} \\ \hline
8\cdot 7 & 56
\end{array}$$
$$56\stackrel{?}{=}56 \quad \text{TRUE}$$

The solution is 7.

40. 12

41. $84=7x$

$\frac{84}{7}=\frac{7x}{7}$ Dividing both sides by 7

$12=1\cdot x$

$12=x$

Check:
$$\begin{array}{c|c}
\multicolumn{2}{c}{84=7x} \\ \hline
84 & 7\cdot 12
\end{array}$$
$$84\stackrel{?}{=}84 \quad \text{TRUE}$$

The solution is 12.

42. 5

43.
$$-x=38$$
$$-1\cdot x=38$$
$$-1\cdot(-1\cdot x)=-1\cdot 38$$
$$1\cdot x=-38$$
$$x=-38$$

Check:
$$\begin{array}{c|c}
\multicolumn{2}{c}{-x=38} \\ \hline
-(-38) & 38
\end{array}$$
$$38\stackrel{?}{=}38 \quad \text{TRUE}$$

The solution is –38.

44. –100

45. $-t=-8$

The equation states that the opposite of t is the opposite of 8. Thus, $t=8$. We could also do this exercise as follows.

$-t=-8$

$-1(-t)=-1(-8)$ Multiplying both sides by –1

$t=8$

Check:
$$\begin{array}{c|c}
\multicolumn{2}{c}{-t=-8} \\ \hline
-(8) & -8
\end{array}$$
$$-8\stackrel{?}{=}-8 \quad \text{TRUE}$$

The solution is 8.

46. $-68=-r$

Using the reasoning in Exercise 47, we see that $r=68$. We can also multiply both sides of the equation by –1 to get this result. The solution is 68.

47. $-7x=49$
$$\frac{-7x}{-7}=\frac{49}{-7}$$
$$1\cdot x=-7$$
$$x=-7$$

Check:
$$\begin{array}{c|c}
\multicolumn{2}{c}{-7x=49} \\ \hline
-7(-7) & 49
\end{array}$$
$$49\stackrel{?}{=}49 \quad \text{TRUE}$$

The solution is –7.

48. –9

49. $0.2m=10$
$$\frac{0.2m}{0.2}=\frac{10}{0.2}$$
$$m=50$$

Check:
$$\begin{array}{c|c}
\multicolumn{2}{c}{0.2m=10} \\ \hline
0.2(50) & 10
\end{array}$$
$$10\stackrel{?}{=}10 \quad \text{TRUE}$$

The solution is 50.

50. 150

51. $-1.2x = 0.24$

$$\frac{-1.2x}{-1.2} = \frac{0.24}{-1.2}$$

$$x = -0.2$$

Check: $-1.2x = 0.24$

$$\begin{array}{c|c} -1.2(-0.2) & 0.24 \end{array}$$

$$0.24 \stackrel{?}{=} 0.24 \quad \text{TRUE}$$

The solution is –0.2.

52. –0.5

53. $-1.3a = -10.4$

$$\frac{-1.3a}{-1.3} = \frac{-10.4}{-1.3}$$

$$a = 8$$

Check: $-1.3a = -10.4$

$$\begin{array}{c|c} -1.3(8) & -10.4 \end{array}$$

$$-10.4 \stackrel{?}{=} -10.4 \quad \text{TRUE}$$

The solution is 8.

54. 6

55. $\frac{y}{8} = 11$

$$\frac{1}{8} \cdot y = 11$$

$$8\left(\frac{1}{8}\right) \cdot y = 8 \cdot 11$$

$$y = 88$$

Check: $\frac{y}{8} = 11$

$$\begin{array}{c|c} \frac{88}{8} & 11 \end{array}$$

$$11 \stackrel{?}{=} 11 \quad \text{TRUE}$$

The solution is 88.

56. 52

57. $\frac{4}{5}x = 16$

$$\frac{5}{4} \cdot \frac{4}{5}x = \frac{5}{4} \cdot 16$$

$$x = \frac{5 \cdot \cancel{4} \cdot 4}{\cancel{4} \cdot 1}$$

$$x = 20$$

Check: $\frac{4}{5}x = 16$

$$\begin{array}{c|c} \frac{4}{5} \cdot 20 & 16 \end{array}$$

$$16 \stackrel{?}{=} 16 \quad \text{TRUE}$$

The solution is 20.

58. $\frac{3}{4}x = 27$

$$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3} \cdot 27$$

$$1 \cdot x = \frac{4 \cdot \cancel{3} \cdot 3 \cdot 3}{\cancel{3} \cdot 1}$$

$$x = 36$$

59. $\frac{-x}{6} = 9$

$$-\frac{1}{6} \cdot x = 9$$

$$-6\left(-\frac{1}{6}\right) \cdot x = -6 \cdot 9$$

$$x = -54$$

Check: $\frac{-x}{6} = 9$

$$\begin{array}{c|c} \frac{-(-54)}{6} & 9 \\ \frac{54}{6} & \end{array}$$

$$9 \stackrel{?}{=} 9 \quad \text{TRUE}$$

The solution is –54.

60. $\frac{-t}{4} = 8$

$$-\frac{1}{4} \cdot t = 8$$

$$-4\left(-\frac{1}{4}\right) \cdot t = -4 \cdot 8$$

$$t = -32$$

61. $\frac{1}{9} = \frac{z}{-5}$

$$\frac{1}{9} = -\frac{1}{5} \cdot z$$

$$-5 \cdot \frac{1}{9} = -5 \cdot \left(-\frac{1}{5} \cdot z\right)$$

$$-\frac{5}{9} = z$$

Check: $\frac{1}{9} = \frac{z}{-5}$

$$\begin{array}{c|c} \frac{1}{9} & \frac{-5/9}{-5} \\ & -\frac{5}{9} \cdot \frac{1}{-5} \end{array}$$

$$\frac{1}{9} \stackrel{?}{=} \frac{1}{9} \quad \text{TRUE}$$

The solution is $-\frac{5}{9}$.

62. $-\frac{6}{7}$

63. $-\frac{3}{5}r = -\frac{3}{5}$

The solution of the equation is the number that is multiplied by $-\frac{3}{5}$ to get $-\frac{3}{5}$. That number is 1. We could also do this exercise as follows:

$$-\frac{3}{5}r = -\frac{3}{5}$$

$$-\frac{5}{3} \cdot \left(-\frac{3}{5}r\right) = -\frac{5}{3}\left(-\frac{3}{5}\right)$$

$$r = 1$$

Check: $$\begin{array}{c|c} \multicolumn{2}{c}{-\frac{3}{5}r = -\frac{3}{5}} \\ \hline -\frac{3}{5}\cdot 1 & -\frac{3}{5} \end{array}$$
$$-\frac{3}{5} \stackrel{?}{=} -\frac{3}{5} \quad \text{TRUE}$$
The solution is 1.

64. $$\begin{aligned} -\frac{2}{5}y &= -\frac{4}{15} \\ -\frac{5}{2}\left(-\frac{2}{5}y\right) &= -\frac{5}{2}\cdot\left(-\frac{4}{15}\right) \\ y &= -\frac{\cancel{5}\cdot\cancel{2}\cdot 2}{\cancel{2}\cdot 3\cdot\cancel{5}} \\ y &= \frac{2}{3} \end{aligned}$$

65. $$\begin{aligned} \frac{-3r}{2} &= -\frac{27}{4} \\ -\frac{3}{2}r &= -\frac{27}{4} \\ -\frac{2}{3}\cdot\left(-\frac{3}{2}r\right) &= -\frac{2}{3}\cdot\left(-\frac{27}{4}\right) \\ r &= \frac{\cancel{2}\cdot\cancel{3}\cdot 3\cdot 3}{\cancel{3}\cdot\cancel{2}\cdot 2} \\ r &= \frac{9}{2} \end{aligned}$$

Check: $$\begin{array}{c|c} \multicolumn{2}{c}{\frac{-3r}{2} = -\frac{27}{4}} \\ \hline -\frac{3}{2}\cdot\frac{9}{2} & -\frac{27}{4} \end{array}$$
$$-\frac{27}{4} \stackrel{?}{=} -\frac{27}{4} \quad \text{TRUE}$$
The solution is $\frac{9}{2}$.

66. $$\begin{aligned} \frac{5x}{7} &= -\frac{10}{14} \\ \frac{5}{7}x &= -\frac{10}{14} \\ \frac{7}{5}\cdot\frac{5}{7}x &= \frac{7}{5}\cdot\left(-\frac{10}{14}\right) \\ x &= -\frac{7\cdot 5\cdot 2}{5\cdot 2\cdot 7} \\ x &= -1 \end{aligned}$$

67. $$\begin{aligned} 4.5 + t &= -3.1 \\ 4.5 + t - 4.5 &= -3.1 - 4.5 \\ t &= -7.6 \end{aligned}$$
The solution is –7.6.

68. $$\begin{aligned} \frac{3}{4}x &= 18 \\ x &= \frac{4}{3}\cdot 18 \\ x &= 24 \end{aligned}$$

69. $$\begin{aligned} -8.2x &= 20.5 \\ \frac{-8.2x}{-8.2} &= \frac{20.5}{-8.2} \\ x &= -2.5 \end{aligned}$$
The solution is –2.5.

70. –5.5

71. $$\begin{aligned} x - 4 &= -19 \\ x - 4 + 4 &= -19 + 4 \\ x &= -15 \end{aligned}$$
The solution is –15.

72. $$\begin{aligned} y - 6 &= -14 \\ y - 6 + 6 &= -14 + 6 \\ y &= -8 \end{aligned}$$

73. $$\begin{aligned} t - 3 &= 8 \\ t - 3 + 3 &= -8 + 3 \\ t &= -5 \end{aligned}$$
The solution is –5.

74. $$\begin{aligned} t - 9 &= -8 \\ t &= -8 + 9 \\ t &= 1 \end{aligned}$$

75. $$\begin{aligned} -12x &= 14 \\ \frac{-12x}{-12} &= \frac{14}{-12} \\ 1\cdot x &= -\frac{7}{6} \\ x &= -\frac{7}{6} \end{aligned}$$
The solution is $-\frac{7}{6}$.

76. $$\begin{aligned} -15x &= 20 \\ \frac{-15x}{-15} &= \frac{20}{-15} \\ x &= -\frac{4}{3} \end{aligned}$$

77. $$\begin{aligned} 48 &= -\frac{3}{8}y \\ -\frac{8}{3}\cdot 48 &= -\frac{8}{3}\left(-\frac{3}{8}y\right) \\ -\frac{8\cdot\cancel{3}\cdot 16}{\cancel{3}} &= y \\ -128 &= y \end{aligned}$$
The solution is –128.

78. $$\begin{aligned} 14 &= t + 27 \\ 14 - 27 &= t + 27 - 27 \\ -13 &= t \end{aligned}$$

79. $$\begin{aligned} a - \frac{1}{6} &= -\frac{2}{3} \\ a - \frac{1}{6} + \frac{1}{6} &= -\frac{2}{3} + \frac{1}{6} \\ a &= -\frac{4}{6} + \frac{1}{6} \\ a &= -\frac{3}{6} \\ a &= -\frac{1}{2} \end{aligned}$$
The solution is $-\frac{1}{2}$.

80. $$-\frac{x}{6}=\frac{2}{9}$$
$$-6\left(-\frac{x}{6}\right)=-6\cdot\frac{2}{9}$$
$$x=-\frac{4}{3}$$

81. $$-24=\frac{8x}{5}$$
$$-24=\frac{8}{5}x$$
$$\frac{5}{8}(-24)=\frac{5}{8}\cdot\frac{8}{5}x$$
$$-\frac{5\cdot\cancel{8}\cdot 3}{\cancel{8}\cdot 1}=x$$
$$-15=x$$
The solution is –15.

82. $$\frac{1}{5}+y=-\frac{3}{10}$$
$$y=-\frac{3}{10}-\frac{2}{10}=-\frac{5}{10}$$
$$y=-\frac{1}{2}$$

83. $$-\frac{4}{3}t=-12$$
$$-\frac{3}{4}\left(-\frac{4}{3}t\right)=-\frac{3}{4}(-12)$$
$$t=\frac{3\cdot\cancel{4}\cdot 3}{\cancel{4}}$$
$$t=9$$
The solution is 9.

84. $$\frac{17}{35}=-x$$

The opposite of x is $\frac{17}{35}$, so $x=-\frac{17}{35}$. We could also multiply both sides of the equation by –1 to get this result. The solution is $-\frac{17}{35}$.

85. $$-483.297=-794.053+t$$
$$-483.297+794.053=-794.053+t+794.053$$
$$310.756=t \quad \text{Using a calculator}$$
The solution is 310.756.

86. Using a calculator we find that the solution is –8655.

87. *Writing Exercise*. For an equation $x+a=b$, add the opposite of a (or subtract a) on both sides of the equation. For an equation $ax=b$, multiply by $1/a$ (or divide by a) on both sides of the equation.

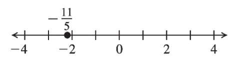
88. *Writing Exercise*. Equivalent expressions have the same value for all possible replacements for the variables. Equivalent equations have the same solution(s).

89. $\frac{1}{3}y-7$

90. $6(2x+11)=12x+66$

91. $35a+55c+5=5(7a+11c+1)$

92.



93. *Writing Exercise*. Yes, it will form an equivalent equation by the addition principle. It will not help to solve the equation, however. The multiplication principle should be used to solve the equation.

94. *Writing Exercise*. Since $a-c=b-c$ can be rewritten as $a+(-c)=b+(-c)$, it is not necessary to state a subtraction principle.

95. $$mx=11.6m$$
$$\frac{mx}{m}=\frac{11.6m}{m}$$
$$x=11.6$$
The solution is 11.6.

96. $$x-4+a=a$$
$$x-4=0$$
$$x=4$$

97. $$cx+5c=7c$$
$$cx+5c-5c=7c-5c$$
$$cx=2c$$
$$\frac{cx}{c}=\frac{2c}{c}$$
$$x=2$$
The solution is 2.

98. $$c\cdot\frac{21}{a}=\frac{7cx}{2a}$$
$$\frac{2a}{7c}\cdot c\cdot\frac{21}{a}=\frac{2a}{7c}\cdot\frac{7cx}{2a}$$
$$\frac{2\cdot\cancel{a}\cdot\cancel{c}\cdot 3\cdot\cancel{7}}{\cancel{7}\cdot\cancel{c}\cdot\cancel{a}}=x$$
$$6=x$$

99. $$7+|x|=30$$
$$-7+7+|x|=-7+30$$
$$|x|=23$$
x represents a number whose distance from 0 is 23. Thus $x=-23$ or $x=23$.

100. $$ax-3a=5a$$
$$ax=8a$$
$$x=8$$

101. $$t-3590=1820$$
$$t-3590+3590=1820+3590$$
$$t=5410$$
$$t+3590=5410+3590$$
$$t+3590=9000$$

102. $$n+268=124$$
$$n+268-268=124-268$$
$$n=-144$$
$$n-268=-144-268$$
$$n-268=-412$$

103. To "undo" the last step, divide 225 by 0.3.

$225 \div 0.3 = 750$

Now divide 750 by 0.3.

$750 \div 0.3 = 2500$

The answer should be 2500 not 225.

104. *Writing Exercise*. No; –5 is a solution of $x^2 = 25$ but not of $x = 5$.

Exercise Set 2.2

1. To isolate x in $x - 4 = 7$, we would use the *Addition principle*. Add 4 to both sides of the equation.

2. To isolate x in $5x = 8$, we would use the *Multiplication principle*. Multiplying both sides of the equation by $\frac{1}{5}$, or divide both sides by 5.

3. To clear fractions or decimals, we use the *Multiplication principle*.

4. To remove parentheses, we use the *Distributive law*.

5. To solve $3x - 1 = 8$, we use the *Addition principle* first. Add 1 to both sides of the equation.

6. To solve $5(x-1) + 3(x+7) = 2$, we use the *Distributive law* first. Distribute 5 in the first set of parentheses and 3 in the second set.

7.
$$\begin{aligned} 3x - 1 &= 7 \\ 3x - 1 + 1 &= 7 + 1 \quad \text{Adding 1 to both sides} \\ 3x &= 7 + 1 \end{aligned}$$
Choice (c) is correct.

8.
$$\begin{aligned} 4x + 5x &= 12 \\ 9x &= 12 \quad \text{Combining like terms} \end{aligned}$$
Choice (e) is correct.

9.
$$\begin{aligned} 6(x-1) &= 2 \\ 6x - 6 &= 2 \quad \text{Using the distributive law} \end{aligned}$$
Choice (a) is correct.

10.
$$\begin{aligned} 7x &= 9 \\ \frac{7x}{7} &= \frac{9}{7} \quad \text{Dividing both sides by 7} \\ x &= \frac{9}{7} \end{aligned}$$
Choice (f) is correct.

11.
$$\begin{aligned} 4x &= 3 - 2x \\ 4x + 2x &= 3 - 2x + 2x \quad \text{Adding } 2x \text{ to both sides} \\ 4x + 2x &= 3 \end{aligned}$$
Choice (b) is correct.

12.
$$\begin{aligned} 8x - 5 &= 6 - 2x \\ 8x - 5 + 5 &= 6 - 2x + 5 \quad \text{Adding 5 to both sides} \\ 8x &= 6 - 2x + 5 \\ 8x + 2x &= 6 - 2x + 5 + 2x \quad \text{Adding } 2x \text{ to both sides} \\ 8x + 2x &= 6 + 5 \end{aligned}$$
Choice (d) is correct.

13.
$$\begin{aligned} 2x + 9 &= 25 \\ 2x + 9 - 9 &= 25 - 9 \quad \text{Subtracting 9 from both sides} \\ 2x &= 16 \quad \text{Simplifying} \\ \frac{2x}{2} &= \frac{16}{2} \quad \text{Dividing both sides by 2} \\ x &= 8 \quad \text{Simplifying} \end{aligned}$$

Check:
$$\begin{array}{r|l} \multicolumn{2}{c}{2x + 9 = 25} \\ \hline 2 \cdot 8 + 9 & 25 \\ 16 + 9 & \end{array}$$
$$25 \stackrel{?}{=} 25 \quad \text{TRUE}$$

The solution is 8.

14.
$$\begin{aligned} 5z + 2 &= 57 \\ 5z &= 55 \\ z &= 11 \end{aligned}$$

15.
$$\begin{aligned} 7t - 8 &= 27 \\ 7t - 8 + 8 &= 27 + 8 \quad \text{Adding 8 to both sides} \\ 7t &= 35 \\ \frac{7t}{7} &= \frac{35}{7} \quad \text{Dividing both sides by 7} \\ t &= 5 \end{aligned}$$

Check:
$$\begin{array}{r|l} \multicolumn{2}{c}{7t - 8 = 27} \\ \hline 7 \cdot 5 - 8 & 27 \\ 35 - 8 & \end{array}$$
$$27 \stackrel{?}{=} 27 \quad \text{TRUE}$$

The solution is 5.

16.
$$\begin{aligned} 6x - 5 &= 2 \\ 6x &= 7 \\ x &= \frac{7}{6} \end{aligned}$$

17.
$$\begin{aligned} 3x - 9 &= 1 \\ 3x - 9 + 9 &= 1 + 9 \\ 3x &= 10 \\ \frac{3x}{3} &= \frac{10}{3} \\ x &= \frac{10}{3} \end{aligned}$$

Check:
$$\begin{array}{r|l} \multicolumn{2}{c}{3x - 9 = 1} \\ \hline 3 \cdot \frac{10}{3} - 9 & 1 \\ 10 - 9 & \end{array}$$
$$1 \stackrel{?}{=} 1 \quad \text{TRUE}$$

The solution is $\frac{10}{3}$.

18.
$$\begin{aligned} 5x - 9 &= 41 \\ 5x &= 50 \\ x &= 10 \end{aligned}$$

19.
$$\begin{aligned} 8z + 2 &= -54 \\ 8z + 2 - 2 &= -54 - 2 \\ 8z &= -56 \\ \frac{8z}{8} &= \frac{-56}{8} \\ z &= -7 \end{aligned}$$

Check:
$$\begin{array}{r|l} \multicolumn{2}{c}{8z+2=-54} \\ \hline 8(-7)+2 & -54 \\ -56+2 & \\ \end{array}$$
$$-54 \stackrel{?}{=} -54 \quad \text{TRUE}$$

The solution is –7.

20. $4x+3=-21$
$$4x=-24$$
$$x=-6$$

21.
$$-37=9t+8$$
$$-37-8=9t+8-8$$
$$-45=9t$$
$$\frac{-45}{9}=\frac{9t}{9}$$
$$-5=t$$

Check:
$$\begin{array}{r|l} \multicolumn{2}{c}{-37=9t+8} \\ \hline -37 & 9\cdot(-5)+8 \\ & -45+8 \\ \end{array}$$
$$-37 \stackrel{?}{=} -37 \quad \text{TRUE}$$

The solution is –5.

22. $-39=1+5t$
$$-40=5t$$
$$-8=t$$

23.
$$12-t=16$$
$$-12+12-t=-12+16$$
$$-t=4$$
$$\frac{-t}{-1}=\frac{4}{-1}$$
$$t=-4$$

Check:
$$\begin{array}{r|l} \multicolumn{2}{c}{12-t=16} \\ \hline 12-(-4) & 16 \\ 12+4 & \\ \end{array}$$
$$16 \stackrel{?}{=} 16 \quad \text{TRUE}$$

The solution is –4.

24. $9-t=21$
$$-t=12$$
$$t=-12$$

25.
$$-6z-18=-132$$
$$-6z-18+18=-132+18$$
$$-6z=-114$$
$$\frac{-6z}{-6}=\frac{-114}{-6}$$
$$z=19$$

Check:
$$\begin{array}{r|l} \multicolumn{2}{c}{-6z-18=-132} \\ \hline -6\cdot 19-18 & -132 \\ -114-18 & \\ \end{array}$$
$$-132 \stackrel{?}{=} -132 \quad \text{TRUE}$$

The solution is 19.

26. $-7x-24=-129$
$$-7x=-105$$
$$x=15$$

27. $5.3+1.2n=1.94$
$$1.2n=-3.36$$
$$\frac{1.2n}{1.2}=\frac{-3.36}{1.2}$$
$$n=-2.8$$

Check:
$$\begin{array}{r|l} \multicolumn{2}{c}{5.31+1.2n=1.94} \\ \hline 5.3+1.2(-2.8) & 1.94 \\ 5.3+(-3.36) & \\ \end{array}$$
$$1.94 \stackrel{?}{=} 1.94 \quad \text{TRUE}$$

The solution is –2.8.

28. $6.4-2.5n=2.2$
$$-2.5n=-4.2$$
$$n=1.68$$

29.
$$32-7x=11$$
$$-32+32-7x=-32+11$$
$$-7x=-21$$
$$\frac{-7x}{-7}=\frac{-21}{-7}$$
$$x=3$$

Check:
$$\begin{array}{r|l} \multicolumn{2}{c}{32-7x=11} \\ \hline 32-7\cdot 3 & 11 \\ 32-21 & \\ \end{array}$$
$$11 \stackrel{?}{=} 11 \quad \text{TRUE}$$

The solution is 3.

30. $27-6x=99$
$$-6x=72$$
$$x=-12$$

31.
$$\frac{3}{5}t-1=8$$
$$\frac{3}{5}t-1+1=8+1$$
$$\frac{3}{5}t=9$$
$$\frac{5}{3}\cdot\frac{3}{5}t=\frac{5}{3}\cdot 9$$
$$t=\frac{5\cdot\cancel{3}\cdot 3}{\cancel{3}\cdot 1}$$
$$t=15$$

Check:
$$\begin{array}{r|l} \multicolumn{2}{c}{\frac{3}{5}t-1=8} \\ \hline \frac{3}{5}\cdot 15-1 & 8 \\ 9-1 & \\ \end{array}$$
$$8 \stackrel{?}{=} 8 \quad \text{TRUE}$$

The solution is 15.

32. $\frac{2}{3}t-1=5$
$$\frac{2}{3}t=6$$
$$\frac{3}{2}\cdot\frac{2}{3}t=\frac{3}{2}\cdot 6$$
$$t=9$$

33.
$$6+\frac{7}{2}x=-15$$
$$-6+6+\frac{7}{2}x=-6-15$$
$$\frac{7}{2}x=-21$$
$$\frac{2}{7}\cdot\frac{7}{2}x=\frac{2}{7}(-21)$$
$$x=-\frac{2\cdot 3\cdot \cancel{7}}{\cancel{7}\cdot 1}$$
$$x=-6$$

Check:
$$\begin{array}{c|c} \multicolumn{2}{c}{6+\frac{7}{2}x=-15} \\ \hline 6+\frac{7}{2}(-6) & -15 \\ 6+(-21) & \end{array}$$
$$-15\stackrel{?}{=}-15 \quad \text{TRUE}$$

The solution is –6.

34. $6+\frac{5}{4}x=-4$
$$\frac{5}{4}x=-10$$
$$x=\frac{4}{5}(-10)$$
$$x=-8$$

35.
$$-\frac{4a}{5}-8=2$$
$$-\frac{4a}{5}-8+8=2+8$$
$$-\frac{4a}{5}=10$$
$$-\frac{5}{4}\left(-\frac{4a}{5}\right)=-\frac{5}{4}\cdot 10$$
$$a=-\frac{5\cdot 5\cdot \cancel{2}}{2\cdot \cancel{2}}$$
$$a=-\frac{25}{2}$$

Check:
$$\begin{array}{c|c} \multicolumn{2}{c}{-\frac{4a}{5}-8=2} \\ \hline -\frac{4}{5}\left(-\frac{25}{2}\right)-8 & 2 \\ 10-8 & \end{array}$$
$$2\stackrel{?}{=}2 \quad \text{TRUE}$$

The solution is $-\frac{25}{2}$.

36. $-\frac{8a}{7}-2=4$
$$-\frac{8a}{7}=6$$
$$x=-\frac{\cancel{2}\cdot 3\cdot 7}{\cancel{2}\cdot 2\cdot 2}$$
$$a=-\frac{21}{4}$$

37. $6x+10x=18$
$$16x=18 \quad \text{Combining like terms}$$
$$\frac{16x}{16}=\frac{18}{16}$$
$$x=\frac{9}{8}$$

Check:
$$\begin{array}{c|c} \multicolumn{2}{c}{6x+10x=18} \\ \hline 6\left(\frac{9}{8}\right)+10\left(\frac{9}{8}\right) & 18 \\ \frac{27}{4}+\frac{45}{4} & \end{array}$$
$$18\stackrel{?}{=}18 \quad \text{TRUE}$$

The solution is $\frac{9}{8}$.

38. $-3z+8z=45$
$$5z=45$$
$$z=9$$

39. $4x-6=6x$
$$-6=6x-4x \quad \text{Subtracting } 4x \text{ from both sides}$$
$$-6=2x \quad \text{Simplifying}$$
$$\frac{-6}{2}=\frac{2x}{2} \quad \text{Dividing both sides by 2}$$
$$-3=x$$

Check:
$$\begin{array}{c|c} \multicolumn{2}{c}{4x-6=6x} \\ \hline 4(-3)-6 & 6(-3) \\ -12-6 & -18 \end{array}$$
$$-18\stackrel{?}{=}-18 \quad \text{TRUE}$$

The solution is –3.

40. $7n=2n+4$
$$5n=4$$
$$n=\frac{4}{5}$$

41.
$$2-5y=26-y$$
$$2-5y+y=26-y+y \quad \text{Adding } y \text{ to both sides}$$
$$2-4y=26 \quad \text{Simplifying}$$
$$-2+2-4y=-2+26 \quad \text{Adding } -2 \text{ to both sides}$$
$$-4y=24 \quad \text{Simplifying}$$
$$\frac{-4y}{-4}=\frac{24}{-4} \quad \text{Dividing both sides by } -4$$
$$y=-6$$

Check:
$$\begin{array}{c|c} \multicolumn{2}{c}{2-5y=26-y} \\ \hline 2-5(-6) & 26-(-6) \\ 2+30 & 26+6 \end{array}$$
$$32\stackrel{?}{=}32 \quad \text{TRUE}$$

The solution is –6.

42. $6x-5=7+2x$
$$4x=12 \quad \text{Simplifying}$$
$$x=3$$

43. $6x+3=2x+3$
$6x-2x=3-3$
$4x=0$
$\frac{4x}{4}=\frac{0}{4}$
$x=0$

Check: $\begin{array}{c|l} \multicolumn{2}{c}{6x+3=2x+3} \\ \hline 6\cdot 0+3 & 2\cdot 0+3 \\ 0+3 & 0+3 \end{array}$
$3 \stackrel{?}{=} 3$ TRUE

The solution is 0.

44. $5y+3=2y+15$
$3y=12$
$y=4$

45. $5-2x=3x-7x+25$
$5-2x=-4x+25$
$4x-2x=25-5$
$2x=20$
$\frac{2x}{2}=\frac{20}{2}$
$x=10$

Check: $\begin{array}{c|l} \multicolumn{2}{c}{5-2x=3x-7x+25} \\ \hline 5-2\cdot 10 & 3\cdot 10-7\cdot 10+25 \\ 5-20 & 30-70+25 \\ -15 & -40+25 \end{array}$
$-15 \stackrel{?}{=} -15$ TRUE

The solution is 10.

46. $10-3x=x-2x+40$
$10-3x=-x+40$
$-2x=30$
$x=-15$

47. $7+3x-6=3x+5-x$
$3x+1=2x+5$ Combining like terms on each side
$3x-2x=5-1$
$x=4$

Check: $\begin{array}{c|l} \multicolumn{2}{c}{7+3x-6=3x+5-x} \\ \hline 7+3\cdot 4-6 & 3\cdot 4+5-4 \\ 7+12-6 & 12+5-4 \\ 19-6 & 17-4 \end{array}$
$13 \stackrel{?}{=} 13$ TRUE

The solution is 4.

48. $5+4x-7=4x-2-x$
$4x-2=3x-2$
$x=0$

49. $\frac{2}{3}+\frac{1}{4}t=2$

The number 12 is the least common denominator, so we multiply by 12 on both sides.

$12\left(\frac{2}{3}+\frac{1}{4}t\right)=12\cdot 2$
$12\cdot\frac{2}{3}+12\cdot\frac{1}{4}t=24$
$8+3t=24$
$3t=24-8$
$3t=16$
$t=\frac{16}{3}$

Check: $\begin{array}{c|l} \multicolumn{2}{c}{\frac{2}{3}+\frac{1}{4}t=2} \\ \hline \frac{2}{3}+\frac{1}{4}\left(\frac{16}{3}\right) & 2 \\ \frac{2}{3}+\frac{4}{3} & \end{array}$
$2 \stackrel{?}{=} 2$ TRUE

The solution is $\frac{16}{3}$.

50. $-\frac{5}{6}+x=-\frac{1}{2}-\frac{2}{3}$

The least common denominator is 6.

$-5+6x=-3-4$
$-5+6x=-7$
$6x=-7+5$
$6x=-2$
$x=-\frac{1}{3}$

51. $\frac{2}{3}+4t=6t-\frac{2}{15}$

The number 15 is the least common denominator, so we multiply by 15 on both sides.

$15\left(\frac{2}{3}+4t\right)=15\left(6t-\frac{2}{15}\right)$
$15\cdot\frac{2}{3}+15\cdot 4t=15\cdot 6t-15\cdot\frac{2}{15}$
$10+60t=90t-2$
$10+2=90t-60t$
$12=30t$
$\frac{12}{30}=t$
$\frac{2}{5}=t$

Check: $\begin{array}{c|l} \multicolumn{2}{c}{\frac{2}{3}+4t=6t-\frac{2}{15}} \\ \hline \frac{2}{3}+4\cdot\frac{2}{5} & 6\cdot\frac{2}{5}-\frac{2}{15} \\ \frac{2}{3}+\frac{8}{5} & \frac{12}{5}-\frac{2}{15} \\ \frac{10}{15}+\frac{24}{15} & \frac{36}{15}-\frac{2}{15} \end{array}$
$\frac{34}{15} \stackrel{?}{=} \frac{34}{15}$ TRUE

The solution is $\frac{2}{5}$.

52. $\frac{1}{2}+4m=3m-\frac{5}{2}$

The least common denominator is 2.

$$\begin{aligned}1+8m&=6m-5\\2m&=-6\\m&=-3\end{aligned}$$

53. $\frac{1}{3}x+\frac{2}{5}=\frac{4}{5}+\frac{3}{5}x-\frac{2}{3}$

The number 15 is the least common denominator, so we multiply by 15 on both sides.

$$\begin{aligned}15\left(\frac{1}{3}x+\frac{2}{5}\right)&=15\left(\frac{4}{5}+\frac{3}{5}x-\frac{2}{3}\right)\\15\cdot\frac{1}{3}x+15\cdot\frac{2}{5}&=15\cdot\frac{4}{5}+15\cdot\frac{3}{5}x-15\cdot\frac{2}{3}\\5x+6&=12+9x-10\\5x+6&=2+9x\\5x-9x&=2-6\\-4x&=-4\\\frac{-4x}{-4}&=\frac{-4}{-4}\\x&=1\end{aligned}$$

Check:
$$\begin{array}{c|c}\multicolumn{2}{c}{\frac{1}{3}x+\frac{2}{5}=\frac{4}{5}+\frac{3}{5}x-\frac{2}{3}}\\\hline \frac{1}{3}\cdot1+\frac{2}{5} & \frac{4}{5}+\frac{3}{5}\cdot1-\frac{2}{3}\\ \frac{1}{3}+\frac{2}{5} & \frac{4}{5}+\frac{3}{5}-\frac{2}{3}\\ \frac{5}{15}+\frac{6}{15} & \frac{12}{15}+\frac{9}{15}-\frac{10}{15}\\ \frac{11}{15} & \frac{11}{15}\end{array}$$
$$\frac{11}{15}\stackrel{?}{=}\frac{11}{15}\quad\text{TRUE}$$

The solution is 1.

54. $1-\frac{2}{3}y=\frac{9}{5}-\frac{1}{5}y+\frac{3}{5}$

The least common denominator is 15.

$$\begin{aligned}15-10y&=27-3y+9\\15-10y&=36-3y\\-7y&=21\\y&=-3\end{aligned}$$

55. $2.1x+45.2=3.2-8.4x$

Greatest number of decimal places is 1

$10(2.1x+45.2)=10(3.2-8.4x)$

Multiplying by 10 to clear decimals

$$\begin{aligned}10(2.1x)+10(45.2)&=10(3.2)-10(8.4x)\\21x+452&=32-84x\\21x+84x&=32-452\\105x&=-420\\x&=\frac{-420}{105}\\x&=-4\end{aligned}$$

Check:
$$\begin{array}{c|c}\multicolumn{2}{c}{2.1x+45.2=3.2-8.4x}\\\hline 2.1(-4)+45.2 & 3.2-8.4(-4)\\ -8.4+45.2 & 3.2+33.6\end{array}$$
$$36.8\stackrel{?}{=}36.8\quad\text{TRUE}$$

The solution is –4.

56.
$$\begin{aligned}0.91-0.2z&=1.23-0.6z\\91-20z&=123-60z\\40z&=32\\z&=\frac{4}{5},\text{ or }0.8\end{aligned}$$

57. $0.76+0.21t=0.96t-0.49$

Greatest number of decimal places is 2

$100(0.76+0.21t)=100(0.96t-0.49)$

Multiplying by 100 to clear decimals

$$\begin{aligned}100(0.76)+100(0.21t)&=100(0.96t)-100(0.49)\\76+21t&=96t-49\\76+49&=96t-21t\\125&=75t\\\frac{125}{75}&=t\\\frac{5}{3}&=t,\text{ or}\\1.\overline{6}&=t\end{aligned}$$

The answer checks. The solution is $\frac{5}{3}$, or $1.\overline{6}$.

58.
$$\begin{aligned}1.7t+8-1.62t&=0.4t-0.32+8\\170t+800-162t&=40t-32+800\\8t+800&=40t+768\\-32t&=-32\\t&=1\end{aligned}$$

59. $\frac{2}{5}x-\frac{3}{2}x=\frac{3}{4}x+3$

The least common denominator is 20.

$$\begin{aligned}20\left(\frac{2}{5}x-\frac{3}{2}x\right)&=20\left(\frac{3}{4}x+3\right)\\20\cdot\frac{2}{5}x-20\cdot\frac{3}{2}x&=20\cdot\frac{3}{4}x+20\cdot3\\8x-30x&=15x+60\\-22x&=15x+60\\-22x-15x&=60\\-37x&=60\\x&=-\frac{60}{37}\end{aligned}$$

Check:
$$\begin{array}{c|c}\multicolumn{2}{c}{\frac{2}{5}x-\frac{3}{2}x=\frac{3}{4}x+3}\\\hline \frac{2}{5}\left(-\frac{60}{37}\right)-\frac{3}{2}\left(-\frac{60}{37}\right) & \frac{3}{4}\left(-\frac{60}{37}\right)+3\\ -\frac{24}{37}+\frac{90}{37} & -\frac{45}{37}+\frac{111}{37}\end{array}$$
$$\frac{66}{37}\stackrel{?}{=}\frac{66}{37}\quad\text{TRUE}$$

The solution is $-\frac{60}{37}$.

60. $\frac{5}{16}y+\frac{3}{8}y=2+\frac{1}{4}y$

The least common denominator is 16.

$$\begin{aligned}5y+6y&=32+4y\\11y&=32+4y\\7y&=32\\y&=\frac{32}{7}\end{aligned}$$

61. $$\begin{aligned} \frac{1}{3}(2x-1) &= 7 \\ 3\cdot\frac{1}{3}(2x-1) &= 3\cdot 7 \\ 2x-1 &= 21 \\ 2x &= 22 \\ x &= 11 \end{aligned}$$

Check: $$\begin{array}{c|c} \multicolumn{2}{c}{\frac{1}{3}(2x-1)=7} \\ \hline \frac{1}{3}(2\cdot 11-1) & 7 \\ \frac{1}{3}\cdot 21 & \\ 7 \stackrel{?}{=} 7 & \text{TRUE} \end{array}$$

The solution is 11.

62. $$\begin{aligned} \frac{1}{5}(4x-1) &= 7 \\ \frac{5}{1}\cdot\frac{1}{5}(4x-1) &= 5\cdot 7 \\ 4x-1 &= 35 \\ 4x &= 36 \\ x &= 9 \end{aligned}$$

63. $$\begin{aligned} 7(2a-1) &= 21 \\ 14a-7 &= 21 && \text{Using the distributive law} \\ 14a &= 21+7 && \text{Adding 7} \\ 14a &= 28 \\ a &= 2 && \text{Dividing by 14} \end{aligned}$$

Check: $$\begin{array}{c|c} \multicolumn{2}{c}{7(2a-1)=21} \\ \hline 7(2\cdot 2-1) & 21 \\ 7(4-1) & \\ 7\cdot 3 & \\ 21 \stackrel{?}{=} 21 & \text{TRUE} \end{array}$$

The solution is 2.

64. $$\begin{aligned} 5(3-3t) &= 30 \\ 15-15t &= 30 \\ -15t &= 15 \\ t &= -1 \end{aligned}$$

65. We can write $11=11(x+1)$ as $11\cdot 1=11(x+1)$. Then $1=x+1$, or $x=0$. The solution is 0.

66. $$\begin{aligned} 9 &= 3(5x-2) \\ 9 &= 15x-6 \\ 15 &= 15x \\ 1 &= x \end{aligned}$$

67. $$\begin{aligned} 2(3+4m)-6 &= 48 \\ 6+8m-6 &= 48 \\ 8m &= 48 && \text{Combining like terms} \\ m &= 6 \end{aligned}$$

Check: $$\begin{array}{c|c} \multicolumn{2}{c}{2(3+4m)-6=48} \\ \hline 2(3+4\cdot 6)-6 & 48 \\ 2(3+24)-6 & \\ 2\cdot 27-6 & \\ 54-6 & \\ 48 \stackrel{?}{=} 48 & \text{TRUE} \end{array}$$

The solution is 6.

68. $$\begin{aligned} 3(5+3m)-8 &= 7 \\ 15+9m-8 &= 7 \\ 9m+7 &= 7 \\ 9m &= 0 \\ m &= 0 \end{aligned}$$

69. $2x=x+x$
$2x=2x$ TRUE
All real numbers are solutions.

70. $5x-x=x+3x$
$4x=4x$ TRUE
Identity; the solution set is all real numbers.

71. $$\begin{aligned} 2r+8 &= 6r+10 \\ 2r+8-10 &= 6r+10-10 \\ 2r-2 &= 6r && \text{Combining like terms} \\ -2r+2r-2 &= -2r+6r \\ -2 &= 4r \\ \frac{-2}{4} &= \frac{4r}{4} \\ -\frac{1}{2} &= r \end{aligned}$$

Check: $$\begin{array}{c|c} \multicolumn{2}{c}{2r+8=6r+10} \\ \hline 2\left(-\frac{1}{2}\right)+8 & 6\left(-\frac{1}{2}\right)+10 \\ -1+8 & -3+10 \\ \multicolumn{2}{c}{7 \stackrel{?}{=} 7 \quad \text{TRUE}} \end{array}$$

The solution is $-\frac{1}{2}$.

72. $$\begin{aligned} 3b-2 &= 7b+4 \\ 3b-6 &= 7b \\ -6 &= 4b \\ \frac{-3}{2} &= b \end{aligned}$$

73. $$\begin{aligned} 4y-4+y+24 &= 6y+20-4y \\ 5y+20 &= 2y+20 \\ 5y-2y &= 20-20 \\ 3y &= 0 \\ y &= 0 \end{aligned}$$

Check: $$\begin{array}{c|c} \multicolumn{2}{c}{4y-4+y+24=6y+20-4y} \\ \hline 4\cdot 0-4+0+24 & 6\cdot 0+20-4\cdot 0 \\ 0-4+0+24 & 0+20-0 \\ \multicolumn{2}{c}{20 \stackrel{?}{=} 20 \quad \text{TRUE}} \end{array}$$

The solution is 0.

74. $$\begin{aligned} 5y-10+y &= 7y+18-5y \\ 6y-10 &= 2y+18 \\ 4y &= 28 \\ y &= 7 \end{aligned}$$

75. $$\begin{aligned} 3(x+4) &= 3(x-1) \\ 3x+12 &= 3x-3 \\ 3x-3x+12 &= 3x-3x-3 \\ 12 &= -3 \quad \text{FALSE} \end{aligned}$$

There is no solution.

76.
$$\begin{aligned}5(x-7)&=3(x-2)+2x\\5x-35&=3x-6+2x\\5x-35&=5x-6\\5x-5x-35&=5x-5x-6\\-35&=-6\quad\text{FALSE}\end{aligned}$$
There is no solution.

77.
$$\begin{aligned}19-3(2x-1)&=7\\19-6x+3&=7\\22-6x&=7\\-6x&=7-22\\-6x&=-15\\x&=\frac{15}{6}\\x&=\frac{5}{2}\end{aligned}$$
Check: $19-3(2x-1)=7$
$$\begin{array}{r|l}19-3\left(2\cdot\frac{5}{2}-1\right) & 7\\19-3(5-1)&\\19-3(4)&\\19-12&\end{array}$$
$7\stackrel{?}{=}7$ TRUE

The solution is $\frac{5}{2}$.

78.
$$\begin{aligned}5(d+4)&=7(d-2)\\5d+20&=7d-14\\34&=2d\\17&=d\end{aligned}$$

79.
$$\begin{aligned}2(3t+1)-5&=t-(t+2)\\6t+2-5&=t-t-2\\6t-3&=-2\\6t&=-2+3\\6t&=1\\t&=\frac{1}{6}\end{aligned}$$
Check: $2(3t+1)-5=t-(t+2)$
$$\begin{array}{r|l}2\left(3\cdot\frac{1}{6}+1\right)-5 & \frac{1}{6}-\left(\frac{1}{6}+2\right)\\2\left(\frac{1}{2}+1\right)-5 & \frac{1}{6}-\frac{1}{6}-2\\2\cdot\frac{3}{2}-5 & -2\end{array}$$
$-2\stackrel{?}{=}-2$ TRUE

The solution is $\frac{1}{6}$.

80.
$$\begin{aligned}4x-(x+6)&=5(3x-1)+8\\4x-x-6&=15x-5+8\\3x-6&=15x+3\\3x-9&=15x\\-9&=12x\\-\frac{3}{4}&=x\end{aligned}$$

81.
$$\begin{aligned}19-(2x+3)&=2(x+3)+x\\19-2x-3&=2x+6+x\\16-2x&=3x+6\\16-6&=3x+2x\\10&=5x\\2&=x\end{aligned}$$
Check: $19-(2x+3)=2(x+3)+x$
$$\begin{array}{r|l}19-(2\cdot2+3) & 2(2+3)+2\\19-(4+3) & 2\cdot5+2\\19-7 & 10+2\end{array}$$
$12\stackrel{?}{=}12$ TRUE

The solution is 2.

82.
$$\begin{aligned}13-(2c+2)&=2(c+2)+3c\\13-2c-2&=2c+4+3c\\11-2c&=5c+4\\7&=7c\\1&=c\end{aligned}$$

83.
$$\begin{aligned}\frac{3}{4}(3t-4)&=15\\\frac{4}{3}\cdot\frac{3}{4}(3t-4)&=\frac{4}{3}\cdot15\\3t-4&=20\\3t&=24\\t&=8\end{aligned}$$
Check: $\frac{3}{4}(3t-4)=15$
$$\begin{array}{r|l}\frac{3}{4}(3\cdot8-4) & 15\\\frac{3}{4}\cdot(24-4)&\\\frac{3}{4}\cdot20&\end{array}$$
$15\stackrel{?}{=}15$ TRUE

The solution is 8.

84.
$$\begin{aligned}\frac{3}{2}(2x+5)&=-\frac{15}{2}\\\frac{2}{3}\cdot\frac{3}{2}(2x+5)&=\frac{2}{3}\left(-\frac{15}{2}\right)\\2x+5&=-5\\2x&=-10\\x&=-5\end{aligned}$$

85.
$$\begin{aligned}2\left(\frac{5}{3}-\frac{1}{12}x\right)&=-1\\2\cdot\frac{5}{3}-2\cdot\frac{1}{12}x&=-1\\\frac{10}{3}-\frac{1}{6}x&=-1\\6\left(\frac{10}{3}-\frac{1}{6}x\right)&=6(-1)\\6\cdot\frac{10}{3}-6\cdot\frac{1}{6}x&=-6\\20-x&=-6\\20-20-x&=-6-20\\-x&=-26\\\frac{-x}{-1}&=\frac{-26}{-1}\\x&=26\end{aligned}$$

Check:
$$2\left(\frac{5}{3}-\frac{1}{12}x\right)=-1$$
$$\begin{array}{r|l} 2\left(\frac{5}{3}-\frac{1}{12}\cdot 26\right) & -1 \\ 2\left(\frac{10}{6}-\frac{13}{6}\right) & \\ 2\left(-\frac{3}{6}\right) & \\ -1 \stackrel{?}{=} -1 & \text{TRUE} \end{array}$$

The solution is 26.

86.
$$\begin{aligned} 5\left(\frac{7}{10}x-\frac{1}{2}\right)&=-6 \\ \frac{7}{2}x-\frac{5}{2}&=-6 \\ 2\left(\frac{7}{2}x-\frac{5}{2}\right)&=2(-6) \\ 7x-5&=-12 \\ 7x&=-7 \\ x&=-1 \end{aligned}$$

87.
$$\begin{aligned} \frac{1}{6}\left(\frac{3}{4}x-2\right)&=-\frac{1}{5} \\ 30\cdot\frac{1}{6}\left(\frac{3}{4}x-2\right)&=30\left(-\frac{1}{5}\right) \\ 5\left(\frac{3}{4}x-2\right)&=-6 \\ \frac{15}{4}x-10&=-6 \\ \frac{15}{4}x&=4 \\ 4\cdot\frac{15}{4}x&=4\cdot 4 \\ 15x&=16 \\ x&=\frac{16}{15} \end{aligned}$$

Check:
$$\frac{1}{6}\left(\frac{3}{4}x-2\right)=-\frac{1}{5}$$
$$\begin{array}{r|l} \frac{1}{6}\left(\frac{3}{4}\cdot\frac{16}{15}-2\right) & -\frac{1}{5} \\ \frac{1}{6}\left(\frac{4}{5}-2\right) & \\ \frac{1}{6}\left(-\frac{6}{5}\right) & \\ -\frac{1}{5} \stackrel{?}{=} -\frac{1}{5} & \text{TRUE} \end{array}$$

The solution is $\frac{16}{15}$.

88.
$$\begin{aligned} \frac{2}{3}\left(\frac{7}{8}-4x\right)-\frac{5}{8}&=\frac{3}{8} \\ \frac{7}{12}-\frac{8}{3}x-\frac{5}{8}&=\frac{3}{8} \\ 14-64x-15&=9 \quad \text{Multiplying by 24} \\ -64x-1&=9 \\ -64x&=10 \\ x&=-\frac{10}{64} \\ x&=-\frac{5}{32} \end{aligned}$$

89.
$$\begin{aligned} 0.7(3x+6)&=1.1-(x-3) \\ 2.1x+4.2&=1.1-x+3 \\ 2.1x+4.2&=-x+4.1 \\ 10(2.1x+4.2)&=10(-x+4.1) \quad \text{Clearing decimals} \\ 21x+42&=-10x+41 \\ 21x&=-10x+41-42 \\ 21x&=-10x-1 \\ 31x&=-1 \\ x&=-\frac{1}{31} \end{aligned}$$

The check is left to the student. The solution is $-\frac{1}{31}$.

90.
$$\begin{aligned} 0.9(2x-8)&=4-(x+5) \\ 1.8x-7.2&=4-x-5 \\ 1.8x-7.2&=-1-x \\ 18x-72&=-10-10x \\ 28x&=62 \\ x&=\frac{31}{14} \end{aligned}$$

91.
$$\begin{aligned} 3(x+5)&=3(5+x) \\ 3x+3\cdot 5&=3\cdot 5+3x \\ 3x+15&=15+3x \quad \text{TRUE} \end{aligned}$$

All real numbers are solutions. The solution set is $\mathbb{R}$. The equation is an identity.

92.
$$\begin{aligned} 5(x-7)&=3(x-2)+2x \\ 5x-35&=3x-6+2x \\ 5x-35&=5x-6 \quad \text{FALSE} \end{aligned}$$

There is no solution. The solution set is $\varnothing$. The equation is a contradiction.

93.
$$\begin{aligned} 4(x-3)&=5(x+6) \\ 4x-4\cdot 3&=5x+5\cdot 6 \\ 4x-12&=5x+30 \\ 4x-12-30&=5x+30-30 \\ 4x-42&=5x \\ 4x-4x-42&=5x-4x \\ -42&=x \end{aligned}$$

The solution is –42. The solution set is {–42}. The equation is a conditional equation.

94.
$$\begin{aligned} 12-3(x+4)&=6(x+1) \\ 12-3x-12&=6x+6 \\ -3x&=6x+6 \\ -3x-6x&=6x+6-6x \\ -9x&=6 \\ x&=-\frac{2}{3} \end{aligned}$$

The solution is $-\frac{2}{3}$. The solution set is $\left\{-\frac{2}{3}\right\}$. The equation is a conditional equation.

95.
$$\begin{aligned} 4+7x&=7(x+1) \\ 7x+4&=7x+1 \quad \text{FALSE} \end{aligned}$$

There is no solution. The solution set is $\varnothing$. The equation is a contradiction.

96. $3(t+2)+t=2(3+2t)$
$$3t+3\cdot 2+t=2\cdot 3+2\cdot 2t$$
$$4t+6=6+4t \quad \text{TRUE}$$
All real numbers are solutions. The solution set is $\mathbb{R}$.
The equation is an identity.

97. $3-(x-2)=5(2x+1)$
$$3-x+2=5\cdot 2x+5\cdot 1$$
$$-x+5=10x+5$$
$$-x+5-5=10x+5-5$$
$$-x-10x=10x-10x$$
$$-11x=0$$
$$\frac{-11x}{-11}=\frac{0}{-11}$$
$$x=0$$
The solution is 0. The solution set is $\{0\}$.
The equation is a conditional equation.

98. $4(3x+2)=5-(2x-3)$
$$4\cdot 3x+4\cdot 2=5-2x+3$$
$$12x+8=-2x+8$$
$$12x+8-8=-2x+8-8$$
$$12x+2x=-2x+2x$$
$$14x=0$$
$$x=0$$
The solution is 0. The solution set is $\{0\}$.
The equation is a conditional equation.

99. $\frac{2}{5}x-\left(\frac{1}{5}-x\right)=4$
$$\frac{2}{5}x-\frac{1}{5}+x=4$$
$$\frac{7}{5}x-\frac{1}{5}+\frac{1}{5}=4+\frac{1}{5}$$
$$\frac{7}{5}x=\frac{21}{5}$$
$$\frac{5}{7}\cdot\frac{7}{5}x=\frac{5}{7}\cdot\frac{21}{5}$$
$$x=3$$
The solution is 3. The solution set is $\{3\}$.
The equation is a conditional equation.

100. $\frac{1}{3}-\left(5-\frac{2}{3}x\right)=4$
$$\frac{1}{3}-5+\frac{2}{3}x=4$$
$$\frac{2}{3}x-\frac{14}{3}=4$$
$$3\cdot\frac{2}{3}x-3\cdot\frac{14}{3}=3(4)$$
$$2x-14=12$$
$$2x-14+14=12+14$$
$$2x=26$$
$$x=13$$
The solution is 13. The solution set is $\{13\}$.
The equation is a conditional equation.

101. $a+(a-3)=(a+2)-(a+1)$
$$a+a-3=a+2-a-1$$
$$2a-3=1$$
$$2a=1+3$$
$$2a=4$$
$$a=2$$

Check: $a+(a-3)=(a+2)-(a+1)$

$2+(2-3)$	$(2+2)-(2+1)$
$2-1$	$4-3$

$$1 \stackrel{?}{=} 1 \quad \text{TRUE}$$
The solution is 2.

102. $0.8-4(b-1)=0.2+3(4-b)$
$$0.8-4b+4=0.2+12-3b$$
$$8-40b+40=2+120-30b$$
$$48-40b=122-30b$$
$$-74=10b$$
$$-7.4=b$$

103. *Writing Exercise*. By adding $t-13$ to both sides of $45-t=13$ we have $32=t$. This approach is preferable since we found the solution in just one step.

104. *Writing Exercise*. Since the rules for order of operations tell us to multiply (and divide) before we add (and subtract), we "undo" multiplications and additions in the opposite order when we solve equations. That is, we add or subtract first and then multiply or divide to isolate the variable.

105. $\frac{2}{9}+\frac{1}{6}=\frac{4}{18}+\frac{3}{18}=\frac{7}{18}$

106. 1

107.
$$\begin{array}{r} 0.111\ldots \\ 9\overline{)1.000} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$
$\frac{1}{9}=0.\overline{1}$, so $-\frac{1}{9}=-0.\overline{1}$.

108. 16

109. *Writing Exercise*. Multiply by 100 to clear decimals. Next multiply by 12 to clear fractions. (These steps could be reversed.) Then proceed as usual. The procedure could be streamlined by multiplying by 1200 to clear decimals and fractions in one step.

110. *Writing Exercise*. First multiply both sides of the equation by $\frac{1}{3}$ to "eliminate" the 3. Then proceed as shown:
$$3x+4=-11$$
$$\frac{1}{3}(3x+4)=\frac{1}{3}(-11)$$
$$x+\frac{4}{3}=-\frac{11}{3}$$
$$x=-\frac{15}{3}$$
$$x=-5$$

111. $8.43x - 2.5(3.2 - 0.7x) = -3.455x + 9.04$
$$8.43x - 8 + 1.75x = -3.455x + 9.04$$
$$10.18x - 8 = -3.455x + 9.04$$
$$10.18x + 3.455x = 9.04 + 8$$
$$13.635x = 17.04$$
$$x = 1.\overline{2497},\text{ or } \frac{1136}{909}$$
The solution is $1.\overline{2497}$, or $\frac{1136}{909}$.

112. Since we are using a calculator we will not clear the decimals.
$$0.008 + 9.62x - 42.8 = 0.944x + 0.0083 - x$$
$$9.62x - 42.792 = -0.056x + 0.0083$$
$$9.676x = 42.8003$$
$$x \approx 4.423346424$$

113. $-2[3(x-2)+4] = 4(5-x) - 2x$
$$-2[3x - 6 + 4] = 20 - 4x - 2x$$
$$-2[3x - 2] = 20 - 6x$$
$$-6x + 4 = 20 - 6x$$
$$4 = 20 \quad \text{Adding } 6x \text{ to both sides}$$
This is a contradiction. No solution.

114. $0 = t - (-6) - (-7t)$
$$0 = t + 6 + 7t$$
$$0 = 6 + 8t$$
$$-6 = 8t$$
$$-\frac{3}{4} = t$$

115. $2(7-x) - 20 = 7x - 3(2+3x)$
$$14 - 2x - 20 = 7x - 6 - 9x$$
$$-2x - 6 = -2x - 6 \quad \text{TRUE}$$
Identity; the solution set is all real numbers.

116. $5(x-7) = 3(x-2) + 2x$
$$5x - 35 = 3x - 6 + 2x$$
$$5x - 35 = 5x - 6$$
$$-35 = -6 \quad \text{FALSE}$$
Contradiction; there is no solution.

117. $2x(x+5) - 3(x^2 + 2x - 1) = 9 - 5x - x^2$
$$2x^2 + 10x - 3x^2 - 6x + 3 = 9 - 5x - x^2$$
$$-x^2 + 4x + 3 = 9 - 5x - x^2$$
$$4x + 3 = 9 - 5x \quad \text{Adding } x^2$$
$$4x + 5x = 9 - 3$$
$$9x = 6$$
$$x = \frac{2}{3}$$
The solution is $\frac{2}{3}$.

118. $9 - 3x = 2(5-2x) - (1-5x)$
$$9 - 3x = 10 - 4x - 1 + 5x$$
$$9 - 3x = 9 + x$$
$$9 - 9 = x + 3x$$
$$0 = 4x$$
$$0 = x$$
The solution is 0.

119. $[7 - 2(8 \div (-2))]x = 0$
Since $7 - 2(8 \div (-2)) \neq 0$ and the product on the left side of the equation is 0, then x must be 0.

120.
$$\frac{5x+3}{4} + \frac{25}{12} = \frac{5+2x}{3}$$
$$12\left(\frac{5x+3}{4} + \frac{25}{12}\right) = 12\left(\frac{5+2x}{3}\right)$$
$$12\left(\frac{5x+3}{4}\right) + 12 \cdot \frac{25}{12} = 4(5+2x)$$
$$3(5x+3) + 25 = 4(5+2x)$$
$$15x + 9 + 25 = 20 + 8x$$
$$15x + 34 = 20 + 8x$$
$$7x = -14$$
$$x = -2$$
The solution is –2.

121. Let x represent the number of miles.
Translating we have:
$$\frac{3}{200}x + \frac{1}{4}(2) = 8$$
Solve the equation for x.
$$\frac{3}{200}x + \frac{1}{4}(2) = 8$$
$$\frac{3}{200}x + \frac{1}{2} = 8$$
$$200\left(\frac{3}{200}x + \frac{1}{2}\right) = 200(8)$$
$$3x + 100 = 1600$$
$$3x = 1500$$
$$\frac{3x}{3} = \frac{1500}{3}$$
$$x = 500$$
He will drive 500 miles.

Exercise Set 2.3

1. False. For example, π represents a constant.
2. True
3. The distance around a circle is its *circumference*.
4. An equation that uses two or more letters to represent a relationship among quantities is a *formula*.
5. We substitute 60 for t and calculate d.
$$d = 1.6t = 1.6 \cdot 60 = 96$$
The optimal viewing distance is 96 in., or 8 ft.
6. $B = 30 \cdot 1800 = 54{,}000$ Btu's.
7. We substitute 21,345 for n and calculate f.
$$f = \frac{n}{15} = \frac{21{,}345}{15} = 1423$$
There are 1423 full-time equivalent students.
8. $M = \frac{1}{5} \cdot 10 = 2$ mi.

9. We substitute 13,500 for c, 1 for r and 50 for L and calculate f.

$$\begin{aligned} f &= \frac{2r+c}{2L} \\ &= \frac{2(1)+13{,}500}{2(50)} \\ &= \frac{13{,}502}{100} \\ &= 135.02 \end{aligned}$$

The frequency is about 135 hertz.

10. $r = \frac{tmap}{hs} = \frac{512(0.05)(100)(0.15)}{4(30)} = 3.2$

11. Substitute 1 for t and calculate n.

$$\begin{aligned} n &= 0.5t^4 + 3.45t^3 - 96.65t^2 + 347.7t \\ &= 0.5(1)^4 + 3.45(1)^3 - 96.65(1)^2 + 347.7(1) \\ &= 0.5 + 3.45 - 96.65 + 347.7 \\ &= 255 \end{aligned}$$

255 mg of ibuprofen remain in the bloodstream.

12. $N = 7^2 - 7 = 49 - 7 = 42$ games

13. $A = bh$

$\frac{A}{h} = \frac{bh}{h}$ Dividing both sides by h

$\frac{A}{h} = b$

14. $\frac{A}{b} = h$

15. $I = Prt$

$\frac{I}{rt} = \frac{Prt}{rt}$ Dividing both sides by rt

$\frac{I}{rt} = P$

16. $\frac{I}{Pr} = t$

17. $H = 65 - m$

$H + m = 65$ Adding m to both sides

$m = 65 - H$ Subtracting H from both sides

18. $g = 216 - a$

19. $P = 2l + 2w$

$P - 2w = 2l + 2w - 2w$ Subtracting $2w$ from both sides

$P - 2w = 2l$

$\frac{P-2w}{2} = \frac{2l}{2}$ Dividing both sides by 2

$\frac{P-2w}{2} = l$, or $\frac{P}{2} - w = l$

20. $P = 2l + 2w$

$P - 2l = 2w$

$\frac{P-2l}{2} = w$, or

$\frac{P}{2} - l = w$

21. $A = \pi r^2$

$\frac{A}{r^2} = \frac{\pi r^2}{r^2}$

$\frac{A}{r^2} = \pi$

22. $\frac{A}{\pi} = r^2$

23. $A = \frac{1}{2}bh$

$2A = 2 \cdot \frac{1}{2}bh$ Multiplying both sides by 2

$2A = bh$

$\frac{2A}{b} = \frac{bh}{b}$ Dividing both sides by h

$\frac{2A}{b} = h$

24. $A = \frac{1}{2}bh$

$2A = bh$

$\frac{2A}{h} = b$

25. $E = mc^2$

$\frac{E}{m} = \frac{mc^2}{m}$ Dividing both sides by m

$\frac{E}{m} = c^2$

26. $\frac{E}{c^2} = m$

27. $Q = \frac{c+d}{2}$

$2Q = 2 \cdot \frac{c+d}{2}$ Multiplying both sides by 2

$2Q = c + d$

$2Q - c = c + d - c$ Subtracting c from both sides

$2Q - c = d$

28. $A = \frac{a+b+c}{3}$

$3A = 3 \cdot \frac{a+b+c}{3}$

$3A = a + b + c$

$3A - a - c = a + b + c - a - c$ Subtracting a and c from both sides

$3A - a - c = b$

29. $p - q + r = 2$

$p + r = 2 + q$

$p + r - 2 = q$

30. $p = \frac{r-q}{2}$

$2 \cdot p = 2 \cdot \frac{r-q}{2}$ Multiplying both sides by 2

$2p = r - q$

$q + 2p = r$

$q = r - 2p$

31. $w = \frac{r}{f}$
$f \cdot w = f \cdot \frac{r}{f}$ Multiplying both sides by f
$fw = r$

32. $D = \frac{c}{w}$
$w \cdot D = w \cdot \frac{c}{w}$ Multiplying both sides by w
$Dw = c$

33. $H = \frac{TV}{550}$
$\frac{550}{V} \cdot H = \frac{550}{V} \cdot \frac{TV}{550}$ Multiplying both sides by $\frac{550}{V}$
$\frac{550H}{V} = T$

34. $P = \frac{ab}{c}$
$\frac{c}{a} \cdot P = \frac{c}{a} \cdot \frac{ab}{c}$
$\frac{Pc}{a} = b$

35. $2x - y = 1$
$2x - y + y - 1 = 1 + y - 1$ Adding $y-1$ to both sides
$2x - 1 = y$

36. $3x - y = 7$
$3x - 7 = y$

37. $2x + 5y = 10$
$5y = -2x + 10$
$y = \frac{-2x+10}{5}$, or $y = -\frac{2}{5}x + 2$

38. $3x + 2y = 12$
$2y = -3x + 12$
$y = \frac{-3x+12}{2}$, or $y = -\frac{3}{2}x + 6$

39. $4x - 3y = 6$
$-3y = -4x + 6$
$y = \frac{-4x+6}{-3}$, or $y = \frac{4}{3}x - 2$

40. $5x - 4y = 8$
$-4y = -5x + 8$
$y = \frac{-5x+8}{-4}$, or $y = \frac{5}{4}x - 2$

41. $9x + 8y = 4$
$8y = -9x + 4$
$y = \frac{-9x+4}{8}$, or $y = -\frac{9}{8}x + \frac{1}{2}$

42. $x + 10y = 2$
$10y = -x + 2$
$y = \frac{-x+2}{10}$, or $y = -\frac{1}{10}x + \frac{1}{5}$

43. $3x - 5y = 8$
$-5y = -3x + 8$
$y = \frac{-3x+8}{-5}$, or $y = \frac{3}{5}x - \frac{8}{5}$

44. $7x - 6y = 7$
$-6y = -7x + 7$
$y = \frac{-7x+7}{-6}$, or $y = \frac{7}{6}x - \frac{7}{6}$

45. $R = \frac{3}{4}r + 25$
$R - 25 = \frac{3}{4}r$
$r = \frac{4}{3}(R - 25)$, or $r = \frac{4R-100}{3}$

46. $M = \frac{5}{9}n + 18$
$M - 18 = \frac{5}{9}n$
$\frac{9}{5}(M - 18) = n$

47. $z = 13 + 2(x + y)$
$z - 13 = 2(x + y)$
$z - 13 = 2x + 2y$
$z - 13 - 2y = 2x$
$\frac{z - 13 - 2y}{2} = x$
$\frac{1}{2}z - \frac{13}{2} - y = x$

48. $A = 115 + \frac{1}{2}(p + s)$
$A - 115 = \frac{1}{2}(p + s)$
$2(A - 115) = p + s$
$2(A - 115) - p = s$

49. $t = 27 - \frac{1}{4}(w - l)$
$t - 27 = -\frac{1}{4}(w - l)$
$-4(t - 27) = w - l$ Multiplying by -4
$-4(t - 27) = w - l$
$-4(t - 27) - w = -l$
$4(t - 27) + w = l$ Multiplying by -1

50. $m = 19 - 5(x - n)$
$m - 19 = -5(x - n)$
$-\frac{1}{5}(m - 19) = x - n$
$-\frac{1}{5}(m - 19) - x = -n$
$\frac{m-19}{5} + x = n$, or $n = \frac{m - 19 + 5x}{5}$

51. $A = at + bt$
$A = t(a + b)$ Factoring
$\frac{A}{a+b} = t$ Dividing both sides by $a + b$

52.
$$S = rx + sx$$
$$S = x(r+s)$$
$$\frac{S}{r+s} = x$$

53.
$$A = \frac{1}{2}ah + \frac{1}{2}bh$$
$$2A = 2\left(\frac{1}{2}ah + \frac{1}{2}bh\right)$$
$$2A = ah + bh$$
$$2A = h(a+b)$$
$$\frac{2A}{a+b} = h$$

54.
$$A = P + Prt$$
$$A = P(1+rt)$$
$$\frac{A}{1+rt} = P$$

55.
$$R = r + \frac{400(W-L)}{N}$$
$$N \cdot R = N\left(r + \frac{400(W-L)}{N}\right) \quad \text{Multiplying both sides by } N$$
$$NR = Nr + 400(W-L)$$
$$NR = Nr + 400W - 400L$$
$$NR + 400L = Nr + 400W \quad \text{Adding } 400L \text{ to both sides}$$
$$400L = Nr + 400W - NR \quad \text{Adding } -NR \text{ to both sides}$$
$$L = \frac{Nr + 400W - NR}{400}, \text{ or } L = W - \frac{N(R-r)}{400}$$

56.
$$S = \frac{360A}{\pi r^2}$$
$$Sr^2 = \frac{360A}{\pi}$$
$$r^2 = \frac{360A}{\pi S}$$

57. *Writing Exercise*. Given the formula for converting inches to centimeters, solve for inches. This yields a formula for converting centimeters to inches.

58. *Writing Exercise*. Answers may vary. A person who knows the interest rate, the amount of interest to earn, and how long money is in the bank wants to know how much money to invest.

59. $(-2)^5 = -32$

60. $-98 \div \frac{1}{2} = -196$

61. $4.2(-11.75)(0) = 0$

62. $-2+5-(-4)-17 = -2+5+4-17$
$= 3+4-17 = 7-17 = -10$

63. $20 \div (-4) \cdot 2 - 3$
$= -5 \cdot 2 - 3$ Dividing and
$= -10 - 3$ multiplying from left to right
$= -13$ Subtracting

64. $5|8-(2-7)| = 5|8-(-5)| = 5|13| = 5 \cdot 13 = 65$

65. *Writing Exercise*. Answers may vary. A decorator wants to have a carpet cut for a bedroom. The perimeter of the room is 54 ft and its length is 15 ft. How wide should the carpet be?

66. *Writing Exercise*. Since h occurs on both sides of the formula, Eva has not solved the formula for h. The letter being solved for should be alone on one side of the equation with no occurrence of that letter on the other side.

67.
$$g = 0.0778n + 4.55s - 2.2029$$
$$3.8 = 0.0778(5) + 4.55s - 2.2029$$
$$3.8 = 0.389 + 4.55s - 2.2029$$
$$3.8 = 4.55s - 1.8139$$
$$5.6139 = 4.55s$$
$$1.23 \approx s$$
The average number of syllables per word should be about 1.23.

68. To find the number of 100 meter rises in h meters we divide: $\frac{h}{100}$. Then
$$T = t - \frac{h}{100}$$
Note that $12 \text{ km} = 12 \text{ km} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 12{,}000 \text{ m}$.

Thus, we have
$$T = t - \frac{h}{100}, \; 0 \le h \le 12{,}000.$$

69. First we substitute 54 for A and solve for s to find the length of a side of the cube.
$$A = 6s^2$$
$$54 = 6s^2$$
$$9 = s^2$$
$$3 = s \quad \text{Taking the positive square root}$$
Now we substitute 3 for s in the formula for the volume of a cube and compute the volume.
$$V = s^3 = 3^3 = 27$$
The volume of the cube is 27 in^3.

70.
$$8 \text{ ft} = 96 \text{ in.}$$
$$700 = \frac{96g^2}{800}$$
$$560{,}000 = 96g^2$$
$$\frac{560{,}000}{96} = g^2$$
$$76.4 \approx g$$
The girth is about 76.4 in.

71.
$$c = \frac{w}{a} \cdot d$$
$$ac = a \cdot \frac{w}{a} \cdot d$$
$$ac = wd$$
$$a = \frac{wd}{c}$$

72. $\frac{y}{z} \div \frac{z}{t} = 1$
$\frac{y}{z} \cdot \frac{t}{z} = 1$
$\frac{yt}{z^2} = 1$
$\frac{z^2}{t} \cdot \frac{yt}{z^2} = \frac{z^2}{t} \cdot 1$
$y = \frac{z^2}{t}$

73. $ac = bc + d$
$ac - bc = d$
$c(a-b) = d$
$c = \frac{d}{a-b}$

74. $qt = r(s+t)$
$qt = rs + rt$
$qt - rt = rs$
$t(q-r) = rs$
$t = \frac{rs}{q-r}$

75. $3a = c - a(b+d)$
$3a = c - ab - ad$
$3a + ab + ad = c$
$a(3+b+d) = c$
$a = \frac{c}{3+b+d}$

76. We subtract the minimum output for a well-insulated house with a square feet from the minimum output for a poorly-insulated house with a square feet. Let S represent the number of BTU's saved.
$S = 50a - 30a$
$S = 20a$

Mid-Chapter Review

1. $2x + 3 - 3 = 10 - 3$
$2x = 7$
$\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 7$
$x = \frac{7}{2}$
The solution is $\frac{7}{2}$.

2. $6 \cdot \frac{1}{2}(x-3) = 6 \cdot \frac{1}{3}(x-4)$
$3(x-3) = 2(x-4)$
$3x - 9 + 9 = 2x - 8 + 9$
$3x - 9 = 2x - 8$
$3x = 2x + 1$
$3x - 2x = 2x + 1 - 2x$
$x = 1$

3. $x - 2 = -1$
$x - 2 + 2 = -1 + 2$
$x = 1$
The solution is 1.

4. $2 - x = -1$
$-x = -3$
$-1(-x) = -1(-3)$
$x = 3$

5. $3t = 5$
$\frac{3t}{3} = \frac{5}{3}$
$t = \frac{5}{3}$
The solution is $\frac{5}{3}$.

6. $-\frac{3}{2}x = 12$
$-\frac{2}{3}\left(-\frac{3}{2}x\right) = -\frac{2}{3} \cdot 12$
$x = -8$

7. $\frac{y}{8} = 6$
$8 \cdot \frac{y}{8} = 8 \cdot 6$
$y = 48$
The solution is 48.

8. $0.06x = 0.03$
$\frac{0.06x}{0.06} = \frac{0.03}{0.06}$
$x = 0.5$

9. $3x - 7x = 20$
$-4x = 20$
$\frac{-4x}{-4} = \frac{20}{-4}$
$x = -5$
The solution is –5.

10. $9x - 7 = 17$
$9x = 24$
$x = \frac{8}{3}$

11. $4(t-3) - t = 6$
$4t - 12 - t = 6$
$3t - 12 = 6$
$3t - 12 + 12 = 6 + 12$
$3t = 18$
$\frac{3t}{3} = \frac{18}{3}$
$t = 6$
The solution is 6.

12. $8n - (3n - 5) = 5 - n$
$8n - 3n + 5 = 5 - n$
$5n + 5 = 5 - n$
$6n + 5 = 5$
$6n = 0$
$n = 0$

13. $$\frac{9}{10}y-\frac{7}{10}=\frac{21}{5}$$
$$10\left(\frac{9}{10}y-\frac{7}{10}\right)=10\left(\frac{21}{5}\right)$$
$$9y-7=42$$
$$9y-7+7=42+7$$
$$9y=49$$
$$\frac{9y}{9}=\frac{49}{9}$$
$$y=\frac{49}{9}$$
The solution is $\frac{49}{9}$.

14. $$2(t-5)-3(2t-7)=12-5(3t+1)$$
$$2t-10-6t+21=12-15t-5$$
$$-4t+11=7-15t$$
$$11t+11=7$$
$$11t=-4$$
$$t=-\frac{4}{11}$$

15. $$\frac{2}{3}(x-2)-1=-\frac{1}{2}(x-3)$$
$$6\cdot\left(\frac{2}{3}(x-2)-1\right)=6\cdot\left(-\frac{1}{2}(x-3)\right)$$
$$4(x-2)-6=-3(x-3)$$
$$4x-8-6=-3x+9$$
$$4x-14=-3x+9$$
$$4x-14+3x=-3x+9+3x$$
$$7x-14=9$$
$$7x=9+14$$
$$7x=23$$
$$\frac{7x}{7}=\frac{23}{7}$$
$$x=\frac{23}{7}$$
The solution is $\frac{23}{7}$.

16. $$E=wA$$
$$\frac{E}{w}=A$$

17. $$Ax+By=C$$
$$By=C-Ax$$
$$\frac{By}{B}=\frac{C-Ax}{B}$$
$$y=\frac{C-Ax}{B}$$

18. $$at+ap=m$$
$$a(t+p)=m$$
$$a=\frac{m}{t+p}$$

19. $$m=\frac{F}{a}$$
$$a\cdot m=a\cdot\frac{F}{a}$$
$$am=F$$
$$\frac{am}{m}=\frac{F}{m}$$
$$a=\frac{F}{m}$$

20. $$v=\frac{b-f}{t}$$
$$t\cdot v=t\cdot\frac{b-f}{t}$$
$$tv=b-f$$
$$tv+f=b$$

Exercise Set 2.4

1. To convert from percent notation to decimal notation, move the decimal point two places to the *left* and drop the percent symbol.
2. The percent symbol, %, means "per *hundred*."
3. The expression 1.3% is written in *percent* notation.
4. The word "of" in a percent problem generally refers to the *base* amount.
5. The *sale* price is the original price minus the discount.
6. The symbol ≈ means "is *approximately* equal to."
7. "What percent of 57 is 23?" can be translated as $n\cdot 57=23$, so choice (d) is correct.
8. "What percent of 23 is 57?" can be translated as $n\cdot 23=57$, so choice (c) is correct.
9. "23 is 57% of what number?" can be translated as $23=0.57y$, so choice (e) is correct.
10. "57 is 23% of what number?" can be translated as $57=0.23y$, so choice (b) is correct.
11. "57 is what percent of 23?" can be translated as $n\cdot 23=57$, so choice (c) is correct.
12. "23 is what percent of 57?" can be translated as $n\cdot 57=23$, so choice (d) is correct.
13. "What is 23% of 57?" can be translated as $a=(0.23)57$, so choice (f) is correct.
14. "What is 57% of 23?" can be translated as $a=(0.57)23$, so choice (a) is correct.
15. "23% of what number is 57?" can be translated as $57=0.23y$, so choice (b) is correct.
16. "57% of what number is 23?" can be translated as $23=0.57y$, so choice (e) is correct.

17. $38\% = 38.0\%$
38% 0.38.0
Move the decimal point 2 places to the left.
$38\% = 0.38$

18. $23\% = 0.23$

19. $5\% = 5.0\%$
5% 0.05.0
Move the decimal point 2 places to the left.
$5\% = 0.05$

20. $3\% = 0.03$

21. $2.5\% = 2.50\%$
2.5% 0.02.50
Move the decimal point 2 places to the left.
$2.5\% = 0.025$

22. $11.1\% = 0.111$

23. $40\% = 40.0\%$
40% 0.40.0
Move the decimal point 2 places to the left.
$40\% = 0.40$, or 0.4

24. $10\% = 0.10$, or 0.1

25. 6.25% 0.06.25
Move the decimal point 2 places to the left.
$6.25\% = 0.0625$

26. $8.375\% = 0.08375$

27. 0.2% 0.00.2
Move the decimal point 2 places to the left.
$0.2\% = 0.002$

28. $0.8\% = 0.008$

29. $175\% = 175.0\%$ 1.75.0
Move the decimal point 2 places to the left.
$175\% = 1.75$

30. $250\% = 2.50$, or 2.5

31. 0.37
First move the decimal point two places to the right; then write a % symbol: 0.37. → 37%

32. $0.51 = 51\%$

33. 0.076
First move the decimal point two places to the right; then write a % symbol: 0.07.6 → 7.6%

34. $0.047 = 4.7\%$

35. 0.7
First move the decimal point two places to the right; then write a % symbol: 0.70. → 70%

36. $0.01 = 10\%$

37. 0.0009
First move the decimal point two places to the right; then write a % symbol: 0.00.09 → 0.09%

38. $0.0056 = 0.56\%$

39. 1.06
First move the decimal point two places to the right; then write a % symbol: 1.06. → 106%

40. $1.08 = 108\%$

41. $\frac{3}{5}$ $\left(\text{Note: } \frac{3}{5} = 0.6\right)$
Move the decimal point two places to the right; then write a % symbol: 0.60. → 60%

42. $\frac{3}{2} = 1.50 = 150\%$

43. $\frac{8}{25}$ $\left(\text{Note: } \frac{8}{25} = 0.32\right)$
First move the decimal point two places to the right; then write a % symbol: 0.32. → 32%

44. $\frac{5}{8} = 0.625 = 62.5\%$

45. ***Translate***.

$$\begin{array}{ccccc} \underbrace{\text{What percent}} & \text{of} & 76 & \text{is} & 19? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ y & \cdot & 76 & = & 19 \end{array}$$

We solve the equation and then convert to percent notation.

$$y \cdot 76 = 19$$
$$y = \frac{19}{76}$$
$$y = 0.25 = 25\%$$

The answer is 25%.

46. Solve and convert to percent notation:

$$x \cdot 125 = 30$$
$$x = 0.24 = 24\%$$

47. ***Translate***.

$$\begin{array}{ccccc} 14 & \text{is} & 30\% & \text{of} & \underbrace{\text{what number}}? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 14 & = & 30\% & \cdot & y \end{array}$$

We solve the equation.

$$14 = 0.3y \quad (30\% = 0.3)$$
$$\frac{14}{0.3} = y$$
$$46.\overline{6} = y$$

The answer is $46.\overline{6}$, or $46\frac{2}{3}$, or $\frac{140}{3}$.

48. Solve: $54 = 24\% \cdot x$
$225 = x$

49. ***Translate***.

$$\begin{array}{ccccc} 0.3 & \text{is} & 12\% & \text{of} & \underbrace{\text{what number}}? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 0.3 & = & 12\% & \cdot & y \end{array}$$

We solve the equation.

$$0.3 = 0.12y \quad (12\% = 0.12)$$
$$\frac{0.3}{0.12} = y$$
$$2.5 = y$$

The answer is 2.5.

50. Solve: $7 = 175\% \cdot x$
$4 = x$

51. ***Translate***.

$$\begin{array}{ccccc} \underbrace{\text{What number}} & \text{is} & 1\% & \text{of} & \text{one million}? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ y & = & 1\% & \cdot & 1{,}000{,}000 \end{array}$$

We solve the equation.

$$y = 0.01 \cdot 1{,}000{,}000 \quad (1\% = 0.01)$$
$$y = 10{,}000 \quad \text{Multiplying}$$

The answer is 10,000.

52. Solve: $x = 35\% \cdot 240$
$x = 84$

53. ***Translate***.

$$\begin{array}{ccccc} \underbrace{\text{What percent}} & \text{of} & 60 & \text{is} & 75? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ y & \cdot & 60 & = & 75 \end{array}$$

We solve the equation and then convert to percent notation.

$$y \cdot 60 = 75$$
$$y = \frac{75}{60}$$
$$y = 1.25 = 125\%$$

The answer is 125%.

54. Any number is 100% of itself, so 70 is 100% of 70. We could also do this exercise as follows: Solve and convert to percent notation:

$$x \cdot 70 = 70$$
$$x = 1 = 100\%$$

55. ***Translate***.

$$\begin{array}{ccccc} \text{What} & \text{is} & 2\% & \text{of} & 40? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ x & = & 2\% & \cdot & 40 \end{array}$$

We solve the equation.

$$x = 0.02 \cdot 40 \quad (2\% = 0.02)$$
$$x = 0.8 \quad \text{Multiplying}$$

The answer is 0.8.

56. Solve: $z = 40\% \cdot 2$
$z = 0.8$

57. Observe that 25 is half of 50. Thus, the answer is 0.5, or 50%. We could also do this exercise by translating to an equation.

Translate.

$$\begin{array}{ccccc} 25 & \text{is} & \underbrace{\text{what percent}} & \text{of} & 50? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 25 & = & y & \cdot & 50 \end{array}$$

We solve the equation and convert to percent notation.

$$25 = y \cdot 50$$
$$\frac{25}{50} = y$$
$$0.5 = y, \text{ or } 50\% = y$$

The answer is 50%.

58. Solve: $0.8 = 2\% \cdot x$
$40 = x$

59. ***Translate***.

$$\begin{array}{ccccc} \text{What percent} & \text{of} & 69 & \text{is} & 23? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ y & \cdot & 69 & = & 23 \end{array}$$

We solve the equation and convert to percent notation.

$$y \cdot 69 = 23 y = \frac{23}{69} y = 0.33\overline{3} = 33.\overline{3}\% \text{ or } 33\frac{1}{3}\%$$

The answer is $33.\overline{3}\%$ or $33\,1/3\%$.

60. Solve: $x \cdot 40 = 9$
$x = 0.225 = 22.5\%$

61. First we reword and translate, letting a represent users aged 18–29 years, in millions.

$$\begin{array}{ccccc} \text{What} & \text{is} & 39\% & \text{of} & 80? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ a & = & 0.39 & \cdot & 80 \end{array}$$

$a = 0.39 \cdot 80 = 31.2$

There are about 31.2 million users aged 18–29 years.

62. Solve: $a = 0.30 \cdot 80$
$a = 24$ million users aged 30-39 years

63. First we reword and translate, letting a represent users aged 40–49 years, in millions.

$$\begin{array}{ccccc} \text{What} & \text{is} & 18\% & \text{of} & 80? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ a & = & 0.18 & \cdot & 80 \end{array}$$

$a = 0.18 \cdot 80 = 14.4$

There are about 14.4 million users aged 40–49 years.

64. Solve: $a = 0.12 \cdot 80$
$a = 9.6$ million users aged 50-64 years

65. First we reword and translate, letting c represent the number of credits Rishi has completed.

$$\begin{array}{ccccc} \text{What} & \text{is} & 60\% & \text{of} & 125? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ c & = & 0.6 & \cdot & 125 \end{array}$$

$c = 0.6 \cdot 125 = 75$

Rishi has completed 75 credits.

66. Solve: $c = 0.2 \cdot 125$
$c = 25$ credits

67. First we reword and translate, letting b represent the number of at-bats.

$$\begin{array}{ccccc} 169 & \text{is} & 31.9\% & \text{of} & \underbrace{\text{what number}}? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 169 & = & 0.319 & \cdot & b \end{array}$$

$\dfrac{169}{0.319} = b$

$530 \approx b$

Yuli Gurriel had 530 at-bats.

68. Solve: $246 = 64.4\% \cdot p$
$p \approx 382$ attempts

69. a. First we reword and translate, letting p represent the unknown percent.

$$\begin{array}{ccccc} \underbrace{\text{What percent}} & \text{of} & \$25 & \text{is} & \$4? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ p & \cdot & 25 & = & 4 \end{array}$$

$\dfrac{p \cdot 25}{25} = \dfrac{4}{25}$

$p = 0.16 = 16\%$

The tip was 16% of the cost of the meal.

b. We add to find the total cost of the meal, including tip:
$\$25 + \$4 = \$29$

70. a. Solve: $12.76 = p \cdot 58$
$0.22 = p$

The tip was 22% of the meal's cost.

b. $\$58 + \$12.76 = \$70.76$

71. To find the percent of households that included a dog, we first reword and translate, letting d represent the unknown percent.

$$\begin{array}{ccccc} \underbrace{\text{38 million}} & \text{is} & \underbrace{\text{what percent}} & \text{of} & \underbrace{\text{87 million}}? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 38 & = & d & \cdot & 87 \end{array}$$

$\dfrac{38}{87} = d$

$0.437 \approx d$

$43.7\% \approx d$

About 43.7% of households included a dog.

To find the percent of households without a dog, we subtract:
$100\% - 43.7\% = 56.3\%$

About 56.3% of households did not include a dog.

72. Solve: $890.97 = t \cdot 1368.51$
$0.651 = t$

About 65.1% of trips were 5 miles or less. We subtract to find what percent were 5 miles or more.
$100\% - 65.1\% = 34.9\%$

About 91.1% of students were enrolled in other schools.

73. Let $I =$ the amount of interest Vio will pay. Then we have:

$$\begin{array}{ccccc} I & \text{is} & 6.8\% & \text{of} & \$2400. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ I & = & 0.068 & \cdot & \$2400 \\ I & = & \$163.20 & & \end{array}$$

Vio will pay \$163.20 interest.

74. Let $I =$ the amount of interest Brianna will pay.
Solve: $I = 4.50\% \cdot \$3500$
$I = \$157.50$

75. A self-employed person must earn 120% as much as a non-self-employed person. Let $a =$ the amount Tia would need to earn, in dollars per hour, on her own for a comparable income.

$$\begin{array}{ccccc} a & \text{is} & 120\% & \text{of} & \$24. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ a & = & 1.2 & \cdot & 24 \\ a & = & 28.80 & & \end{array}$$

Tia would need to earn \$28.80 per hour on her own.

76. Let $a =$ the amount Rik would need to earn, in dollars per hour, on his own for a comparable income.
Solve: $a = 1.2(\$28)$
$a = \$33.60$ per hour

77. We reword and translate.

$$\begin{array}{ccccc} \underbrace{\text{What percent}} & \text{of} & 2.6 & \text{is} & 12? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ p & \cdot & 2.6 & = & 12 \end{array}$$

$p \cdot 2.6 = 12$

$p \approx 462 = 462\%$

The actual cost exceeds the initial estimate by about 462%.

78. Solve: $p \cdot 20.91 = 0.75$
$p \approx 0.036$

The short course record is faster by about 3.6%.

79. First we reword and translate.

$$\begin{array}{ccccc} \text{What} & \text{is} & 25.1\% & \text{of} & 4049.2? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ a & = & 0.251 & \cdot & 4049.2 \end{array}$$

Solve. Multiply.

$a = 0.251 \cdot 4049.2$

$a = 1016.3492 \approx 1016.3$

About 1016.3 thousand associate degrees were awarded.

80. Let a = the area of Arizona.
Solve: $a = 19\% \cdot 586{,}400$
$a = 111{,}416 \text{ mi}^2$

81. Let m = the number of e-mails that are spam and viruses. Then we have:

$$\begin{array}{ccccc} \underbrace{\text{What percent}} & \text{of} & 320 & \text{is} & 145? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ p & \cdot & 320 & = & 145 \end{array}$$

$$p = \frac{145}{320}$$
$$p \approx 0.453 = 45.3\%$$

About 45.3% of e-mail is spam and viruses.

82. Let p = the percent of undiagnosed diabetics.
Solve: $8.5 = p \cdot 37.3$
$p \approx 0.228$, or about 22.8%

83. Let n = number of people who had 4 or more e-mail addresses.
Then we have:

$$\begin{array}{ccccccc} \underbrace{\text{people}} & \text{is} & 91.8\% & \text{of} & 28\% & \text{of} & 302 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ n & = & 0.918 & \cdot & 0.28 & \cdot & 302 \end{array}$$

$n = 0.918 \cdot 0.28 \cdot 302$
$n \approx 77.6$
About 77.6 million people in the United States had 4 or more e-mail addresses.

84. Let n = the number of scholarships awarded.
Solve: $n = 0.114 \cdot 0.404 \cdot 634{,}852$
$n \approx 29{,}239$ scholarships

85. Let c represent the average cost of brewing a cup of coffee at home.

$$\begin{array}{ccccc} \$4.90 & \text{is} & 445\% & \text{of} & \underbrace{\text{what number}}? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4.90 & = & 4.45 & \cdot & c \end{array}$$

$$\frac{4.90}{4.45} = c$$
$$1.10 \approx c$$

The average cost is \$1.10.

86. Let s = the sodium content, in mg, in a serving of the dry roasted peanuts.
Solve: $95 = 50\% \cdot s$
$s \approx 190$ mg

87. a. In the survey report, 40% of all sick days on Monday or Friday sounds excessive. However, for a traditional 5-day business week, 40% is the same as $\frac{2}{5}$. That is, just 2 days out of 5.

b. In the FBI statistics, 26% of home burglaries occurring between Memorial Day and Labor Day sounds excessive. However, 26% of a 365-day year is 73 days, For the months of June, July, and August there are at least 90 days. So 26% is less than the rate for other times during the year, or less than expected for a 90-day period.

88. *Writing Exercise*. \$12 is $13\frac{1}{3}\%$ of \$90. Gavin would be considered to be stingy, since the standard tip rate is 15% to 20%.

89. The opposite of $-\frac{1}{3}$ is $\frac{1}{3}$.

90. –3

91. $-(-(-12)) = -12$

92. $(-3x)^2 = 9x^2$

93. *Writing Exercise*. The book is marked up \$30. Since Campus Bookbuyers paid \$30 for the book, this is a 100% markup.

94. *Writing Exercise*. No, the increase is not the same as the decrease because of the bases for the percents are different.

For example, if the salary is \$50,000, then a 10% decrease is 50,000(0.10) = \$5000, resulting in a salary of \$50,000 – \$5000 = \$45,000.

An increase of 10% of the salary of \$45,000 is 45,000(0.10) = \$4500, resulting in a salary of \$45,000 + 4500 = \$49,000, which is less than the original salary of \$50,000.

95. *Writing Exercise*. The ending salary is the same either way. If s is the original salary, the new salary after a 5% raise followed by an 8% raise is $1.08(1.05s)$. If the raises occur in the opposite order, the new salary is $1.05(1.08s)$. It would be better to receive the 8% raise first, because this increase yields a higher new salary the first year than a 5% raise. By the commutative and associative laws of multiplication we see that these are equal.

96. Let n = the number of new light passenger vehicles sold.
Solve: $443{,}840 = 0.1775 \cdot 0.73 \cdot n$

$$n = \frac{443{,}840}{0.1775 \cdot 0.73} \approx 3{,}425{,}352 \text{ vehicles}$$

97. Since $2 \text{ ft} = 2 \times 1 \text{ ft} = 2 \times 12 \text{ in.} = 24 \text{ in.}$, we can express 2 ft 6 in. as 24 in. + 6 in., or 30 in. We reword and translate. Let a = Boone's final adult height.

$$\begin{array}{ccccc} \underbrace{30 \text{ in.}} & \text{is} & 75\% & \text{of} & \underbrace{\text{adult height}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 30 & = & 0.75 & \cdot & a \end{array}$$

$$\frac{30}{0.75} = a$$
$$40 = a$$

Note that 40 in. = 36 in. + 4 in. = 3 ft 4 in.
Boone's final adult height will be 3 ft 4 in.

98. The dropout rate will decrease by 63 – 51, or 12 per thousand over 5 years (2014 to 2019).

$$\frac{63}{1000}-\frac{51}{1000}=\frac{12}{1000}$$
$$\frac{12}{1000}\div 5\text{ years}=\frac{2.4}{1000}=0.0024=0.24\%$$

The dropout rate is about 0.24% per year.

Assuming that the dropout rate will continue to decline by the same amount each year, $\frac{2.4}{1000}$, the estimates for 2018 and 2020 can be calculated as follows. If the dropout rate drops by about $\frac{2.4}{1000}$ per year, then the dropout rate in 2020 is $\frac{51}{1000}-\frac{2.4}{1000}=\frac{48.6}{1000}\approx\frac{49}{1000}$, or about 49 students per thousand. So from 2019 to 2018, the dropout rate increases by about $\frac{2.4}{1000}$ per year. Then the dropout rate in 2018 is $\frac{51}{1000}+\frac{2.4}{1000}=\frac{53.4}{1000}\approx\frac{53}{1000}$, or about 53 students per thousand.

Thus, we estimate the drop out rate to be 0.24% per year. We estimate that the drop out rate in 2018 is 53 per thousand, and the drop out rate in 2020 is 49 per thousand.

99. Using the formula for the area A of a rectangle with length l and width w, $A=l\cdot w$, we first find the area of the photo.

$$A=8\text{ in.}\times 6\text{ in.}=48\text{ in}^2$$

Next we find the area of the photo that will be visible using a mat intended for a 5-in. by 7-in. photo.

$$A=7\text{ in.}\times 5\text{ in.}=35\text{ in}^2$$

Then the area of the photo that will be hidden by the mat is $48\text{ in}^2-35\text{ in}^2$, or 13 in^2.

We find what percentage of the area of the photo this represents.

$$\underbrace{\text{What percent}}_{p}\ \underset{\cdot}{\text{of}}\ \underbrace{48\text{ in}^2}_{48}\ \underset{=}{\text{is}}\ \underbrace{13\text{ in}^2}_{13}?$$

$$\frac{p\cdot 48}{48}=\frac{13}{48}$$
$$p\approx 0.27=27\%$$

The mat will hide about 27% of the photo.

100. *Writing Exercise*. Suppose Jorge has x dollars of taxable income. If he makes a \$50 tax-deductible contribution, then he pays tax of $0.3(x-\$50)$, or $0.3x-\$15$ and his assets are reduced by $0.3x-\$15+\$50$, or $0.3x+\$35$. If he makes a \$40 non-tax-deductible contribution, he pays tax of $0.3x$ and his assets are reduced by $0.3x+\$40$. Thus, it costs him less to make a \$50 tax-deductible contribution.

Exercise Set 2.5

1. In order, the steps are:
1) Familiarize.
2) Translate.
3) Carry out.
4) Check.
5) State.

2. To solve an equation, use the step *Carry out*.

3. To write the answer clearly, use the step *State*.

4. To make and check a guess, use the step *Familiarize*.

5. To reword the problem, use the step *Translate*.

6. To make a table, use the step *Familiarize*.

7. To recall a formula, use the step *Familiarize*.

8. To compare the answer with a prediction from an earlier step, use the step *Check*.

9. ***Familiarize***. Let n = the number. Then three less than two times the number is $2n-3$.

Translate.

$$\underbrace{\text{Three less than twice a number}}_{2n-3}\ \underset{=}{\text{is}}\ \underset{19}{19.}$$

Carry out. We solve the equation.

$$\begin{aligned}2n-3&=19\\2n&=22&&\text{Adding 3}\\n&=11&&\text{Dividing by 2}\end{aligned}$$

Check. Twice 11 is 22, and three fewer is 19. The answer checks.

State. The number is 11.

10. Let n = the number.
Solve: $10n-2=78$
$n=8$

11. ***Familiarize***. Let a = the number. Then "five times the sum of 3 and twice some number" translates to $5(2a+3)$.

Translate.

$$\underbrace{\text{Five times the sum of 3 and twice some number}}_{5(2a+3)}\ \underset{=}{\text{is}}\ \underset{70}{70.}$$

Carry out. We solve the equation.

$$\begin{aligned}5(2a+3)&=70\\10a+15&=70&&\text{Using the distributive law}\\10a&=55&&\text{Subtracting 15}\\a&=\frac{11}{2}&&\text{Dividing by 10}\end{aligned}$$

Check. The sum of $2\cdot\frac{11}{2}$ and 3 is 14, and $5\cdot 14=70$. The answer checks.

State. The number is $\frac{11}{2}$.

12. Let x = the number.
Solve:
$$\begin{aligned} 2(3x+4) &= 34 \\ 6x+8 &= 34 \\ 6x &= 26 \\ x &= \frac{13}{3} \end{aligned}$$

13. ***Familiarize***. Let d = the kayaker's distance, in miles, from the finish. Then the distance from the start line is $4d$.

Translate.
$$\begin{array}{ccccc} \underbrace{\text{Distance from finish}} & \text{plus} & \underbrace{\text{distance from start}} & \text{is} & 20.5 \text{ mi.} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ d & + & 4d & = & 20.5 \end{array}$$

Carry out. We solve the equation.
$$\begin{aligned} d+4d &= 20.5 \\ 5d &= 20.5 \\ d &= 4.1 \end{aligned}$$

Check. If the kayakers are 4.1 mi from the finish, then they are $4\cdot(4.1)$, or 16.4 mi from the start. Since $4.1 + 16.4$ is 20.5, the total distance, the answer checks.

State. The kayakers had traveled approximately 16.4 mi.

14. Let d = the distance from Nome, in miles. Then $2d$ = the distance from Anchorage.
Solve:
$$\begin{aligned} d+2d &= 1049 \\ d &= \frac{1049}{3} \end{aligned}$$
The musher has traveled $2\cdot\frac{1049}{3}$, or $699\frac{1}{3}$ mi.

15. ***Familiarize***. Let d = the distance, in miles, that Marcus Ericsson had traveled to the given point after the start. Then the distance from the finish line was $500 - d$ miles.

Translate.
$$\begin{array}{ccccc} \underbrace{\text{Distance to finish}} & \text{plus} & \underbrace{\text{20 mi more}} & \text{was} & \underbrace{\text{distance to start.}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 500-d & + & 20 & = & d \end{array}$$

Carry out. We solve the equation.
$$\begin{aligned} 500-d+20 &= d \\ 520-d &= d \\ 520 &= 2d \\ 260 &= d \end{aligned}$$

Check. If Marcus Ericsson was 260 mi from the start, he was 500 – 260, or 240 mi from the finish. Since 240 is 20 more than 260, the answer checks.

State. Ericsson had traveled 260 mi at the given point.

16. Let d = the distance Kyle Larson had traveled, in miles, at the given point.
Solve:
$$\begin{aligned} 220-d+80 &= d \\ d &= 150 \text{ mi} \end{aligned}$$

17. ***Familiarize***. Let n = the number of the smaller apartment number. Then $n+1 =$ the number of the larger apartment number.

Translate.
$$\begin{array}{ccccc} \underbrace{\text{Smaller number}} & \text{plus} & \underbrace{\text{larger number}} & \text{is} & 2409 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ n & + & (n+1) & = & 2409 \end{array}$$

Carry out. We solve the equation.
$$\begin{aligned} n+(n+1) &= 2409 \\ 2n+1 &= 2409 \\ 2n &= 2408 \\ n &= 1204 \end{aligned}$$
If the smaller apartment number is 1204, then the other number is $1204+1$, or 1205.

Check. 1204 and 1205 are consecutive numbers whose sum is 2409. The answer checks.

State. The apartment numbers are 1204 and 1205.

18. Let n = the number of the smaller apartment number. Then $n+1 =$ the number of the larger apartment number.
Solve:
$$\begin{aligned} n+(n+1) &= 1419 \\ n &= 709 \end{aligned}$$
The apartment numbers are 709 and $709+1$, or 709 and 710.

19. ***Familiarize***. Let n = the smaller house number. Then $n+2 =$ the larger number.

Translate.
$$\begin{array}{ccccc} \underbrace{\text{Smaller number}} & \text{plus} & \underbrace{\text{larger number}} & \text{is} & 572. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ n & + & (n+2) & = & 572 \end{array}$$

Carry out. We solve the equation.
$$\begin{aligned} n+(n+2) &= 572 \\ 2n+2 &= 572 \\ 2n &= 570 \\ n &= 285 \end{aligned}$$
If the smaller number is 285, then the larger number is $285+2$, or 287.

Check. 285 and 287 are consecutive odd numbers and $285+287=572$. The answer checks.

State. The house numbers are 285 and 287.

20. Let n = the smaller house number. Then $n+2=$ the larger number.
Solve: $n+(n+2)=794$
$n=396$
The house numbers are 396 and 398.

21. ***Familiarize***. Let x = the first page number. Then $x+1=$ the second page number, and $x+2=$ the third page number.

Translate.

$$\underbrace{\text{The sum of three consecutive page numbers}} \quad \text{is} \quad 99.$$
$$\downarrow \qquad\qquad \downarrow \quad \downarrow$$
$$x+(x+1)+(x+2) \quad = \quad 99$$

Carry out. We solve the equation.
$$x+(x+1)+(x+2)=99$$
$$3x+3=99$$
$$3x=96$$
$$x=32$$
If x is 32, then $x+1$ is 33 and $x+2=34$.

Check. 32, 33, and 34 are consecutive integers, and $32+33+34=99$. The result checks.

State. The page numbers are 32, 33, and 34.

22. Let x, $x+1$, and $x+2$ represent the first, second, and third page numbers, respectively.
Solve: $x+(x+1)+(x+2)=60$
$x=19$
If x is 19, then $x+1$ is 20, and $x+2$ is 21. The page numbers are 19, 20, and 21.

23. ***Familiarize***. Let m = the number of Cross River Gorillas. Then $m+400=$ the number of Mountain Gorillas.

Translate.

$$\underbrace{\text{Cross River}} \quad \text{plus} \quad \underbrace{\text{Mountain}} \quad \text{is} \quad 1020.$$
$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \quad \downarrow$$
$$m \quad + \quad (m+400) \quad = \quad 1020$$

Carry out. We solve the equation.
$$m+(m+400)=1020$$
$$2m+400=1020$$
$$2m=620$$
$$m=310$$
If m is 310, then $m+400$ is 710.

Check. 710 is 400 more than 310, and $710+310=1020$. The answer checks.

State. There were 710 Mountain Gorillas and 310 Cross River Gorillas.

24. Let y = Kane Tanaka's years lived.
Then $y+3=$ Jeanne Calment's years lived.
Solve: $y+(y+3)=241$
$y=119$
If y is 119, then $a+3$ is 122. The Kane Tanaka lived 119 years, and Jeanne Calment lived 122 years.

25. ***Familiarize***. Familiarize. Let d = the number of dollars lost, in millions on *Town & Country*. Then $d+15.9$ is the number dollars lost, in millions on *Mars Needs Moms*.

Translate.

$$\underbrace{\text{Town \& Country}} \quad \text{plus} \quad \underbrace{\text{Mars Needs Moms}} \quad \text{is} \quad 205.1$$
$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \quad \downarrow$$
$$d \quad + \quad d+15.9 \quad = \quad 205.1$$

Carry out. We solve the equation.
$$d+d+15.9=205.1$$
$$2d=189.2$$
$$d=94.6$$
If d is 94.6, then $d+15.9$ is $94.6+15.9=110.5$.

Check. Their total is $94.6+110.5=205.1$. The answer checks.

State. *Town & Country* lost \$94.6 million and *Mars Needs Moms* lost \$110.5 million.

26. Let d = the number of structures destroyed in 2021. Then $d+11{,}932$ is the number destroyed in 2020.
Solve: $d+(d+11{,}932)=23{,}876$
$d=5972$ structures in 2021
If $d=5972$, then $d+11{,}932=17{,}904$ structures.
Then there were 17,904 structures destroyed in 2020 and 5972 structures destroyed in 2021.

27. ***Familiarize***. The page numbers are consecutive integers. If we let x = the smaller number, then $x+1=$ the larger number.

Translate. We reword the problem.

$$\underbrace{\text{First integer}} \quad + \quad \underbrace{\text{Second integer}} \quad = \quad 281$$
$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \quad \downarrow$$
$$x \quad + \quad (x+1) \quad = \quad 281$$

Carry out. We solve the equation.
$$x+(x+1)=281$$
$$2x+1=281 \quad \text{Combining like terms}$$
$$2x=280 \quad \text{Adding } -1 \text{ on both sides}$$
$$x=140 \quad \text{Dividing on both sides by 2}$$

Check. If $x=140$, then $x+1=141$. These are consecutive integers, and $140+141=281$. The answer checks.

State. The page numbers are 140 and 141.

28. Let s = the length of the shortest side, in mm. Then $s+2$ and $s+4$ represent the lengths of the other two sides.
Solve: $s+(s+2)+(s+4)=195$
$s=63$
If $s=63$, then $s+2=65$ and $s+4=67$. The lengths of the sides are 63 mm, 65 mm, and 67 mm.

29. ***Familiarize***. Let w = the width, in meters. Then $w+4$ is the length. The perimeter is twice the length plus twice the width.

Translate.

$$\underbrace{\text{Twice the width}}_{\downarrow \atop 2w} \quad \underbrace{\text{plus}}_{\downarrow \atop +} \quad \underbrace{\text{twice the length}}_{\downarrow \atop 2(w+4)} \quad \underbrace{\text{is}}_{\downarrow \atop =} \quad \underbrace{92.}_{\downarrow \atop 92}$$

Carry out. We solve the equation.

$$\begin{aligned} 2w+2(w+4)&=92\\ 2w+2w+8&=92\\ 4w&=84\\ w&=21 \end{aligned}$$

Then $w+4=21+4=25$.

Check. The length, 25 m, is 4 more than the width, 21 m. The perimeter is $2\cdot 21\text{ m}+\ 2\cdot 25\text{ m}=$ $42\text{ m}+50\text{ m}=92\text{ m}$. The answer checks.

State. The length of the garden is 25 m and the width is 21 m.

30. Let $w=$ the width of the rectangle, in feet. Then $w+60=$ the length.

Solve: $2(w+60)+2w=520$
$$w=100$$

The length is 160 ft, the width is 100 ft, and the area is 16,000 ft^2.

31. ***Familiarize***. Let w = the width, in inches. Then $2w=$ the length. The perimeter is twice the length plus twice the width. We express $10\frac{1}{2}$ as 10.5.

Translate.

$$\underbrace{\text{Twice the length}}_{\downarrow \atop 2\cdot 2w} \quad \underbrace{\text{plus}}_{\downarrow \atop +} \quad \underbrace{\text{twice the width}}_{\downarrow \atop 2w} \quad \underbrace{\text{is}}_{\downarrow \atop =} \quad \underbrace{10.5\text{ in.}}_{\downarrow \atop 10.5}$$

Carry out. We solve the equation.

$$\begin{aligned} 2\cdot 2w+2w&=10.5\\ 4w+2w&=10.5\\ 6w&=10.5\\ w&=1.75,\ \text{or } 1\tfrac{3}{4} \end{aligned}$$

Then $2w=2(1.75)=3.5$, or $3\frac{1}{2}$.

Check. The length, $3\frac{1}{2}$ in., is twice the width, $1\frac{3}{4}$ in. The perimeter is $2\left(3\frac{1}{2}\text{ in.}\right)+2\left(1\frac{3}{4}\text{ in.}\right)=$ $7\text{ in.}+3\frac{1}{2}\text{ in.}=10\frac{1}{2}\text{ in.}$ The answer checks.

State. The actual dimensions are $3\frac{1}{2}$ in. by $1\frac{3}{4}$ in.

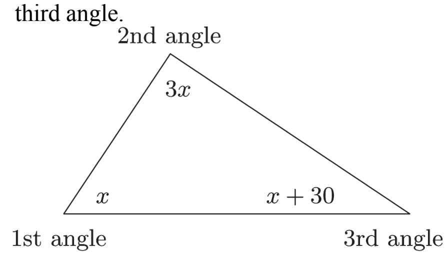
32. Let w = the width, in feet. Then $3w+6=$ the length.

Solve: $2(3w+6)+2w=124$
$$w=14$$

Then $3w+6=3\cdot 14+6=42+6=48$.

The billboard is 48 ft long and 14 ft wide.

33. ***Familiarize***. We draw a picture. We let $x=$ the measure of the first angle. Then $3x=$ the measure of the second angle, and $x+30=$ the measure of the third angle.



Recall that the measures of the angles of any triangle add up to 180º.

Translate.

$$\underbrace{\text{Measure of first angle}}_{\downarrow \atop x} \ \underset{\downarrow \atop +}{+} \ \underbrace{\text{measure of second angle}}_{\downarrow \atop 3x} \ \underset{\downarrow \atop +}{+} \ \underbrace{\text{measure of third angle}}_{\downarrow \atop x+30} \ \underbrace{\text{is}}_{\downarrow \atop =} \ \underbrace{180^\circ.}_{\downarrow \atop 180}$$

Carry out. We solve the equation.

$$\begin{aligned} x+3x+(x+30)&=180\\ 5x+30&=180\\ 5x&=150\\ x&=30 \end{aligned}$$

Possible answers for the angle measures are as follows:

First angle: $x=30°$

Second angle: $3x=3(30)°=90°$

Third angle: $x+30°=30°+30°=60°$

Check. Consider 30º, 90º and 60º. The second angle is three times the first, and the third is 30º more than the first. The sum of the measures of the angles is 180º. These numbers check.

State. The measure of the first angle is 30º, the measure of the second angle is 90º, and the measure of the third angle is 60º.

34. Let x = the measure of the first angle. Then $4x$ = the measure of the second angle, and $x+4x-45$, or $5x-45=$ the measure of the third angle.

Solve: $x+4x+(5x-45)=180$
$$x=22.5$$

If x is 22.5, then $4x$ is 90, and $5x-45$ is 67.5, so the measures of the first, second, and third angles are 22.5º, 90º, and 67.5º, respectively.

35. ***Familiarize***. Let x = the measure of the first angle. Then $4x =$ the measure of the second angle, and $x+4x+5=5x+5$ is the measure of the third angle.

Translate.

$$\underbrace{\text{Measure of first angle}} + \underbrace{\text{measure of second angle}} + \underbrace{\text{measure of third angle}} \text{ is } \underbrace{180^\circ}$$
$$x + 4x + (5x+5) = 180$$

Carry out. We solve the equation.

$$\begin{aligned} x+4x+(5x+5) &= 180 \\ 10x+5 &= 180 \\ 10x &= 175 \\ x &= 17.5 \end{aligned}$$

If $x = 17.5$, then $4x = 4(17.5) = 70$, and $5x+5 = 5(17.5)+5 = 87.5+5 = 92.5$.

Check. Consider 17.5º, 70º, and 92.5º. The second is four times the first, and the third is 5° more than the sum of the other two. The sum of the measures of the angles is 180º. These numbers check.

State. The measure of the second angle is 70º.

36. Let x = the measure of the first angle. Then $3x$ = the measure of the second angle, and $x+3x+10=4x+10=$ the measure of the third angle.

Solve: $x+3x+(4x+10)=180$

$x = 21.25$

If $x = 21.25$, then $3x = 64.75$, and $4x+10 = 95°$.

The measure of the third angle is 95º.

37. ***Familiarize***. Let b = the length of the bottom section of the rocket, in feet. Then $\frac{1}{6}b =$ the length of the top section, and $\frac{1}{2}b =$ the length of the middle section.

Translate.

$$\underbrace{\text{Length of top section}} + \underbrace{\text{length of middle section}} + \underbrace{\text{length of bottom section}} \text{ is } \underbrace{240 \text{ ft}}$$
$$\frac{1}{6}b + \frac{1}{2}b + b = 240$$

Carry out. We solve the equation. First we multiply by 6 on both sides to clear the fractions.

$$\begin{aligned} \frac{1}{6}b+\frac{1}{2}b+b &= 240 \\ 6\left(\frac{1}{6}b+\frac{1}{2}b+b\right) &= 6\cdot 240 \\ 6\cdot\frac{1}{6}b+6\cdot\frac{1}{2}b+6\cdot b &= 1440 \\ b+3b+6b &= 1440 \\ 10b &= 1440 \\ b &= 144 \end{aligned}$$

Then $\frac{1}{6}b = \frac{1}{6}\cdot 144 = 24$ and $\frac{1}{2}b = \frac{1}{2}\cdot 144 = 72$.

Check. 24 ft is $\frac{1}{6}$ of 144 ft, and 72 ft is $\frac{1}{2}$ of 144 ft. The sum of the lengths of the sections is 24 ft + 72 ft + 144 ft = 240 ft. The answer checks.

State. The length of the top section is 24 ft, the length of the middle section is 72 ft, and the length of the bottom section is 144 ft.

38. Let s = the part of the sandwich Jing gets, in inches. Then the lengths of Demi's and Lennox's portions are $\frac{1}{2}s$ and $\frac{3}{4}s$, respectively.

Solve: $s+\frac{1}{2}s+\frac{3}{4}s=18$

$s = 8$

Then $\frac{1}{2}s = \frac{1}{2}\cdot 8 = 4$ and $\frac{3}{4}s = \frac{3}{4}\cdot 8 = 6$.

Jing gets 8 in., Demi gets 4 in., and Lennox gets 6 in.

39. ***Familiarize***. Let r = the speed downstream. Then $r - 10$ = the speed upstream. Then, since $d = r\cdot t$, we multiply to find each distance.

Downstream distance $= r(2)$ mi;

Upstream distance $= (r-10)(3)$ mi.

Translate.

$$\underbrace{\text{Distance downstream}} \text{ plus } \underbrace{\text{distance upstream}} \text{ is } \underbrace{\text{total distance.}}$$
$$r(2) + (r-10)(3) = 30$$

Carry out. We solve the equation.

$$\begin{aligned} 2r+3(r-10) &= 30 \\ 2r+3r-30 &= 30 \\ 5r-30 &= 30 \\ 5r &= 60 \\ r &= 12 \end{aligned}$$

$r - 10 = 2$

Check. Distance = speed × time.

Distance downstream $= 12(2) = 24$ mi

Distance upstream $= (12-10)(3) = 6$ mi

The total distance is 24 mi + 6 mi, or 30 mi. The answer checks.

State. The speed downstream was 12 mph.

40. Let r = the speed of the bus. Then $r + 50$ = the speed of the train.

Solve: $r\cdot\frac{1}{3}+(r+50)\cdot\frac{1}{2}=37.5$

$r = 15$ km/h

41. ***Familiarize***. Let d = the distance Amista ran. Then $17 - d$ = the distance Amista walked. Then, since $t=\frac{d}{r}$, we divide to find each time.

Time running $= \frac{d}{12}$ hr;

Time walking $= \frac{17-d}{5}$ hr;

Translate.

$$\begin{array}{ccc} \underbrace{\text{Time running}} & \text{is} & \underbrace{\text{time walking.}} \\ \downarrow & \downarrow & \downarrow \\ \dfrac{d}{12} & = & \dfrac{17-d}{5} \end{array}$$

Carry out. We solve the equation.

$$\begin{aligned} 60\cdot\frac{d}{12} &= 60\cdot\frac{17-d}{5} \\ 5d &= 12(17-d) \\ 5d &= 204-12d \\ 17d &= 204 \\ d &= 12 \end{aligned}$$

$17-d=5$

Check. Time = distance ÷ speed.

Time running $=\dfrac{12}{12}=1$ hr;

Time walking $=\dfrac{17-12}{5}=1$ hr;

The time running is the same as the time walking
The answer checks.

State. Amista ran for 1 hour.

42. Let t = the time driving on the interstate. Then $3t$ = the time driving on the Blue Ridge Parkway.
Solve: $70\cdot t+40\cdot(3t)=285$

$$t=1\frac{1}{2}\text{ hr}$$

43. Let $p=$ the percent decrease. The population decreased by $2.83-2.10=0.73$.
Rewording and Translating:

$$\begin{array}{ccccc} \underbrace{\text{Population decrease}} & \text{is} & \underbrace{\text{what percent}} & \text{of} & \underbrace{\text{original population.}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 0.73 & = & p & \cdot & 2.83 \end{array}$$

$$\begin{aligned} \frac{0.73}{2.83} &= p \\ 0.258 &\approx p \end{aligned}$$

The percent decrease was about 25.8%.

44. Let $p=$ the percent increase. The population increased by $543-278=265$.
Rewording and Translating:

$$\begin{array}{ccccc} \underbrace{\text{Population increase}} & \text{is} & \underbrace{\text{what percent}} & \text{of} & \underbrace{\text{original population.}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 265 & = & p & \cdot & 278 \end{array}$$

$$\begin{aligned} \frac{265}{278} &= p \\ 0.953 &\approx p \end{aligned}$$

The percent increase was about 95.3%.

45. Let $p=$ the percent increase. The budget increased by $\$1{,}800{,}000-\$1{,}600{,}000=\$200{,}000$.
Rewording and Translating:

$$\begin{array}{ccccc} \underbrace{\text{Budget increase}} & \text{is} & \underbrace{\text{what percent}} & \text{of} & \underbrace{\text{original budget.}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 200{,}000 & = & p & \cdot & 1{,}600{,}000 \end{array}$$

$$\begin{aligned} \frac{200{,}000}{1{,}600{,}000} &= p \\ 0.125 &= p \end{aligned}$$

The percent increase is 12.5%.

46. Let $p=$ the percent. The number of jobs increased by $1{,}816{,}200-1{,}800{,}000=16{,}200$.
Solve: $16{,}200=p\cdot 1{,}800{,}000$

$$\begin{aligned} 0.009 &= p \\ 0.9\% &= p \end{aligned}$$

47. Let $b=$ the bill without tax.
Rewording and Translating:

$$\begin{array}{ccccc} \underbrace{\text{The bill}} & \text{plus} & \underbrace{\text{tax}} & \text{is} & \$1310.75. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ b & + & 0.07b & = & 1310.75 \end{array}$$

$$\begin{aligned} 1.07b &= 1310.75 \\ b &= \frac{1310.75}{1.07} \\ b &= 1225 \end{aligned}$$

The bill without tax is \$1225.

48. Let $c=$ the cost without tax
Solve: $c+0.06c=16{,}536$

$$\$15{,}600=c$$

49. Let $s=$ the sales tax.
Rewording and Translating:

$$\begin{array}{ccccc} \underbrace{\text{amount spent}} & \text{plus} & \underbrace{\text{sales tax}} & \text{is} & \$4960.80. \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ s & + & 0.06s & = & 4960.80 \end{array}$$

$$\begin{aligned} 1.06s &= 4960.80 \\ s &= \frac{4960.80}{1.06} \\ s &= 4680 \end{aligned}$$

$4960.80-4680=280.80$
The sales tax is \$280.80.

50. Let $c=$ the cost without tax
Solve: $c+0.04c=7115.68$

$$\$6842=c$$

Then $\$7115.68-\$6842=\$273.68$.

51. ***Familiarize***. Let p = the regular price of the chair. At 30% off, Henley paid $(100-30)\%$, or 70% of the regular price.

Translate.

$$\begin{array}{ccccc} \$224 & \text{is} & 70\% & \text{of} & \underbrace{\text{the regular price.}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 224 & = & 0.70 & \cdot & p \end{array}$$

Chapter 1: Review of Whole Numbers and Integers

Vocabulary Review

1. The _________ number system determines the value of a digit by its position in a number.
2. When a number is approximated based upon a specified place, using the first digit to the right of that number, it is called a _________ number.
3. _________ are numbers, which are being added together, and the result of this addition is called the _________ or _________.
4. To check one's accuracy without recalculating, an _________ is often made.
5. The first number in a subtraction problem is the _________ and the number being subtracted is the _________. The result of the process is called the _________.
6. In multiplication, the number being multiplied is the _________ that is multiplied by the _________ and the result is called the _________.
7. In division, the number being divided is the _________that is divided by the _________ and the result is called the _________.

Skill Builders

8. Read or write the words for the following numbers: 5,649; 18,715; 564,252; and 4,826,006.
9. Round the numbers in Exercise 8 to the nearest thousand.
10. Add the following numbers: 45,628; 166,555; 3,243; 29,678; and 132.
11. Estimate the sum for Exercise 10 by rounding each addend to the nearest hundred.
12. Subtract the following: $873,492 minus $426,888.
13. Multiply the following: $678 times 3,444.
14. Round the result of Exercise 13 to the nearest million dollar.
15. Divide $8,777,302 by 144. Round to the nearest dollar.
16. Round the result of Exercise 15 to the nearest thousand dollars.
17. If the quotient of 48,000 and 6 is subtracted from the product of 25 and 480, what is the difference?
18. Estimate the product of 98 and 32 by rounding each factor to the nearest ten.

Simplify.

19. $153 + (-38)$ **20.** $(-24) + (-32)$ **21.** $154 - (-12)$

22. $17(-32)$ **23.** $(-15)(-7)$ **24.** $-36 \div 9$

Applications

25. An upcoming special on legal-sized note pads will require the stock clerk to shrink-wrap the pads in groups of four. His boss has given him five large boxes of pads to package. Each box holds 144 pads. How many promotional bundles of four pads will the stock clerk be able to assemble?

26. An area supervisor has told a retail sales associate that she should be averaging $75 in (SPH) sales per hour. The associate is trying to determine her average sales for her morning four-hour shift. She has had the following sales thus far: $58, $119, $75, $125, and $89. Is she meeting her SPH goal? Round the average to the nearest whole dollar.

27. Sales for a small shoe store were $1,428 for the day. Refunds in the following amounts were made from the cash drawer: $48, $26, and $55. What were the net sales for the store that day?

28. A local retailer offers prepaid long distance phone cards at 3 cents per minute. A small business owner normally pays his local telephone service 9 cents per minute for any long distance calls. The store owner averages about 500 minutes a month in long distance calls to his vendors. How much money could he save each month if he canceled his local long distance service and used the prepaid cards instead?

29. The floor sales manager just completed a walk-through of his four departments. He is trying to do a quick estimate of total sales for the day in hundreds of dollars. The men's department sales totaled $22,392. Shoes indicated $1,438 in sales. Small accessories reported $345. The boys' department sales were $826. What are the estimated total sales for the four departments in the hundreds of dollars? (Note: round each addend prior to adding.)

30. Mrs. Nickerson earns $79,000 each year. She is going to buy a new SUV for $46,000. She will pay taxes of $2,800 and license fees of $1,760. She plans to make a down payment of $12,000 and finance the remainder at her credit union. How much will she finance?

31. Sunbelt Seafood had a net loss of –$3,518 in January and –$2,143 in February. In March they had a net profit of $4,517. Find the net profit or loss for the quarter.

32. Shapley's Lighting had a loss of –$2,500 each month in the second quarter of the year. What was their loss for the quarter?

INSTRUCTOR'S RESOURCE MANUAL

ELEMENTARY AND INTERMEDIATE ALGEBRA: CONCEPTS AND APPLICATIONS

EIGHTH EDITION

Bittinger/Ellenbogen/Johnson

Pearson

Pearson

ISBN-13: 978-0-13-797736-9

Contents

INTRODUCTION

Dear Faculty:

The Bittinger/Ellenbogen/Johnson team at Pearson is very excited that you will be using *Elementary and Intermediate Algebra: Concepts and Applications*, Eighth Edition. We know that whether you are teaching this course for the first time or the tenth time, you will face many challenges, including how to prepare for class, how to make the most effective use of your class time, how to present the material to your students in a manner that will make sense to them, how best to assess your students, and the list goes on.

This manual is designed to make your job easier. Inside these pages are words of advice from experienced instructors, general and content-specific teaching tips, a list of the topics covered within the *Elementary and Intermediate Algebra: Concepts and Applications*, Eighth Edition, mini lectures, chapter tests, extra practice exercises, and final exams.

We would like to thank the following professors for sharing their advice and teaching tips. This manual would not be what it is without their valuable contributions.

Dean Barchers, Red Rocks Community College
Viola Lee Bean, Boise State University
Susan M. Caldiero, Cosumnes River College
Tim Chappell, Penn Valley Community College
Beth Fraser, Middlesex Community College–Lowell
Nikki D. Handley, Odessa College
Marty Hodges, Colorado Technical University
Matthew Hudock, St. Philip's College
Michelle Jackson, Bowling Green Community College
Cindy Katz, St. Philip's College
David Keller, Kirkwood Community College
Paul W. Lee, St. Philip's College
Ben Mayo, Yakima Valley Community College
Christian Miller, Glendale Community College
Donald W. Solomon, University of Wisconsin–Milwaukee
Sharon Testone, Onondaga Community College
Malissa Trent, Northeast State Community College

It is also important to know that you have a very valuable resource available to you in your Pearson sales representative. You can locate your representative by logging on to www.pearsonhighered.com/replocator and typing in your zip code. Please feel free to contact your representative if you have any questions relating to our text or if you need additional supplements.

We know that teaching this course can be challenging. We hope that this and the other resources we have provided will help to minimize the amount of time it takes you to meet those challenges.

Good luck in your course!

The Bittinger/Ellenbogen/Johnson team

General, First-Time Advice

We asked the contributing professors for words of advice to instructors who are teaching this course for the first time or for the first time in a long while. Their responses can be found on the following pages.

Tim Chappell, *Penn Valley Community College*

1. Create an atmosphere of learning in the classroom from day one. Relate to students some of your experiences in learning mathematics, including both good and bad. Students need to see you as a fellow learner as well as an instructor. They also need to see you as a person. Inject your personality into the course. Use jokes, personal stories, or mathematical games to keep it interesting.
2. Put students into groups. Even if you don't use group work extensively, group them on the first day. Have them make introductions and complete a brief group assignment. Encourage the groups to take advantage of the few minutes before and after class starts to check homework with each other.
3. Carefully develop a list of homework problems. The assigned problems should match the level and type of problems you expect students to master. Avoid large homework sets. Instead, I give the students a small homework set that I call journal homework. To receive credit for these problems, the students must work each problem step by step and provide explanation for each step. I select odd-numbered problems from the text for journal homework so there are answers in the back. This homework is graded full or no credit. Students learn quickly that an answer with no supported work is not an acceptable answer. This helps them to include steps on test work which helps them earn partial credit on the tests. If students struggle with the journal homework or if they just want more practice, they are encouraged to work additional problems in that section as daily homework.
4. Require students to make corrections to tests. This reinforces the fact that any skills or concepts not mastered now will come back to haunt them in the next math class. I constantly remind students that the goal of this class is not just to pass, but to understand as much as possible so that they can be successful in college algebra and higher courses. Students should not only rework the problem successfully on separate paper, but they should also explain why they missed the problem the first time.
5. After the first test, I have students do some self-analysis by answering three questions. How did they prepare for the class daily and for the test? Does the test score reflect what they know and can do? Are there changes that they need to make? Students are not graded on the content of their responses. I read and keep these papers. I find the papers helpful when a student comes to my office for help. I can ask them if they have been successful in the changes they needed to make.
6. Students lack basic survival skills such as organization and identifying the key concepts of the course. I require students to keep journal notebooks containing notes, daily homework, journal homework, tests, corrections to tests, and concept reviews. I provide a concept review for each unit, outlining each of the concepts that they are responsible for learning. Key concepts are in bold print, emphasizing their importance in the course. Students are encouraged to cross-reference between the journal homework problems and the concepts in the concept review.

Nikki D. Handley, *Odessa College*

1. Try to give students a set of steps for working problems. This often gives them a starting place on their own. Otherwise, they get frustrated wondering where to begin on each problem.
2. Repetition: have students work problems the same way again and again. Have them tell you the steps orally on how to work a particular problem. If they can see it and write it and hear it and say it, it tends to make a larger impact.
3. Carefully define and explain like terms and that you must have like terms to add or subtract. However, in multiplying and dividing, anything goes—like or not.

4. Walk around the room while students are working problems and ask students individually if they have questions. They are more apt to ask for clarification on a small step while you are right there by them.
5. Clarify the following on a regular basis: equation and expression, term and factor, associative law and distributive law.
6. Use different colors of markers when working problems. This helps them see each of the steps. I show division in red and addition or sub-traction in blue or purple all semester long.
7. Have patience! Point out places where mistakes are commonly made and explain to students why it is wrong.

Matthew Hudock, *St. Philip's College*

1. Find a full-time faculty member who is teaching the same course(s). See if you can use him or her as a contact to ask questions about the course or about departmental and/or college policies.
2. Get a timeline, sample syllabus, and sample tests for the course either from the department or from your full-time faculty contact person.
3. Get a list of all paperwork details and administrative details that are due during the course of the semester and the associated due dates.
4. Discuss with your faculty contact person the student population that the college is serving. Find out the focus of the school you are teaching at (Is it a feeder school to a major university? Or is it mostly a vocational/technical school?).
5. Discuss the placement test that the college/department uses with your faculty contact person. How good of a job does it do? If you find a student that is misplaced, how do you get that student in the right place?
6. Find out the passing rates and dropout rates for the course(s) you are teaching.
7. Make sure you bring several different colors of markers to class.
8. Be on time, do not let the class out early, and be well dressed.
9. Return graded papers to your students by the next class period.

Ben Mayo, *Yakima Valley Community College*

1. Remember that although your students don't ask a lot of questions at first, they may not understand everything you are telling them. Students frequently feel overwhelmed when dealing with math and therefore aren't able to process new information very quickly.
2. To help make students more comfortable in the class, I joke with them that the more questions they ask, the more I get paid. In other words, I do all I can to encourage them to ask questions.
3. Don't assume that students have retained everything that they learned in their previous math class. In fact, at the community college level, I may have many students who haven't taken courses for several months or even years, so they may have forgotten most of what they had learned previously. However, with patience on the part of the instructor, that information can be reawakened in the students' memories and they can go on to be very successful with the new course.
4. I once had an instructor tell us on the first day of class that he was there to weed the garden. I prefer to tell my students that I am there to turn the weeds into flowers. These students need encouragement and reinforcement; don't treat your students like they are stupid.

Malissa Trent, *Northeast State Community College*

1. You will always wish for more time to cover the material. Do not be lured into the trap of slowing down at the beginning of the semester to accommodate students who are not prepared for your course. These students must seek additional help outside of class. Slowing down generally doesn't benefit the weaker students in the way that you hope it will. The majority of these students need to go back to the previous course, if possible. You will find that slowing down at the beginning of a course will cause you great stress at the end of the course. You will have to cover four sections in one class period and that is neither fun nor productive.
2. Most of your students will not be able to work with fractions. It does not matter how many times they have worked with fractions in the past, they simply cannot deal with them. This is always a surprise to new math teachers since adding fractions is a fourth grade skill. Most of your students will skip fraction problems in the homework whenever possible or will require a calculator to combine fractions. When working algebra problems that include fractions, show every possible step every time.
3. If you have time, try to use games and other activities to engage the students. For example, when I teach factoring, I have my students play factor bingo. During this game, they factor 20

problems on average. They work in teams and enjoy the competition. If I just gave them a worksheet with 20 factoring problems, they would get bored with it and would not complete as many problems in the same amount of time.

4. You must collect and grade homework if you expect them to do it. I collect homework at the end of every chapter and grade it during the chapter test. Homework must be in a three-ring binder and must be orderly and neat. I pre-select 10 problems (usually two per section) and grade those.

5. I also give 3-5 problem quizzes every day at the beginning of class. These include simple problems that are from the current or prior chapters. Since the final exam is comprehensive, I find that the quizzes help keep certain topics from the beginning of the semester fresh in the minds of the students at the end of the semester.

Paul W. Lee, *St. Philip's College*

1. **Time Management**

   I like to make sure that I am able to cover all of the objectives that are included in a particular course that I am teaching. Students are expected to know these objectives when they enroll in the following course, so I want to do my best to ensure they are prepared. The key to covering all of the objectives is to have a tentative course outline. I always prepare my tentative course outline before the semester starts. I begin by taking a calendar that includes all school holidays and I pencil in the days that I will be covering particular sections. I also include any other activities that may take up class time, such as test days, review days, etc. If I expect to finish the objectives for the course during the semester, the tentative outline that I create must include all sections that are to be taught in the course.

   Once the semester begins, the tentative outline can be adjusted slightly. Some sections require more time, while others don't require as much. However, I try not to change the test days, if possible. I also make sure that I don't start falling behind. While it is okay to slightly adjust the material, I use the outline to keep me on track to finish all objectives.

2. **Classroom**

   As we all know, students do not all learn the same way. There are questionnaires that students can fill out to assess their learning style. I like for the students to fill out a learning styles assessment and then let the students brainstorm on ideas that would help them learn based on their particular learning style.

   I prefer not to spend all classroom time lecturing. For math classes, lecturing is not usually the ideal method for most students to learn. I try to begin my classes by answering a couple of homework questions that were assigned from the previous class meeting. This is a good way to review the previous material before beginning anything new. I always limit this portion to the first 5–10 minutes of class. If not, students begin asking more and more questions which may cause us to fall behind on all the material that needs to be covered for the semester.

   After answering homework questions, I begin the new material. I always like to allow the students to work some examples at their desk after I give instruction on a new objective. This gives students the opportunity to work a few problems while they have someone handy to whom they can ask questions. I allow them to either ask me questions or one of their classmates. Sometimes, I encourage the students to form groups to work a particular problem. It is important for students to get to know one another, so that they can study together or just have someone to talk with about the class.

3. **Tests**

   When I first started teaching, testing was one of my biggest concerns. I was afraid that I may make tests too easy, too difficult, too long, too short, etc. I found that other instructors in the department were more than happy to share copies of tests that they had used. I did not use their tests in my class, but their tests did give me ideas about how many questions I should include on the tests and what type of questions should be asked.

TEACHING TIPS CORRELATED TO TEXTBOOK SECTIONS

Following is a listing of the topics included in the Elementary and Intermediate Algebra: Concepts and Applications *text, as well as specific teaching tips provided by the contributing professors.*

1 Introduction to Algebraic Expressions

SECTION TITLES AND TOPICS

1.1 Introduction to Algebra
Evaluating Algebraic Expressions ● Translating to Algebraic Expressions ● Translating to Equations

1.2 The Commutative, Associative, and Distributive Laws
The Commutative Laws ● The Associative Laws ● The Distributive Law ● The Distributive Law and Factoring

1.3 Fraction Notation
Factors and Prime Factorizations ● Multiplication, Division, and Simplification of Fractions ● Addition and Subtraction of Fractions

1.4 Positive and Negative Real Numbers
Whole Numbers and Integers ● The Rational Numbers ● Real Numbers and Order ● Absolute Value

1.5 Addition of Real Numbers
Adding with the Number Line ● Adding without the Number Line ● Problem Solving ● Combining Like Terms

1.6 Subtraction of Real Numbers
Opposites and Additive Inverses ● Subtraction ● Problem Solving

1.7 Multiplication and Division of Real Numbers
Multiplication ● Division

1.8 Exponential Notation and Order of Operations
Exponential Notation ● Order of Operations ● Simplifying and the Distributive Law ● The Opposite of a Sum

TEACHING TIPS

Students will often complain that you are moving too fast through Chapter 1 if you review. Explain to them that the quick pace through Chapter 1 is intentional because the student should already know how to do Chapter 1. If they complain that it's new to them and that they've never seen the material before, suggest to them to drop the class and take the class before Elementary Algebra, which is usually Pre-Algebra.

It's very important that students have this chapter mastered before moving into Chapter 2, especially adding and subtracting positive and negative numbers resulting in negative answers.

Dean Barchers,
Red Rocks Community College

Section 1.1

The chart on "Translating to Algebraic Expressions" (p. 4) is very helpful. Make the students look closely at the subtraction wordings. "Less than" and "subtracted from" reverse the order in which the numbers are listed. In division, discuss carefully the difference between "divided into" and "divided by."

You can't overemphasize the difference between expressions and equations. Once students learn to solve an equation, they often then try to solve everything. I often give the students an analogy from English. I compare an expression to a phrase ("the pretty package") and an equation to a sentence ("The pretty package contained a birthday gift.")

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Beginning on the first day of class, stress the difference between an expression and an equation. Ask students what you can do to each.

David A. Keller,
Kirkwood Community College

◆ ◆ ◆

I also compare translating in algebra to translating into any language. Examples 5, 6, & 7 (pp. 6 & 7) relate the words directly to the translation. This method is an excellent way to set it up, and to give the students a way to approach most translation problems. I would work through several different examples like this on the board with the students.

Beth Fraser,
Middlesex Community College–Lowell

Section 1.2

Distributing on the right causes many more problems for students than distributing on the left.

Additionally, they have difficulty with the words "term" and "factor." Be sure to introduce them here and discuss the differences.

David A. Keller,
Kirkwood Community College

◆ ◆ ◆

Students seem to have more trouble applying the distributive law when the single term is on the right of the polynomial by which it is multiplied.

Explain that although they may not recognize the titles, the commutative, associative, and distributive laws are properties that students commonly use.

Deana Richmond

Section 1.3

Students have problems with the vocabulary in this section. They confuse the words "opposite," "reciprocal," and "inverse."

When doing arithmetic with fractions, always look at the operation first. That will determine the next step.

David A. Keller,
Kirkwood Community College

Section 1.4

I use an alternate picture to supplement the author's approach of showing relationships among different types of numbers (p. 32) to help students see the subset relationships between sets of numbers. To draw it, use concentric circles, with the smallest circle representing the natural numbers, the next representing the whole numbers and so on.

To help students with the inequality symbols, remind them that the alligator (or PacMan) eats the bigger quantity which they may remember from previous math courses.

Students do not see any difference between positive and nonnegative.

David A. Keller,
Kirkwood Community College

Section 1.5

Problems adding real numbers are a crucial part of Chapter 1. If a student can't do these problems without a calculator, chances are very good that they won't do well in the class.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

Do not assume students can work with decimals without a calculator. In my pre-testing of students only about 1 in 4 can solve $0.092 - 1$ without a calculator.

David A. Keller,
Kirkwood Community College

Section 1.6

The "opposite of an opposite" rule is the most important rule in this section. It's crucial to teach students that problems like $4 - (-9)$ are just addition problems.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

Students often have tremendous difficulty sorting out the difference between the various uses of the " – ." When does it mean opposite, when does it mean "negative," and when does it mean "subtract"? They need to see it used repeatedly and carefully when first presented. Start with the idea of an opposite (which if we start with positives, makes sense that the negative sign means the opposite of a positive), then stress the definition of subtraction as adding the opposite.

Beth Fraser,
Middlesex Community College–Lowell

Section 1.7

Students need examples of why dividing by zero is not possible, but zero divided by a nonzero number is always zero. Here's a scenario I use: I will divide 0 pizzas equally among 5 people. How much does each person get? Now, I will divide 5 pizzas equally among 0 people. With a bit of discussion, students realize why the second one is impossible.

David A. Keller,
Kirkwood Community College

Section 1.8

Problems that require the removal of parentheses and the combining of like terms (83–90 and 99–112 on p. 67) are another crucial part of Chapter 1 that students need to have mastered.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

As in Section 1.6, the negative sign may cause problems here in this section. Take time on problems like $(-x)^2$ vs. $-x^2$ especially if they are using calculators. I have seen calculators that assume the parentheses and give $-5^2 = 25$.

The negative sign will also cause problems on examples like: $5t-(3t+4)$. This is another one to spend time on and be sure they understand this concept.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Many students memorize the mnemonic PEMDAS or Please Excuse My Dear Aunt Sally, but they fail to realize that P means any grouping symbol and some of the other finer details.

Compute -3^2 and $(-3)^2$. Discuss the differences.

David A. Keller,
Kirkwood Community College

◆ ◆ ◆

Encourage students to write each step of the simplification process, following the order of operations rules one at a time. This will help avoid errors.

Deana Richmond

2 Equations, Inequalities, and Problem Solving

SECTION TITLES AND TOPICS

TEACHING TIPS

Section 2.1

Stress to students that you are aware that they can do most of the simpler problems in their heads, but they need to use algebra to complete them since problems will get more complicated, making it too difficult to do the math in their heads.

Dean Barchers,
Red Rocks Community College

Section 2.2

Students will have difficulty determining whether to eliminate the constant or the coefficient first. I would suggest that they can remember that they "add" to eliminate the constant first and then "multiply" by the reciprocal of the coefficient. Remembering the word "am" — "a" for add and "m" for multiply. Reinforce this by saying "I am successful in math."

Susan M. Caldiero,
Cosumnes River College

◆ ◆ ◆

In using the principles together, stress the reversal of the order of operations. I often compare it to wrapping and unwrapping a gift. When you wrap, you start with the box, then the paper, then the ribbon. To get to the gift, you must reverse the process—undo the ribbon first, then the paper and then the box.

Another problem is that students need to realize that $-x$ is the same as $-1x$. They often just drop negative signs between rows of work, but won't usually drop a numerical coefficient. I tell students to write in the -1 as a coefficient if they need to.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Students really struggle with problems that include fractions. I always teach students to multiply by the LCD to clear fractions. A common error is to multiply the LCD to both the fraction AND the expression in parentheses.

Example: Some students multiply $4\left(\frac{3}{4}(3t-6)=9\right)$ to get $3(12t-24)=36$.

David A. Keller,
Kirkwood Community College

Section 2.3

This is a very difficult section for students. They don't see the connection between solving formulas and solving equations. Start with a very basic equation to solve, such as $2x+5=3$. Solve this by having the students tell you the steps and write them on the board next to each step. Now substitute letters for the constants and solve the problem opposite the basic equation that you just solved so students can see the correlation. Giving real-world data helps motivate students.

Also, when solving $y=ax+bx-4$, remind students of the distributive property and that is used in reverse to solve for x.

Susan M. Caldiero,
Cosumnes River College

◆ ◆ ◆

When solving for one variable in terms of other variables, get students in the habit of first identifying the variable they are solving for and explain that they are trying to get that variable by itself on one side of the equation. Then go through and ask them how they can "get rid of" all the other variables so that they have the variable they are trying to solve for by itself on one side. Go through two examples each of how to "get rid of" the "other" variables with division, addition or subtraction, reciprocals, multiplication and combinations of all of the operations.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

Formulas requiring factoring are especially challenging. To help, try replacing the variables with numbers. For example, to solve $8=at+bt$ for t, first have students substitute $a=3$ and $b=2$ to see how it works.

David A. Keller,
Kirkwood Community College

◆ ◆ ◆

Remind students that to solve an equation involving two or more variable terms, we use the distributive law to combine like terms.

Example:

$$y=ax+bx-4$$
$$y=(a+b)x-4$$
$$y+4=(a+b)x$$
$$\frac{y+4}{a+b}=x$$

Deana Richmond

Section 2.4

Go to the newspaper or internet and pull some data that uses percentages. Use this information to involve students and make the math relevant. They are so interested when you use real-world data.

Susan M. Caldiero,
Cosumnes River College

◆ ◆ ◆

This section, especially the applications, is very intimidating for students. Start with some examples of shopping: sales tax and percent-off sales.

Ask students: If the price increases by 10% and then decreases by 10%, is the price back to the original price?

For example: One buys a car for $\$25{,}400$, including 5% tax. How much was paid in tax? Most students will say $0.05(25{,}400)$ is the tax paid. Make them tell you how tax is computed in WORDS! Item + Tax = Total, where Tax = Rate $\times$ Item.

David A. Keller,
Kirkwood Community College

Section 2.5

Emphasize to students that there are different types of word problems. As an instructor, pick 7–8 different types of word problems to test them on. Cover 5–6 of them in class with examples and leave at least one or two for them to figure out on their own. Be sure to announce that you are intentionally not covering one or two different types—you want them to figure those out on their own (with resources such as tutoring if they can't get it) because that is the way the real world works when you're a nurse, engineer, etc. A lot of times there won't be example problems to follow in the real world.

Announce that the test covering Chapter 2 will be about 40% word problems, so if they don't master word problems, they will fail the test! This really motivates students to take word problems very seriously. Otherwise they tend to blow them off and fail to realize how often they will see them in other math classes!

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

This is the one area that all students, in all algebra classes will say they "don't get," even before they start the lesson. Take plenty of time on the "Five Steps for Problem Solving in Algebra" (p. 113). This is one of the best examples of how to solve word problems, especially with the stress on familiarization. Giving the students steps to follow is the best help. That is the usual issue—"What do I do next?"

Beth Fraser,
Middlesex Community College–Lowell

Section 2.6

Students will have difficulty recognizing when to reverse the inequality sign when multiplying or dividing by a negative number. Show them how only multiplying or dividing by a negative can change the sign of the answer. Emphasize that the inequality sign will reverse only if the variable has a negative coefficient.

Susan M. Caldiero,
Cosumnes River College

◆ ◆ ◆

The most difficult issue is the multiplication rule for negative numbers. Stress that it is the same as equations, but as soon as you get to a step that says "divide by negative" or "multiply by negative," you reverse the direction of the inequality.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Emphasize that we solve inequalities using the same techniques used to solve equations. The exception is that if we multiply or divide by a negative value, the direction of the inequality symbol must be reversed.

Deana Richmond

Section 2.7

Student *always* find word problems to be the biggest challenge of any algebra class. This class is usually their first main exposure to the more difficult type of word problems. To study for word problems, the students' first task is to understand how to do each type of word problem.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

Encourage students to verify that the final solution makes sense in the context of the problem situation.

Deana Richmond

3 Introduction to Graphing

SECTION TITLES AND TOPICS

TEACHING TIPS

Section 3.1

When graphing points, write the point to be graphed on the board and put an "x" above the x-coordinate and a "y" above the y-coordinate. This consistently reinforces the first number as the x and the second number as the y.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

Teach students that both the horizontal and vertical axes must include labels and scales. The scale along the horizontal axis should be consistent. The scale along the vertical axis should be consistent. They need not be equivalent to each other.

Deana Richmond

Section 3.2

When writing the word "linear" on the board, point out that the first four letters of the word linear are the word "line." You can do so by underlining "line" in linear to help reinforce that a linear graph is a line.

Again, word problems are the students' biggest struggle. Make sure you cover the word problems in this section and work through lots of very detailed examples.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

Students will sometimes want to shortcut and only use two points to graph a line. Stress that yes, it will give them a line, but is it the right one? If three points are a straight line, it is probably correct. If the three points don't form a line, students will also spend a lot of (frustrating) time struggling to figure out which one is wrong. I usually have them find a fourth point, and if three out of four are right, the odd one out is probably the error. Stress the practice of plotting points using a table.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Plan to have numerous discussions about choosing appropriate scales and calculator "viewing windows" especially with applications.

David A. Keller,
Kirkwood Community College

Section 3.3

When graphing using intercepts, emphasize that these are points and insist that student write them as points, or ordered pairs. They sometimes become confused and write the x-intercept and y-intercept as a single point. Instead of $(3, 0)$ and $(0, 4)$, they will graph $(3, 4)$. Also, remind them to divide by the exact coefficient of each variable when solving—negative signs often get dropped!

Show them when it is best to graph using intercepts and when it is better to use the table. Also, students should recognize that horizontal and vertical lines will only have an intercept on ONE axis…it is impossible for these lines to have both an x- and y-intercept (unless it is $x = 0$ or $y = 0$).

Susan M. Caldiero,
Cosumnes River College

◆ ◆ ◆

Spend time on examples like $y = -4$ and $x = 3$. Students have a great tendency to confuse the equations of horizontal and vertical lines. If they understand this concept here, when you add the idea of slopes that are zero and undefined, it can be easier, because they have seen these as special cases.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Students need multiple explanations as to why you let $y = 0$ to find an x-intercept and let $x = 0$ to find a y-intercept. Start by drawing a linear graph and ask the students to circle the intercepts. Most can easily find intercepts on a picture.

David A. Keller,
Kirkwood Community College

Section 3.4

Again, announce how many word problems will be on the test, making at least 40% of the test word problems so students will take them seriously.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

Focus students' attention on the units of the answer. The numbers are the easy part!

David A. Keller,
Kirkwood Community College

◆ ◆ ◆

This section presents a great opportunity to focus on word problems. Students can readily apply the concepts of earlier sections to real-life situations, many of which will be familiar to them.

Deana Richmond

Section 3.5

Use the idea of skiing to point out that slope measures the "steepness" of the line, employing the examples of bunny slope (horizontal line), intermediate slope (slant line) and mogul run (vertical line).

Susan M. Caldiero,
Cosumnes River College

◆ ◆ ◆

It is important to be sure that students understand what "slope" means. Make sure that they can calculate a slope, and can see the difference between positive and negative slopes. If they do, they will be able to spot errors in graphing lines.

Here the ideas of the zero and undefined slopes add to the special facts about the horizontal and vertical lines.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Have students use the slope formula to attempt to calculate the slope of a horizontal line $y = b$ and a vertical line $x = a$. For example, students can identify two points on a horizontal line, and observe that since the y-coordinates are equal, then $m = \frac{y_2 - y_1}{x_2 - x_1} = 0$. Likewise, since any two points on a vertical line have the same x-coordinate, the slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ will be undefined.

Deana Richmond

Section 3.6

Emphasize here that there will be three main ways to graph a line: Using x and y intercepts, using the slope-intercept form and using the point-slope form.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

Show that the slope-intercept form can be used to find the equation of a line containing two points by finding the slope and then solving for b by substituting in the values for x and y with one of the points.

Susan M. Caldiero,
Cosumnes River College

◆ ◆ ◆

This is the most important form of an equation to know. Students should be able to use this method of graphing as well as plotting points. If they can, then they really do understand the definitions of both the y-intercept and the slope.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

I'm always surprised by how many students start graphing by using b as the x-intercept rather than the y-intercept. You should supplement the exercises with graphs and ask for the equation of the lines that are given.

David A. Keller,
Kirkwood Community College

Section 3.7

Emphasize the point-slope form requires a *point and a slope*, and that the slope-intercept form requires a *slope and the y-intercept*. Teach them that the name of the equation emphasizes what is needed to write an equation of a line.

Be sure to blend in some problems from Section 3.6 with this section so students can practice identifying which form they need to graph the line, slope-intercept or point-slope.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

Students struggle to remember the point-slope formula. Show them how similar point-slope form is to slope. Most students remember $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Notice point-slope is the same formula after you multiply by the LCD.

David A. Keller,
Kirkwood Community College

4 Polynomials

SECTION TITLES AND TOPICS

4.1 Exponents and Their Properties
Multiplying Powers with Like Bases ● Dividing Powers with Like Bases ● Zero as an Exponent ● Raising a Power to a Power ● Raising a Product or a Quotient to a Power

4.2 Negative Exponents and Scientific Notation
Negative Integers as Exponents ● Scientific Notation ● Multiplying and Dividing Using Scientific Notation

4.3 Polynomials
Terms ● Types of Polynomials ● Degree and Coefficients ● Combining Like Terms ● Evaluating Polynomials and Applications

4.4 Addition and Subtraction of Polynomials
Addition of Polynomials ● Opposites of Polynomials ● Subtraction of Polynomials ● Problem Solving

4.5 Multiplication of Polynomials
Multiplying Monomials ● Multiplying a Monomial and a Polynomial ● Multiplying Any Two Polynomials

4.6 Special Products
Products of Two Binomials ● Multiplying Sums and Differences of Two Terms ● Squaring Binomials ● Multiplications of Various Types

4.7 Polynomials in Several Variables
Evaluating Polynomials ● Like Terms and Degree ● Addition and Subtraction ● Multiplication

4.8 Division of Polynomials
Dividing by a Monomial ● Dividing by a Binomial

TEACHING TIPS

Section 4.1

When introducing the quotient rule, start by solving an equation like $x+5=7$ and show that when 5 moves across the equal symbol the number changes signs. Then demonstrate the quotient rule and show that when the exponent moves above or below the "division bar," the exponent "changes signs."

Susan M. Caldiero,
Cosumnes River College

◆ ◆ ◆

When looking at examples like $\left(-5x^4\right)^2$, take the time to stress that -5 is squared.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Students will likely ask what happens if the exponent in the denominator is larger than exponent in numerator using quotient rule. The textbook waits to explain this in Section 4.2, but when students ask, I show them right away.

David A. Keller,
Kirkwood Community College

Section 4.2

Students will try to change signs on many of these problems as soon as they see a negative exponent. At this point, consider taking a good amount of time and reviewing the entire set of rules, as summarized in the table "Definitions and Properties of Exponents" (p. 245). Remind them that you are not changing other rules, merely adding a new one.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Remind students there is almost always more than one way to arrive at the correct answer. It's valuable to ask other students to outline a different sequence of steps to arrive at the same answer.

David A. Keller,
Kirkwood Community College

◆ ◆ ◆

Remind students that raising an expression to a negative exponent does not change the sign of the expression.

Deana Richmond

Section 4.3

Here the idea of a coefficient needs to be carefully illustrated, so that students use the correct signs. Problems which ask students to identify a coefficient (for example, Example 3 on p. 250) are a good start. The discussion of the differences among degree, coefficient and term is also time well spent. Being able to combine like terms correctly will depend upon understanding these concepts.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Watch for students adding exponents when they collect like terms.

David A. Keller,
Kirkwood Community College

Section 4.4

The most frequently missed problems in this section are like those that require students to subtract polynomials. Students tend to distribute the negative sign to only the first term. Do many problems of this type to be sure that students understand this concept.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

The biggest problem here will be with the signs in subtraction. Be careful how the vertical problems are presented. Take them through the extra steps as indicated in Example 7 on p. 259 of the text.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Distributing a negative requires constant review.

David A. Keller,
Kirkwood Community College

Section 4.5

When doing this section, emphasize that the students need to really master this section so that they are able to do it backwards, because that's exactly what they will be doing in Chapter 5!

Dean Barchers,
Red Rocks Community College

Section 4.6

Students will want to distribute the exponent for binomial squares. Remind them to use the definition of squaring to show that this is inappropriate. For example, $(a+b)^2 = (a+b)(a+b)$. This will avoid the mistake.

Susan M. Caldiero,
Cosumnes River College

◆ ◆ ◆

FOIL is one of the most important systems for students to be familiar with, especially when they try to factor. Stress the patterns that the product has when it is multiplied, so that you can refer back to it when you reach factoring.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Stress the difference between expressions like $3xy^2$, $3(xy)^2$ and $(3xy)^2$.

David A. Keller,
Kirkwood Community College

Section 4.7

Extend the definitions of like terms and degrees that were presented in Section 4.3 to include polynomials in several variables.

Deana Richmond

Section 4.8

This is by far the most difficult section of the chapter for many students. Be sure to announce that so your students can "gear up" for it.

When doing polynomial division, point out that they are really doing a four-step cycle over and over again:

1. Divide the highest degree term by the highest degree term.
2. Multiply.
3. Subtract.
4. If needed, bring down a term.

When explaining how to do polynomial division, write down a number division problem, such as 523 divided by 21, next to the equation. Do each problem side by side, alternating back and forth between the polynomial division and the number division, step by step, showing how very similar polynomial division is to number division.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

Begin by doing a whole number division problem and talk about the steps, then do a problem using variables. When doing the division, make students write out each step.

Susan M. Caldiero,
Cosumnes River College

◆ ◆ ◆

When working with long division, have the students look for the missing terms first. Many students will find it helpful to include the missing term(s) in the dividend (Example 5 on p. 292).

David A. Keller,
Kirkwood Community College

5 Polynomials and Factoring

SECTION TITLES AND TOPICS

TEACHING TIPS

Section 5.1

Students need to recognize that factoring $2(x+3)+3x(x+3)$ is the same as factoring $2y+3xy$. To accomplish this, have them factor $2y+3xy$ and then underneath it write $2(x+3)+3x(x+3)$ and write both $(x+3)$ and y in colored pen. This helps them see that they can be factored the same way. I would suggest doing two more examples where the common factor is written in a different color and then have them try it.

Susan M. Caldiero,
Cosumnes River College

◆ ◆ ◆

In factoring by grouping, the two areas where students tend to have a problem are with the signs (especially if the sign of the third term is negative), and the use of the number 1. If there is a common factor of 1 or -1, students often will forget to write it out (and lose it in the final step), or will neglect to write it if it is the factor that is left. For example, $6x+3=3(2x)$.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Factoring is the one math topic students have more difficulties with than any other topic. While I don't try to scare students with this, I do emphasize early on the importance of mastering the various patterns. Remind the students that "factors" means "multipliers," nothing more.

Factoring by grouping can be difficult because students often omit the "1."

Marty Hodges,
Colorado Technical University

◆ ◆ ◆

Review the difference between a factor and a term at the very beginning of this section.

David A. Keller,
Kirkwood Community College

Section 5.2

In both Sections 5.2 and 5.3, stress the sign patterns from FOIL. Signs are the biggest error that students will make.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Page 314 has a great explanation of "reverse FOIL." The manner in which gradually more difficult factorizations are arranged in this section will be helpful to the students. I go slowly with this as it takes students some time to mentally digest the sign patterns.

It is also important to show a prime polynomial in order to help the students realize that not all polynomials are factorable.

Marty Hodges,
Colorado Technical University

◆ ◆ ◆

Students need constant reminders to check for common factors of the entire expression first.

David A. Keller,
Kirkwood Community College

Section 5.3

Talk about when it is advantageous to factor using the grouping method and when it is better to use "guess and check."

Susan M. Caldiero,
Cosumnes River College

◆ ◆ ◆

Stress to students that the factorization of a polynomial expression is unique. There are no other prime factors that will multiply to give that polynomial.

Deana Richmond

Section 5.4

Review the special product formulas before beginning to factor them. Remind the students to look for squares in the first and last terms.

Susan M. Caldiero,
Cosumnes River College

◆ ◆ ◆

Often referred to as "special products," simple pattern recognition is essential. I note too that the perfect square trinomial patterns differ only in sign. And after differences of squares, a simple inclusion that "sums of squares do not have a formula for factoring" is easy to demonstrate.

Marty Hodges,
Colorado Technical University

Section 5.5

Have students make a list of perfect cubes and cube roots and learn them. This will help them recognize a perfect cube in a factoring problem.

Nikki D. Handley,
Odessa College

◆ ◆ ◆

When discussing the sum and difference of cubes, I start by asking the students what value of x will make $x^3 - 8 = 0$. After they come up with $x = 2$, we discuss how is if $x = 2$ is a solution then $x - 2$ is a factor. We then divide $x^3 - 8$ by $x - 2$ to get the other factor of $x^2 + 2x + 4$.

Furthermore, I stress that when using the sum and/or difference of cubes, if the resulting trinomial factor has degree of two, then it is prime in the real numbers.

I tell my students that if there is a choice between using a difference of squares and a difference of cubes, they should use the difference of squares. Make sure you compare and contrast factoring $x^6 - 64y^6$ with factoring $x^6 + 64y^6$.

Matthew Hudock,
St. Philip's College

◆ ◆ ◆

To help students memorize the formulas used to factor perfect-square trinomials and differences of squares, call attention to certain patterns, especially in the signs of the terms.

Deana Richmond

Section 5.6

I point out to students that on the test they will not be told which type of factoring to use and so they will have to identify the best strategy. I recommend using the following steps.

First try to factor what you can out of every term. Then use the following identification method to try to factor. If the polynomial has:

2 terms	Factor with the difference of two squares.
	There is no formula to factor the addition of two squares.
3 terms	If $a = 1$, use methods of section 5.2.
	If $a \neq 1$, use methods of section 5.3.
4 terms	Use factoring by grouping.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

This makes an excellent summary and gives you an opportunity to review, and to start to help the students make choices about methods. I like to stress that students learn to consider what is happening in each problem, rather than to mechanically use a system by rote.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

When entering this area, I usually warn students that we are now entering a “mixed bag” of factoring. Previously, each pattern had its own little section—now we’re on our own! No new patterns, but matching any one problem to the proper pattern requires care and sometimes patience.

Marty Hodges,
Colorado Technical University

◆ ◆ ◆

Emphasize that when factoring an expression that is both the difference of squares and the difference of cubes, students should begin by factoring as the difference of squares.

Deana Richmond

Section 5.7

Point out that after you get everything on one side and factor, you will have two expressions multiplied together equaling zero. Point out that the only way you can multiply two expressions together to get zero is if one of those expressions is zero. Write on the board $(\ )(\ )=0$ and ask them what one of the missing expressions must be to get zero. Someone will tell you one of them has to be zero. To reinforce this concept, challenge them to come up with two nonzero numbers multiplied together resulting in zero. They will agree with you that it is impossible to have two nonzero numbers multiplied together resulting in zero. They will also agree with you that at least one term has to be zero. So if you have $(x+3)(x-2)=0$, then the *only* way that the equation can be true is if the first expression $x+3=0$ or if the second expression is $x-2=0$.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

When taking a test on this chapter, students will be confused about when to factor only and when to solve the factors. Emphasize that they can only solve when there is an equal sign in the problem.

Susan M. Caldiero,
Cosumnes River College

◆ ◆ ◆

Stress the difference between equations and expressions, and solving and simplifying or factoring. Once students learn to solve equations, they sometimes develop a habit of solving everything. They will assume an $=0$ after each expression and try to solve it. Or, will just factor equations instead of solving them. They need to be clear on exactly what the terms “factor” and “solve” mean, and when they do which one.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Consider $2x^2+10x+12=0$. After factoring it is $2(x+3)(x+2)=0$. Many students are unsure what to do with the 2. I give two explanations:

(A) You can multiply or divide both sides of the equation by any non-zero constant—therefore, multiply by $\frac{1}{2}$.

(B) The zero product rule says if three factors are multiplied together and the result is 0, then at least one of the three factors must be zero. In this case, one of the factors is 2, which can’t be 0. Therefore, the zero must be one of the other two factors.

David A. Keller,
Kirkwood Community College

Section 5.8

Again, stress the key to this section is identifying the types of word problems and being able to identify what type of problem is given: “right triangle problem,” “area” problem, etc.

Dean Barchers,
Red Rocks Community College

◆ ◆ ◆

Emphasize the need for answers to pass the “reality check.”

David A. Keller,
Kirkwood Community College

◆ ◆ ◆

Emphasize that students will be using the same ideas learned earlier in the chapter. In this section, they must read each question carefully and write a corresponding equation.

Deana Richmond

6 Rational Expressions and Equations

SECTION TITLES AND TOPICS

6.1 Rational Expressions
Restricting Replacement Values ● Simplifying Rational Expressions ● Factors that Are Opposites

6.2 Multiplication and Division
Multiplication ● Division

6.3 Addition, Subtraction, and Least Common Denominators
Addition When Denominators Are the Same ● Subtraction When Denominators Are the Same ● Least Common Multiples and Denominators

6.4 Addition and Subtraction with Unlike Denominators
Adding and Subtracting with LCDs ● When Factors are Opposites

6.5 Complex Rational Expressions
Using Division to Simplify ● Multiplying by the LCD

6.6 Rational Equations
Solving Rational Equations

6.7 Applications Using Rational Equations and Proportions
Problems Involving Work ● Problems Involving Motion ● Problems Involving Proportions

TEACHING TIPS

Section 6.1

Students will decide immediately that they cannot do this section because of their issues with fractions. Keep the first examples as easy as possible.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Showing the division line itself acts in place of parentheses can be illustrated by placing all on one line, like $\frac{(x+2)}{x}$.

Marty Hodges,
Colorado Technical University

◆ ◆ ◆

It is imperative that students understand the difference between factors/factored form and terms/expanded form. Have them do a worksheet in class where they must differentiate between the two and the list the factors or terms.

Expression	Form?	Factors/Terms
$2x^3-14x^2+10$	*Expanded*	*terms*: $2x^3$, $-14x^2$, 10
$(3x^4+5)(2x-18)$	*Factored*	*factors*: $3x^4+5$, $2x-18$

Cindy Katz,
St. Philip's College

◆ ◆ ◆

To confirm students' understanding of factor and term, ask, "Which of these fractions can be simplified—$\frac{ab}{ac}$; $\frac{a+b}{a+c}$; $\frac{a+b}{ac}$ —and why?"

David A. Keller,
Kirkwood Community College

Section 6.2

Encourage liberal use of parentheses; it helps students to remember that only factors, not terms, cancel.

Marty Hodges,
Colorado Technical University

◆ ◆ ◆

When doing arithmetic with algebraic fractions, the first thing to check is what operation is happening. Remember GCF is used when multiplying and dividing; LCD when adding and subtracting. On my exam, I often will choose the same two fractions to include in two separate questions, one multiplication and one addition. 10%–20% of students do not see that the questions are different. Example: $\frac{3}{2x}+\frac{4x}{x^2}$ and $\frac{3}{2x}\cdot\frac{4x}{x^2}$.

David A. Keller,
Kirkwood Community College

Section 6.3

Comparing the LCM and LCD is very useful; often students don't understand that they're essentially the same thing.

Marty Hodges,
Colorado Technical University

◆ ◆ ◆

Include some intermediate examples between the simplest: $\frac{1}{12}$ and $\frac{7}{30}$ and more difficult examples (p. 392).

For example:

Find the LCD of $\frac{2}{5x}$ and $\frac{8}{5}$.

Find the LCD of $\frac{7}{14x^2}$ and $\frac{6}{x^2}$.

Find the LCD of $\frac{1}{16x^3}$ and $\frac{3}{2x}$.

Cindy Katz,
St. Philip's College

◆ ◆ ◆

To build the LCD, factor each denominator and compare. What does one have that another is missing? The LCD contains each factor the greatest number of times it appears in any one denominator.

Deana Richmond

Section 6.4

This section will be the most difficult one in the chapter because addition and subtraction of fractions are difficult for many students. They need strong factoring skills. If there are any weaknesses there, that will compound the issue. All problems in this section will need to be done slowly. I would do just addition first and spend time to be sure students understand this concept, before introducing the subtraction.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

When building the LCD: after factoring denominators, compare—what's this one have that the other one's missing? Get them to match with no "overlaps" and you have that LCD.

Marty Hodges,
Colorado Technical University

◆ ◆ ◆

Remind students often that we add (or subtract) rational expressions as we do fractions; we must have a common denominator.

Deana Richmond

Section 6.5

Here is another place where students have the chance to choose a method. Stress the differences between the methods, and discuss the pros and cons, so that they again get away from rote mechanics.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

It is good to show two methods for simplifying these "fractions within fractions"—and to make a distinction on the appropriateness of each method. Usually, I just refer to these as compound algebraic fractions and compare by showing analogous compound arithmetic fractions.

Marty Hodges,
Colorado Technical University

◆ ◆ ◆

Many students find it helpful to write complex fractions in horizontal form and then follow the order of operations. Example:

$$\frac{\frac{3}{x}+\frac{1}{2x}}{\frac{1}{3x}-\frac{3}{4x}}=\left(\frac{3}{x}+\frac{1}{2x}\right)\div\left(\frac{1}{3x}-\frac{3}{4x}\right).$$

Deana Richmond

Section 6.6

Again, you will see students trying to solve expressions. Stressing vocabulary is important here.

Beth Fraser,
Middlesex Community College–Lowell

◆ ◆ ◆

Make sure students understand the difference between an expression and an equation and the goals of each. We simplify expressions and end up with an expression as a result. We solve equations and end up with a value as a result.

Cindy Katz,
St. Philip's College

◆ ◆ ◆

Although the text does not present this method, some students may prefer to bring all nonzero terms to one side of the equation and write as a single rational expression. The student can then solve by setting each factor in the numerator of the expression to zero.

Deana Richmond

Section 6.7

Using the same steps as in Section 5.8 shows consistency and connectivity. And the text includes a tool that has something here I use as well—working the problem incorrectly to show how careful one must be in interpreting wording. Once the set-up is achieved, one must merely follow the precepts from earlier sections to find the solution.

Marty Hodges,
Colorado Technical University

◆ ◆ ◆

It should be noted that an equation may yield solutions that do not make sense in the context of the original problem.

Deana Richmond

7 Functions and Graphs

SECTION TITLES AND TOPICS

TEACHING TIPS

Section 7.1

This section will be very difficult for most students and should be taught over more than one lecture, if possible. My students always appear to understand the material when I go over it but on further review I notice that they have little comprehension of functions. A problem that always arises is the desire to make the function definition into a variable. For example, when given $f(5)=20$, they will divide both sides by 5 and write $f=4$. You cannot do enough examples like Problems #27–36 on p. 451. Most students at this level don't have enough experience reading graphs in general to fully understand what they are looking for when you ask them to find $f(1)$. The substitution problems (#43–48 on pp. 451–452) are generally the easiest type of problem for most students.

Malissa Trent,
Northeast State Community College

◆ ◆ ◆

After we discuss the concept of functions, I ask students for examples of functions in their lives. We go around the room until everyone has participated. Obviously, I help those who need assistance to find a function in their world.

I then ask students to compute $3\cdot 5$ using their calculators. They quickly get an answer. Then I ask them to compute it again. They act puzzled, and usually at least one student states how this is silly because the calculator is going to give the same answer again for $3\cdot 5$. I tell them I agree and that will happen every time you type in $3\cdot 5$. I then explain that is the meaning of a function. If the calculator quits acting like a function, they should replace the batteries or get a new calculator.

Tim Chappell,
Penn Valley Community College

Section 7.2

Tell students you're changing to code words for x and y. They will now be referred to as domain and range respectively to protect their identity.

Nikki D. Handley,
Odessa College

◆ ◆ ◆

Plan to spend several days on this section, and you will need many supplementary handouts. There is nothing about this section that is simple for the students and **many will never understand domain and range** (reading graphs). If they are able to read the graphs, they will find it difficult to express answers in set notation. I have a graphing packet that I hand out at the beginning of this chapter. It includes 10 pages of graphs that I download from my calculator screen and I have an overhead copy of the packet. We work together on finding the domain and range of most of the graphs in class. I first teach them how to state the domain and range in words. We write down, “All real numbers that are less than 6.” When I feel that the students are starting to understand how I am getting the domain and range, I then work on converting the sentences to set notation.

Piecewise functions are also introduced in this section. You will need to completely explain many examples. They cannot read piecewise functions without help.

Malissa Trent,
Northeast State Community College

◆ ◆ ◆

Emphasize the difference between mathematical domain and problem domain.

I use movie theater prices to introduce piecewise-defined functions. A moviegoer has to find which domain they are in; then they can see the price they have to pay.

Tim Chappell,
Penn Valley Community College

Section 7.3

A review of the slope-intercept form of a line is appropriate before this section. Communicate the importance of defining the independent variable so that the y-intercept is a relevant data point.

Have students list the slope and y-intercept for each of the problems you assign. Be sure to have them explain in context of the problem what the slope and y-intercept represent.

Tim Chappell,
Penn Valley Community College

Section 7.4

When using the Algebra of Functions, help students see that they will get the same answer either way the work the problem. For example, $(f+g)(x)$ gives them the same answer as $f(x)+g(x)$.

Nikki D. Handley,
Odessa College

◆ ◆ ◆

The first part of this section will be easy for most students, but finding the domain of the sum, difference, product, or quotient of two functions will be hard for them. **They still won’t understand domain.** They will have memorized a few rules (such as that the denominator of a fraction cannot equal zero), but mainly seem to guess at the answers. Do not be discouraged if you have to teach set notation again in this section as if you never covered it. The topics in this chapter are much more abstract than anything the students have done in the past and they need constant reinforcement of topics taught earlier in the chapter.

Malissa Trent,
Northeast State Community College

8 Systems of Linear Equations and Problem Solving

SECTION TITLES AND TOPICS

TEACHING TIPS

Section 8.1

I stress the concept of systems of equations in two variables. The main idea here is that these systems represent equations of two lines. When they are not parallel, you have a unique solution. Otherwise, they either are the same line or the system is inconsistent.

Donald W. Solomon,
University of Wisconsin-Milwaukee

Section 8.2

I give a quiz over Sections 8.1 and 8.2. I give four systems, and they have to choose three systems to solve. They must solve one by graphing, one by substitution, and one by elimination. I have already modeled this method selection process in class. I find the quiz helps them to identify the strengths and weaknesses of each method.

Tim Chappell,
Penn Valley Community College

◆ ◆ ◆

When solving by substitution or elimination, I have them check their system for parallel lines before they compute. Then I try to show them how to minimize computational effort.

Donald W. Solomon,
University of Wisconsin-Milwaukee

◆ ◆ ◆

I find that it is helpful to remind students that we are working for ordered pairs that make both equations true *simultaneously*. Although we are not solving graphically in this section, I continue to remind students by using a sketch or a computer generated graph that the point(s) that make(s) both equations true *simultaneously* is (are) the point(s) where the graphs intersect.

Laurie Hurley,
Mt. Mansfield Academy

Section 8.3

I ask my students to estimate the answer to the application first before solving the system. This reinforces their reasoning skills and helps them to quickly identify unreasonable answers.

I help students learn to use tables in increments. First I walk them through a couple of problems from start to finish. Then I give them only the column and row headings, and ask them to fill in the rest.

Tim Chappell,
Penn Valley Community College

◆ ◆ ◆

There are only two classifications of problems in this application section. Students seem to have difficulty identifying patterns. As they are attempting to reason out the problems, I work at pattern identification.

Donald W. Solomon,
University of Wisconsin-Milwaukee

Section 8.4

To demonstrate 3×3 systems visually, I use the walls and floors of the room. I ask students to find the intersection of the west wall, the north wall, and the floor. I usually pick the corner where the trash can is located. You can use this approach to see how you can choose different combinations to still arrive at the same point: west wall and north wall combined with west wall and floor, west wall and north wall combined with north wall and floor, etc.

Tim Chappell,
Penn Valley Community College

◆ ◆ ◆

I find that students struggle with the vocabulary for independent and dependent equations in systems with more than two equations. I relate an independent equation to someone with an independent personality who joins a group of persons. Because the newcomer is independent, she adds something to the conversation. Someone whose personality is entirely dependent on the others will add nothing to the conversation.

Laurie Hurley,
Mt. Mansfield Academy

Section 8.7

I don't treat this as a separate section. I introduce matrices in Sections 8.2 and 8.4. I usually spend twice as much time in Section 8.4 that way, but students really appreciate it in Section 8.5. I require students to solve only some of the problems in Section 8.5 by hand. They get to set up each of the remaining problems as a system, rewrite as a matrix, and then use technology to find the answers.

Tim Chappell,
Penn Valley Community College

9 Inequalities and Problem Solving

SECTION TITLES AND TOPICS

TEACHING TIPS

Section 9.1

Students often forget the rule about reversing the inequality symbol when multiplying or dividing both sides of an inequality by a negative number. Consequently, I try to really emphasize this idea in class by using a simple numeric inequality that the students can easily identify as true. I go through a whole series of problems where I multiply and divide the true inequality by both positive and negative numbers to illustrate that the statement does not remain true when using the negative number unless the inequality symbol is reversed. I believe this helps many students to develop a better understanding of the concept, and increases the number of those who retain it when doing problems.

Michelle Jackson,
Bowling Green Community College

◆ ◆ ◆

In checking solutions to an inequality, there are two checks to make. First, students need to check to make sure they have the critical number right. Second, they need to check that their interval is correct.

Tim Chappell,
Penn Valley Community College

◆ ◆ ◆

Many students will want to reverse the inequality symbol every time they see a negative sign in a problem. For example, given $x+3<12$, they will subtract 3 from both sides and write the next line as $x>9$. Another common mistake occurs in this situation: Given $4x<-12$, they will write $x>-3$.

Malissa Trent,
Northeast State Community College

◆ ◆ ◆

I stress that the algebraic manipulation of inequalities works the same way as it does with equations, except for when you multiply or divide both sides by a negative number—then the inequality reverses.

Donald W. Solomon,
University of Wisconsin-Milwaukee

Section 9.2

One thing that is important when discussing compound inequalities is to make sure that students not only understand that "and" is the same as intersection and "or" is the same as union but they understand the mathematical use of these conjunctions. For example, I use the statement "I am going out to dinner *or* a movie" and ask them what their expectations are. I emphasize that in algebra "or" means that one or the other or both is true, which is slightly different than how we usually mean it in conversation. The mathematical use of "and" matches the conversational use that both are true.

Michelle Jackson,
Bowling Green Community College

◆ ◆ ◆

I rather loudly stress the words "and" and "or" as we work on conjunction and disjunction. It tends to wake them up.

Donald W. Solomon,
University of Wisconsin-Milwaukee

◆ ◆ ◆

To teach basic properties of sets, I play the string game with my students. I divide the room into teams and give each team a list of numbers to use. Then I draw a two-ring Venn diagram with a red circle and a blue circle. Each circle has a specific property that I know, but do not reveal to the students. The circles do not have to have an intersection, but the game works best if they do. Students take turns calling out a number from the list and I put the number in the appropriate place on the diagram (always make a key before you start). They learn which numbers fall in which circle, which numbers are in the intersection, and which numbers are not in either circle. The game continues until someone figures out the rule for each circle (e.g., "odd numbers less than 50," "prime numbers less than 30," "multiples of 3," "divisors of 48").

Malissa Trent,
Northeast State Community College

Section 9.3

I use the absolute-value principle to solve the equations. I use the same principle to find the critical numbers for the inequalities. Then we test intervals to determine which intervals satisfy the inequality.

Tim Chappell,
Penn Valley Community College

◆ ◆ ◆

Absolute-value equations and inequalities are the most difficult concepts for students in this chapter. I recommend dividing the material over two days and covering equations on the first day and the inequalities on the second. I have also found that students do not seem to retain this information well after the chapter is finished. I suggest putting these types of problems on subsequent quizzes as a type of cumulative review. This will help the students master the concept for the final exam.

Michelle Jackson,
Bowling Green Community College

◆ ◆ ◆

It is a long road to convince students that only $|p| < a$ or $|p| \le a$ will give them conjunctions. Each time we do an absolute-value problem, I draw pictures. Eventually it pays off.

Donald W. Solomon,
University of Wisconsin-Milwaukee

◆ ◆ ◆

There are two major mistakes that I see repeated among my students when they are solving absolute-value equations: In the problem $4|x+6|=12$, they will distribute the 4 as if the absolute value bars are parentheses. In the problem $|x+3|-10=-4$ they will write “no solution” before adding the 10 to both sides. You must emphasize the need to isolate the absolute-value before decisions are made regarding the solution(s).

Malissa Trent,
Northeast State Community College

Section 9.4

I require colored pencils for this section. Students who try to do this in ink make huge messes. Although students can vary shading using a pencil, I find that colored pencils work much better and the work tends to be neater when colored pencils are used. To keep supplies cost effective, I recommend that several students share a pack of pencils. Each student will only need three or four colors.

Malissa Trent,
Northeast State Community College

Section 9.5

Neat and accurate graphs enhance understanding considerably, and become increasingly important the more constraints there are. It is worth the time and effort to produce a good graph.

Laurie Hurley,
Mt. Mansfield Academy

10 Exponents and Radicals

SECTION TITLES AND TOPICS

10.1 Radical Expressions and Functions
Square Roots and Square-Root Functions ● Expressions of the form $\sqrt{a^2}$ ● Cube Roots ● Odd and Even nth Roots

10.2 Rational Numbers as Exponents
Rational Exponents ● Negative Rational Exponents ● Laws of Exponents ● Simplifying Radical Expressions

10.3 Multiplying Radical Expressions
Multiplying Radical Expressions ● Simplifying by Factoring ● Multiplying and Simplifying

10.4 Dividing Radical Expressions
Dividing and Simplifying ● Rationalizing Denominators or Numerators with One Term

10.5 Expressions Containing Several Radical Terms
Adding and Subtracting Radical Expressions ● Products of Two or More Radical Terms ● Rationalizing Denominators or Numerators with Two Terms ● Terms with Differing Indices

10.6 Solving Radical Equations
The Principle of Powers ● Equations with Two Radical Terms

10.7 The Distance Formula, the Midpoint Formula, and Other Applications
Using the Pythagorean Theorem ● Two Special Triangles ● The Distance Formula and the Midpoint Formula

10.8 The Complex Numbers
Imaginary Numbers and Complex Numbers ● Addition and Subtraction ● Multiplication ● Conjugates and Division ● Powers of i

TEACHING TIPS

Section 10.1

Here, I like to add that "c-squared equals a" implies that c is the square root of a to help relate the square root concept.

Marty Hodges,
Colorado Technical University

◆ ◆ ◆

Also, make sure they know the perfect squares through at least 225 and their roots.

Beth Fraser,
Middlesex Community College-Lowell

◆ ◆ ◆

When working with nth roots, have the students ask themselves, "What multiplied *by itself* index number of times will result in the radicand?"

Cindy Katz,
St. Philip's College

Section 10.2

Referring to rational exponents simply as "fraction" exponents sometimes helps students see this as an "Aha!"

Marty Hodges,
Colorado Technical University

Section 10.3

In simplifying radical expressions, students will get confused as to which numbers come "out" and which ones stay "under" the radical sign. They will try shortcuts a bit too soon with the simpler examples and then get stuck with the more difficult problems. Encourage them to write out everything for a while.

Beth Fraser,
Middlesex Community College-Lowell

◆ ◆ ◆

Quite often, students will feel more comfortable working with the "numbers" first and then coming back and working with the variables. For example: $\sqrt{32x^{15}} = \sqrt{16 \cdot 2x^{15}} = 4\sqrt{2x^{15}}$. Also, factoring $\sqrt{x^{15}}$ as $\sqrt{x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x}$ and then counting the number of x^2's to bring out of the radical helps some students to better see what's going on.

Cindy Katz,
St. Philip's College

◆ ◆ ◆

Remind the students that the roots must be the same when they are multiplying radical expressions. Also, it is usually easier to multiply the radicands first and then simplify.

Sharon Testone,
Onondaga Community College

Section 10.4

When explaining denominator rationalization, it's easy for students to remember to "Just use the denominator divided by itself as an alternate form of 1."

Marty Hodges,
Colorado Technical University

Section 10.5

When rationalizing denominators, be sure that the students understand the difference between: $\sqrt{x+2}$ and $\sqrt{x}+2$. I emphasize that the sign is under the radical in one case and "free" from the radical in the second case.

Sharon Testone,
Onondaga Community College

◆ ◆ ◆

Stress that when any operation is done with a radical expression, the final results must also be simplified.

Beth Fraser,
Middlesex Community College-Lowell

◆ ◆ ◆

Comparing like radicals to like terms (presented in a previous chapter) makes it easier for students to relate to adding and subtracting radicals.

Marty Hodges,
Colorado Technical University

◆ ◆ ◆

Strongly emphasize the analogy of combining like terms.

Cindy Katz,
St. Philip's College

◆ ◆ ◆

The students manage to multiply radical expressions, but when they divide and rationalize, you will think they never learned FOIL or any other form of the distributive law. Here's your chance to review patterns again.

Donald W. Solomon,
University of Wisconsin-Milwaukee

Section 10.6

Note to the students that we use the same rules as we did when solving linear equations: what you do to one side of an equation, you do to the other. Emphasize the word “side.” Students must be careful squaring binomials—all of a sudden, they have never heard of FOIL.

Beth Fraser,
Middlesex Community College- Lowell

◆ ◆ ◆

It’s critical to check answers in the original equation since squaring may introduce (dare I say) “spare roots”—sorry, no pun intended.

Marty Hodges,
Colorado Technical University

Section 10.7

I like to geometrically illustrate the theorem by drawing a 3-4-5 right triangle on the board and actually counting the squares of the two smaller areas to show how they match the squares in the bigger (hypotenuse) area.

Marty Hodges,
Colorado Technical University

◆ ◆ ◆

Emphasize the difference between $x^2 = n$ (two answers, $\pm\sqrt{n}$) and $x = \sqrt{n}$ (one answer, the principal square root). Explain when each case is used and why this section will only have positive answers even though we get two answers from the original problems.

Cindy Katz,
St. Philip’s College

◆ ◆ ◆

Be sure to spend a sufficient amount of time on the distance formula and the midpoint formula because they will be needed when working with circles. Although the formulas appear quite simple, students often have difficulty with them.

Sharon Testone,
Onondaga Community College

Section 10.8

Encouraging students to refer to the $\sqrt{-1}$ early on simply as “i” helps dispel fears that this is some sort of voodoo math.

Marty Hodges,
Colorado Technical University

◆ ◆ ◆

When adding, subtracting or multiplying complex numbers, compare these operations to working with binomials. For example: $(2x+3)-(6x+4)$ can be compared with $(3+2i)-(4+6i)$. When dividing complex numbers, compare this operation to rationalizing denominators.

Sharon Testone,
Onondaga Community College

◆ ◆ ◆

I like to have students make a Venn diagram of the sets of numbers: complex, real, rational, irrational, integer, whole, natural, imaginary, and pure imaginary numbers, and give examples of each.

Laurie Hurley,
Mt. Mansfield Academy

11 Quadratic Functions and Equations

SECTION TITLES AND TOPICS

11.1 Quadratic Equations
The Principle of Square Roots ● Completing the Square ● Problem Solving

11.2 The Quadratic Formula
Solving Using the Quadratic Formula ● Approximating Solutions

11.3 Studying Solutions of Quadratic Equations
The Discriminant ● Writing Equations from Solutions

11.4 Applications Involving Quadratic Equations
Solving Formulas ● Solving Problems

11.5 Equations Reducible to Quadratic
Equations in Quadratic Form ● Radical Equations and Rational Equations

11.6 Quadratic Functions and Their Graphs
The Graph of $f(x) = ax^2$ ● The Graph of $f(x) = a(x-h)^2$ ● The Graph of $f(x) = a(x-h)^2 + k$

11.7 More About Graphing Quadratic Functions
Graphing $f(x) = ax^2 + bx + c$ ● Finding Intercepts

11.8 Problem Solving and Quadratic Functions
Maximum and Minimum Problems ● Fitting Quadratic Functions to Data

11.9 Polynomial Inequalities and Rational Inequalities
Quadratic and Other Polynomial Inequalities ● Rational Inequalities

TEACHING TIPS

Section 11.1

I remind students that the leading coefficient a is always positive, so they should watch the constant c to determine whether the discriminant will involve a subtraction or an addition.

Tim Chappell,
Penn Valley Community College

Section 11.3

I tell them and show them that the discriminant tells us that complex roots of quadratics appear in conjugate pairs and our factoring of solutions allows us to use the quadratic formula to factor quadratics.

Donald W. Solomon,
University of Wisconsin-Milwaukee

Section 11.4

Communicate the difference between exact and approximate solutions for quadratic equations. Help students identify when approximate solutions are appropriate in applications—which is most of the time.

Tim Chappell,
Penn Valley Community College

Section 11.5

Equations reducible to quadratic represent a great set of pattern exercises that also forces review of other concepts.

Donald W. Solomon,
University of Wisconsin-Milwaukee

Section 11.6

I like to have the students find all intercepts (exactly) for each parabola. I encourage—insist, in fact—that they use the principle of roots to find the x-intercepts when the format is already present, as in $f(x) = a(x-h)^2 + k$.

Donald W. Solomon,
University of Wisconsin-Milwaukee

◆ ◆ ◆

This is a great section for group work. Give each group a family of functions, such as $y = ax^2$ with different a-values, $y = (x-h)^2$ with different h-values, and $y = x^2 + k$ with different k-values. Have each group report back with the rule for their family of functions. Then have students guess the characteristics of functions you create with different a, h, and k values.

Tim Chappell,
Penn Valley Community College

12 Exponential Functions and Logarithmic Functions

SECTION TITLES AND TOPICS

TEACHING TIPS

Section 12.1

We discuss multi-step functions, such as assembly lines and crafts, as examples of composite functions.

Explain that the inverse of a relation can be found by switching the x and y values. Develop this through sets of points to graphs and then to algebra. Explain also that the inverse of a relation can be found by performing the opposite operations in opposite order. That is the only way to undo what the original function did. I use examples such as replacing a faulty part on a car.

Tim Chappell,
Penn Valley Community College

Section 12.2

Be sure to develop e through compounding. Use a spreadsheet or graphing calculator with tables. I use the financial features on the Texas Instruments graphing calculator for various auto loans, home loans, or investments. Students really get into it, and I assign a short paper on loans and investments.

Tim Chappell,
Penn Valley Community College

◆ ◆ ◆

Show students a table comparing linear growth to exponential growth over time. Have them think of examples of things that grow linearly and that grow exponentially.

Laurie Hurley,
Mt. Mansfield Academy

Section 12.3

I have students convert between addition and subtraction, multiplication and division, square roots and squares. Then I introduce logarithms and have them convert between logarithms and exponentials.

Tim Chappell,
Penn Valley Community College

◆ ◆ ◆

Your challenge here is to convince the student that logarithms are exponents.

Donald W. Solomon,
University of Wisconsin-Milwaukee

Section 12.4

I tell my students that a logarithmic expression is essentially an exponent. I then develop the rules for logarithms from the rules for exponents.

Tim Chappell,
Penn Valley Community College

◆ ◆ ◆

Students need to be reminded to consider their domain and to check their answers.

Laurie Hurley,
Mt. Mansfield Academy

Section 12.7

We do most of the problems in this section. I give the students the option of using the doubling/half-life shortcut or working it out from the growth/decay equation.

Tim Chappell,
Penn Valley Community College

13 Conic Sections

SECTION TITLES AND TOPICS

TEACHING TIPS

Section 13.1

I begin this section by having students compare the equations and graphs of parabolas, circles, ellipses and hyperbolas with each other. We look for similarities and differences.

Sharon Testone,
Onondaga Community College

Section 13.2

Compare and contrast ellipses centered at $(0,0)$ with those centered at (h,k). Students usually have trouble with those centered at (h,k).

A nice activity is to have the students make a scale drawing of the earth's elliptical orbit. They are always surprised that it appears to be very close to circular.

Sharon Testone,
Onondaga Community College

Section 13.3

For hyperbolas, emphasize that although asymptotes are not a part of the graph, they are very helpful when constructing the graph.

Sharon Testone,
Onondaga Community College

◆ ◆ ◆

For hyperbolas with their center at the origin, I have students first substitute $x = 0$ and then $y = 0$ to find any intercepts that exist so they can quickly determine if the hyperbola is vertical or horizontal.

Laurie Hurley,
Mt. Mansfield Academy

Section 13.4

I have students work in small groups on the synthesis questions in the Exercises section. Since these concepts are quite difficult for them, it is important to give the students class time to work on the concepts with their classmates.

Sharon Testone,
Onondaga Community College

14 Sequences, Series, and the Binomial Theorem

SECTION TITLES AND TOPICS

14.1 Sequences and Series
Sequences ● Finding the General Term ● Sums and Series ● Sigma Notation

14.2 Arithmetic Sequences and Series
Arithmetic Sequences ● Sum of the First n Terms of an Arithmetic Sequence ● Problem Solving

14.3 Geometric Sequences and Series
Geometric Sequences ● Sum of the First n Terms of a Geometric Sequence ● Infinite Geometric Series ● Problem Solving

14.4 The Binomial Theorem
Binomial Expansion Using Pascal's Triangle ● Binomial Expansion Using Factorial Notation

TEACHING TIPS

Section 14.1

Be sure that the students understand the difference between an infinite sequence and a finite sequence. It may be obvious to instructors, but I have found that it is not obvious to my students.

Many students will not have seen the sigma notation prior to this section; therefore, they will need a thorough explanation here.

Sharon Testone,
Onondaga Community College

Section 14.2

Emphasize that an arithmetic sequence has a common difference "d." Complete several examples with the students. Once again, the synthesis exercises in this section are great for group work.

Sharon Testone,
Onondaga Community College

◆ ◆ ◆

Make sure students are precise in their use of the terms sequence and series. In everyday speech, people often interchange them, which can lead to misunderstandings in this chapter.

Laurie Hurley,
Mt. Mansfield Academy

Section 14.3

I begin this section by reviewing that an arithmetic sequence has a common difference "d" and then I write several geometric sequences on the board and ask the students to find the difference. Eventually, someone sees that there is a common ratio "r" in this case instead of a common difference "d."

Sharon Testone,
Onondaga Community College

Section 14.4

When expanding a binomial like: $\left(2x+\frac{3}{x}\right)^6$, I always have the students let $a=2x$ and $b=\frac{3}{x}$. I then have them expand $(a+b)^6$ and finally substitute the $2x$ for "a," the $\frac{3}{x}$ for "b," and simplify. They seem to have an easier time with the more difficult problems if I approach them in this manner.

Sharon Testone,
Onondaga Community College

MINI LECTURES

Introduction to Algebra

Objectives:

- Evaluating Algebraic Expressions
- Translating to Algebraic Expressions
- Translating to Equations

Examples:

1. Evaluate the expressions when $a = 2$, $b = 3$, and $c = 5$.

   a) $2a$ b) $\frac{a+b}{c}$

   c) $\frac{5ab}{6}$ d) $\frac{c-1}{2}$

2. Translate each phrase to an algebraic expression, using x as the variable.

   a) 6 less than a number

   b) Fifty more than a number

   c) The product of 8 and the sum of a number and 5

   d) Thirteen less than three times a number

3. Translate each problem to an equation. Do not solve.

   a) When 18 is subtracted from a number, the result is 254. Find the number.

   b) Frank had \$100 before spending x dollars on a gift. After he purchased the gift, he had \$63 left. How much did he spend on the gift?

Teaching Notes:

- When a numerical coefficient is 1, the 1 is usually not written (e.g., $1x$ is usually written as x). Students often ignore a term when they do not see a coefficient.
- Students often have difficulty translating phrases with "less than."
- Remind students that expressions evaluate to a number.

Answers: 1a) 4, b) 1, c) 5, d) 2; 2a) $x-6$, b) $x+50$, c) $8(x+5)$, d) $3x-13$; 3a) $x-18=254$, b) $\$100-x=\$63$

The Commutative, Associative, and Distributive Laws

Objectives:

- The Commutative Laws
- The Associative Laws
- The Distributive Law
- The Distributive Law and Factoring

Examples:

1. Use the commutative laws to write an equivalent expression.

   a) $2+x$ b) $3y+z$ c) ab d) $5(y+x)$

2. Use the associative laws to write an equivalent expression.

   a) $(2+x)+m$ b) $a+(5+y)$ c) $(5a)n$ d) $4(2p)$

3. List the terms in $2x+4y+6$.

4. List the factors in each expression.

   a) $11x$ b) $5(a+b)$

5. Multiply.

   a) $3(k+6)$ b) $5(h+2)$ c) $2(5a+2b+c)$

6. Use the distributive law to factor.

   a) $3y+6x$ b) $12n+3$ c) $4x+8y+12z$

7. Name the law (commutative, associative, or distributive) illustrated by each statement.

   a) $3+7=7+3$ b) $3+(6+2)=(3+6)+2$ c) $5(c+d)=5c+5d$

   d) $(5+7)+10=10+(5+7)$ e) $7\left(\frac{3}{7}\right)=\left(\frac{3}{7}\right)7$

Teaching Notes:

- Remind students that the commutative laws deal with the order of addition or multiplication, whereas the associative laws deal with grouping.
- The distributive law involves both multiplication and addition.
- Have students provide examples to show whether or not the commutative laws and associative laws hold for subtraction and division.
- Students have difficulty distinguishing between *terms* and *factors* – terms are separated by addition (keeping in mind that all subtractions can be written as addition of the opposite); factors are multiplied.

Answers: *1a)* $x+2$, *b)* $z+3y$ or $y\cdot 3+z$, *c) ba, d)* $5(x+y)$ or $(y+x)5$; *2a)* $2+(x+m)$, *b)* $(a+5)+y$, *c)* $5(an)$, *d)* $(4\cdot 2)p$; *3a) 2x, 4y, 6; 4a) 11, x, b) 5,* $(a+b)$; *5a)* $3k+18$, *b)* $5h+10$, *c)* $10a+4b+2c$; *6a)* $3(y+2x)$, *b)* $3(4n+1)$, *c)* $4(x+2y+3z)$; *7a) Commutative, b) associative, c) distributive, d) commutative, e) commutative*

Fraction Notation

Objectives:

- Factors and Prime Factorization
- Multiplication, Division, and Simplification of Fractions
- Addition and Subtraction of Fractions

Examples:

1. List the prime numbers, the composite numbers, and the numbers that are neither.

   a) 8, 1, 13, 32, 23, 14, 100 b) 4, 5, 25, 19, 30, 33, 21, 51

2. List all the factors of each number.

   a) 12 b) 63 c) 49 d) 23

3. Find the prime factorization. If a number is prime, state this.

   a) 6 b) 18 c) 24 d) 75

   e) 140 f) 400 g) 81 h) 210

4. Simplify.

   a) $\frac{12}{36}$ b) $\frac{4}{24}$ c) $\frac{20}{35}$ d) $\frac{21}{49}$

   e) $\frac{30}{42}$ f) $\frac{50}{75}$ g) $\frac{18}{90}$ h) $\frac{96}{108}$

5. Perform the indicated operation and, if possible, simplify.

   a) $\frac{2}{3}\cdot\frac{4}{5}$ b) $\frac{3}{7}\div\frac{2}{3}$ c) $\frac{3}{14}+\frac{4}{14}$ d) $\frac{5}{6}-\frac{3}{4}$

   e) $\frac{5}{6}+\frac{7}{8}$ f) $\frac{12}{x}-\frac{8}{x}$ g) $20\div\frac{2}{3}$ h) $\dfrac{\frac{1}{2}}{\frac{3}{4}}$

Teaching Notes:

- Recall that factors are multiplied to give a product; whereas, common factors are those found in both the numerator and denominator.
- Remind students that 1 is neither prime nor composite.
- Some students forget whether they should take the reciprocal of the divisor or the reciprocal of the dividend.
- Some students forget to multiply the numerator when creating equivalent fractions.
- Some students add or subtract denominators along with numerators instead of finding a common denominator.

Answers: 1a) Prime: 13 and 23, composite: 8, 32, 14, and 100, neither: 1, b) prime: 5 and 19, composite: 4, 25, 30, 33, 21, and 51; 2a) 1, 2, 3, 4, 6, 12, b) 1, 3, 7, 9, 21, 63, c) 1, 7, 49, d) 1, 23; 3a) $2\cdot3$, *b)* $2\cdot3\cdot3$, *c)* $2\cdot2\cdot2\cdot3$, *d)* $3\cdot5\cdot5$, *e)* $2\cdot2\cdot5\cdot7$, *f)* $2\cdot2\cdot2\cdot2\cdot5\cdot5$, *g)* $3\cdot3\cdot3\cdot3$, *h)* $2\cdot3\cdot5\cdot7$; *4a)* $\frac{1}{3}$, *b)* $\frac{1}{6}$, *c)* $\frac{4}{7}$, *d)* $\frac{3}{7}$, *e)* $\frac{5}{7}$, *f)* $\frac{2}{3}$, *g)* $\frac{1}{5}$, *h)* $\frac{8}{9}$; *5a)* $\frac{8}{15}$, *b)* $\frac{9}{14}$, *c)* $\frac{1}{2}$, *d)* $\frac{1}{12}$, *e)* $\frac{41}{24}$, *f)* $\frac{4}{x}$, *g) 30, h)* $\frac{2}{3}$

Positive and Negative Real Numbers

Objectives:

- Whole Numbers and Integers
- The Rational Numbers
- Real Numbers and Order

Examples:

1. State which real number corresponds to the situation.
   a) Brent owes his parents \$600. b) Jayne has \$125 in her checking account.

2. Graph each rational number on the number line: $-2,\ 5,\ -4\frac{1}{3},\ 1.5$.

3. Write decimal notation.
   a) $-\frac{3}{4}$ b) $\frac{1}{6}$ c) $-\frac{3}{25}$

4. Graph each irrational number on the number line: $\sqrt{3}, -\sqrt{10}$.

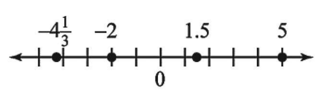
5. Use either < or > for $\square$ to write a true sentence.
   a) $-3 \ \square\ 8$ b) $-\frac{4}{5}\ \square\ -\frac{1}{3}$ c) $10\ \square\ -12$

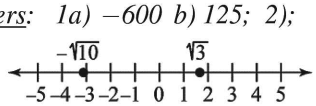
6. Write an inequality with the same meaning.
   a) $-6 \geq -10$ b) $-3 < 5$ c) $4 > 0$

7. Find the absolute value.
   a) $|-3|$ b) $|-6|$ c) $|4|$

Teaching Notes:

- Remind students that integers are rational numbers; any integer can be written as the ratio of itself and 1.
- Decimal numbers that terminate or repeat in a fixed block are both examples of rational numbers – ask students to give examples of both.
- The decimal form of an irrational number neither terminates nor repeats.
- Some students have never seen absolute value before and will need examples.

Answers: *1a) -600 b) 125; 2);* 

3a) -0.75, b) $0.1\overline{6}$, c) -0.12;
4a) 

; 5a) <, b) <, c) >; 6a) $-10 \leq -6$, b) $5 > -3$, c) $0 < 4$; 7a) 3, b) 6, c) 4